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A METHOD FOR DETERMINING RELIABILITY OF A SELECTED STRUCTURAL COMPONENT OF AN AIRCRAFT FROM THE POINT OF VIEW OF FATIGUE PROCESSES

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ABSTRACT

The crack growth dynamics can be described with difference equations which after rearrangement can give a partial differential equation of the Fokker-Planck type (Risken 1984). What results from solving this equation is a probability density function of the crack length increment against the aircraft flying time. The in this way determined reliability enables durability of the aircraft component to be found, for which reliability will not exceed the required value because of the flight safety. From the physical point of view, the dynamics of a fatigue process can be described with the Paris equation. The paper has been concluded with some remarks on practical applications of the suggested method to assess reliability and durability of structural components of an aircraft.

Keywords: fatigue, aircraft, reliability, Fokker-Planck equation, Paris equation.

INTRODUCTION

The literature on issues of fatigue in structural components of aircraft, considered from the point of view of reliability and durability, is extensive and touches many and various questions (Baranowski, 2015; Jacyna-Gołda, 2017; Retsel, 2015; Valis, 2012; Woch, 2017; Werbińska-Wojciechowska 2013,2015; Tchórzewska-Cieślak, 2016; Żurek 2009). The in the paper presented method has been based on:

- the Paris equation that defines the rate of fatigue crack growth in a structural component,
- an actual, service-induced spectrum of loading structural components, obtained in the course of tests,
- difference equations used to describe - with stochastic approach applied - the dynamics of fatigue crack growth in structural components,
- the specificity of aircraft service.

What has been presented in this paper is a method to determine fatigue life of a structural component of an aircraft, with the following assumptions made (Tomaszek, 2013):

- there is an initial crack l_0 in the structural component,
- the component's health/maintenance status has been determined with one parameter only, i.e. the length of a crack therein. The actual value of the parameter has been denoted with l;

- any change in the crack length may only occur in the course of the system/device being operated;
- in the case given consideration the Paris formula takes the following form (Tomaszek, 2016; Zieja, 2016; Żurek, 2014):

$$\frac{dl}{dN_z} = CM_k^m \left(\sigma_{\max}\right)^m \pi^{\frac{m}{2}} l^{\frac{m}{2}}, \tag{1}$$

where:

C, m - material constants,
N_z - a variable that denotes the number of cycles in the assumed load spectrum,
M_k - coefficient of the finiteness of the component's dimensions at the crack location,
σ_{max} - maximum load defined with equation (3);

- the structure's component affecting load in the form of the assumed loading spectrum is a destructive factor. Let us assume that the accepted load spectrum allows for the determination of:
  - the total number of load cycles N_c in the course of one flight assumed a standard cycle,
  - maximum loads within thresholds in the assumed spectrum amount to $\sigma_1^{\max}$, $\sigma_2^{\max}$, …, $\sigma_L^{\max}$ (the assumed number of thresholds in the spectrum is L),
  - the number of repetitions of specific threshold values of the loading during one flight (standard load) n_i, where:

$$N_c = \sum_{i=1}^{L} n_i \,; \tag{2}$$

- maximum values of loads within the assumed thresholds are found in the following way:

$$\sigma_{\max}^i = \sigma_{śr}^i + \sigma_a^i \tag{3}$$

where:

$\sigma_{\max}^i$ - maximum value of the cyclic load within the *i*-th threshold,
$\sigma_{śr}^i$ - average value of the cyclic load within the *i*-th threshold,

$$\sigma_{śr}^i = \frac{\sigma_{\max}^i + \sigma_{\min}^i}{2} \tag{4}$$

where:

$\sigma_{\min}^i$ - minimum value of the cyclic load within the *i*-th threshold,
σ_a^i - the amplitude of the cyclic load within the *i*-th threshold.

- the following frequencies of the occurrence of loads correspond to values thereof within the thresholds $\sigma_{\max}^1$, $\sigma_{\max}^2$, …, $\sigma_{\max}^L$:

$$\frac{n_1}{N_c} = P_1,\ \frac{n_2}{N_c} = P_2,\ \ldots,\ \frac{n_L}{N_c} = P_L. \tag{5}$$

where:

$$P_1 + P_2 + \ldots. + P_L = 1 \tag{6}$$

HOW TO DETERMINE THE PROBABILITY DENSITY FUNCTION OF A CRACK LENGTH IN SOME SELECTED COMPONENT OF AIRCRAFT'S STRUCTURE

For assumptions accepted in Section 1 an attempt will be made to determine the form of the probability crack-length density function dependent on the time of the operational use (flying time) of an aircraft. Relationship (1) may be expressed against the flying time of the aircraft. Therefore, it is assumed that:

$$N_z = \lambda t, \tag{7}$$

where:

λ - the occurrence rate of load cycles upon the component in the assumed spectrum,
t - flying time of the aircraft.

In the case under consideration

$$\lambda = \frac{1}{\Delta t}, \tag{8}$$

where:

Δt - the average duration of the fatigue-load cycle in the assumed spectrum.

The formula to determine Δt can be written down in the form of the following equation:

$$\Delta t = \frac{T}{N_C}, \tag{9}$$

where:

T - time of flight for a standard cycle.

The relationship (1) against the flying time takes the following form:

$$\frac{dl}{dt} = \lambda C M_k^m (\sigma_{\max})^m \pi^{\frac{m}{2}} l^{\frac{m}{2}}. \tag{10}$$

The form of the solution to equation (10) depends on the value of the index exponent *m*. In the case under consideration $m = 2$. Hence, equation (10) takes the following form:

$$\frac{dl}{dt} = \lambda C M_k^2 (\sigma_{\max})^2 \pi l. \tag{11}$$

Thus, the crack increment for the flying time of an aircraft Δt can be described with the following equation:

$$\Delta l = \lambda C M_k^2 (\sigma_{\max})^2 \pi l \Delta t. \tag{12}$$

With the hitherto findings applied, one can set about determining a relationship for the probability density function of a crack length against the flying time of the aircraft.

Let $U_{l,t}$ denote the probability that at time instance *t* (for the flying time of the aircraft equal to *t*) the crack will gain length *l*. For the assumptions made, the dynamics of the crack length growth will be described with the following difference equation (Franck, 2005; Risken, 1984):

$$U_{l,t+\Delta t} = \sum_{i=1}^{L} P_i U_{l-\Delta l_i,t} \ , \tag{13}$$

where:

P_i - probability that the load $\sigma_{\max}^{i}$ occurs, with $P_1 + P_2 + P_3 + ... + P_L = 1$;

Δl_i - crack increment in time Δt for the load equal to $\sigma_i^{\max}$, where $i = 1,2,3,.....,L$:

$$\Delta l_i = CM_k^2 \left(\sigma_{\max}^{i}\right)^2 \pi l \underbrace{\lambda \Delta t}_{1} . \tag{14}$$

Equation (13) in function notation takes the following form:

$$U_{l,t+\Delta t} = \sum_{i=1}^{L} P_i U\left(l - \Delta l_i, t\right) \tag{15}$$

Equation (15) is rearranged now in a partial differential equation. The following approximations have been made (Narayan, 2012; Pham 2006):

$$U(l,t+\Delta t) \cong U(l,t) + \frac{\partial U(l,t)}{\partial t} \Delta t \, , \tag{16}$$

$$U(l-\Delta l_i,t) \cong U(l,t) - \frac{\partial U(l,t)}{\partial l} \Delta l_i + \frac{1}{2} \frac{\partial^2 U(l,t)}{\partial l^2} (\Delta l_i)^2 . \tag{17}$$

Having substituted (16) and (17) into (15), the following equation results:

$$U(l,t) + \frac{\partial U(l,t)}{\partial t} \Delta t = \sum_{i=1}^{L} P_i \left\{ U(l,t) - \frac{\partial U(l,t)}{\partial l} \Delta l_i + \frac{1}{2} \frac{\partial^2 U(l,t)}{\partial l^2} (\Delta l_i)^2 \right\}, \tag{18}$$

$$\frac{\partial U(l,t)}{\partial t} \Delta t = -\frac{\partial U(l,t)}{\partial l} \sum_{i=1}^{L} P_i \Delta l_i + \frac{1}{2} \frac{\partial^2 U(l,t)}{\partial l^2} \sum_{i=1}^{L} P_i (\Delta l_i)^2 .$$

Hence,

$$\frac{\partial U(l,t)}{\partial t} = -\underbrace{\frac{1}{\Delta t} \sum_{i=1}^{L} P_i \Delta l_i}_{\alpha(t)} \frac{\partial U(l,t)}{\partial l} + \frac{1}{2} \underbrace{\frac{1}{\Delta t} \sum_{i=1}^{L} P_i (\Delta l_i)^2}_{\beta(t)} \frac{\partial^2 U(l,t)}{\partial l^2} \, , \tag{19}$$

where:

α(t) - mean increment in crack length within the component of the aircraft's structure per time unit,

β(t) - mean square of the increment in crack length within the component of the aircraft's structure per time unit.

The relationship that determines the *α(t)* coefficient from equation (19) can be rearranged in the following way:

$$\alpha(t) = \frac{1}{\Delta t} \sum_{i=1}^{L} P_i \Delta l_i = \frac{1}{\Delta t} \sum_{i=1}^{L} CM_k^2 P_i \left(\sigma_{\max}^{i}\right)^2 \pi l \lambda \Delta t \, ,$$

$$\alpha(t) = \lambda CM_k^2 \pi l [\underbrace{P_1 \left(\sigma_{\max}^{1}\right)^2 + P_2 \left(\sigma_{\max}^{2}\right)^2 + ... + P_L \left(\sigma_{\max}^{L}\right)^2}_{\mathrm{E}\left[\sigma_{\max}^2\right]}],$$

$$\alpha(t) = \lambda CM_k^2 \pi \mathrm{E}[\sigma_{\max}^2] l \, , \tag{20}$$

where:

$\mathrm{E}[\sigma_{\max}^2]$ - the second moment of loading the structure's component.

What is to be found then is a relationship for the crack length *l*, approached in a deterministic way, as based on the following dependence (21):

$$\frac{dl}{dt} = \lambda C M_k^2 \pi \mathrm{E}[\sigma_{\max}^2] l \,. \tag{21}$$

Hence,

$$\int_{l_0}^{l} \frac{dl}{l} = \int_0^t \lambda C M_k^2 \pi \mathrm{E}[\sigma_{\max}^2] dt \,, \tag{22}$$

that is:

$$l = l_o e^{\lambda C M_k^2 \mathrm{E}[\sigma_{\max}^2] \pi t} \,. \tag{23}$$

After having introduced the following notations:

$$C M_k^2 \pi = C_2 \,, \tag{24}$$

$$C_2 \mathrm{E}[\sigma_{\max}^2] = \overline{C_2} \,, \tag{25}$$

the relationship for the α(t) coefficient takes the following form:

$$\alpha(t) = \lambda \overline{C_2} l_o e^{\lambda \overline{C_2} t} \,. \tag{26}$$

In a similar way, one can find the relationship for the *β(t)* coefficient.

After some rearrangements equation (19) takes the following form:

$$\frac{\partial u(l,t)}{\partial t} = -\alpha(t) \frac{\partial u(l,t)}{\partial l} + \frac{1}{2} \beta(t) \frac{\partial^2 u(l,t)}{\partial l^2} \,. \tag{27}$$

A particular solution of equation (27) is the crack-length density function of the following form:

$$u(l,t) = \frac{1}{\sqrt{2\pi A(t)}} e^{-\frac{(l-B(t))^2}{2A(t)}} \,, \tag{28}$$

where:

B(t) - an average crack length for the aircraft's flying time *t*,
A(t) - crack-length variance for the aircraft's flying time *t*.

Coefficients *B(t)* and *A(t)* for the material constant m=2 are solutions to the integrals:

$$B(t) = \int_0^t \alpha(t) dt = l_0 \left(e^{\lambda \overline{C_2} t} - 1 \right), \tag{29}$$

$$A(t) = \int_0^t \beta(t) dt = \frac{1}{2} l_0^2 \overline{C_2} \omega \left(e^{2\lambda \overline{C_2} t} - 1 \right), \tag{30}$$

where:

$$\omega = \frac{\mathrm{E}[\sigma_{\max}^4]}{(\mathrm{E}[\sigma_{\max}^2])^2} \,.$$

HOW TO DETERMINE RELIABILITY OF A COMPONENT OF THE AIRCRAFT'S STRUCTURE USING THE DENSITY FUNCTION OF A CRACK LENGTH

Reliability of a component of the aircraft's structure can be determined using the density function of a crack length described with equation (28). However, what is to be found first is the value of a permissible crack length for the assumed flight-safety level, with account taken of the aircraft structure's component given consideration. To do this, the stress intensity factor in the following form is used (Knopik, 2016, 2018; Piegdoń 2018):

$$K = M_k \sigma \sqrt{\pi l}\ , \tag{31}$$

where:

M_k - a correcting factor that comprises geometric characteristics, characteristics of the finiteness of the component's dimensions, and of the crack shape.

In the case of critical crack length l_{kr} and critical stress $\sigma_{kr,}$ the factor determined with the dependence (31) becomes a critical quantity K_c and is then termed the material's resistance to cracking:

$$K_c = M_k \sigma_{kr} \sqrt{\pi l_{kr}} \tag{32}$$

With the dependence (32) applied and safety factor introduced, one can find the permissible crack length:

$$l_d = \frac{K_c^2}{k M_k^2 \sigma_{kr}^2 \pi}\ , \tag{33}$$

where:

k - safety factor.

Making use of findings of hitherto considerations, the reliability of the aircraft structure's component can be written down with equation (34):

$$R(t) \cong \int_{-\infty}^{l_d} U(l,t)\,dl\ , \tag{34}$$

where:

$$u(l,t) = \frac{1}{\sqrt{2\pi A(t)}} e^{-\frac{(l-B(t))^2}{2A(t)}}\ .$$

With account taken of equations (29) i (30), the following form of the integrand is arrived at for the dependence (34):

$$U(l,t) = \frac{1}{\sqrt{2\pi(\frac{1}{2} l_0^2 \overline{C_2} \omega (e^{2\lambda \overline{C_2} t} - 1))}} e^{-\frac{(l - l_0 (e^{\lambda \overline{C_2} t} - 1))^2}{l_0^2 \overline{C_2} \omega (e^{2\lambda \overline{C_2} t} - 1)}}\ . \tag{35}$$

What results from the standardisation of the random variable l is a standardised random variable z of mean value equal to zero and variance equal to unity:

$$z = \frac{l - B(t)}{\sqrt{A(t)}}. \qquad (36)$$

After standardisation of this random variable equation (34) takes the following form:

$$R(t) \cong \int_{-\infty}^{\frac{l_d - B(t)}{\sqrt{A(t)}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \qquad (37)$$

Results of calculations based on the dependence (37) against the aircraft flying time can be presented in the form of a table (Table 1).

Table 1 - Results of reliability calculations

AIRCRAFT's FLYING TIME	t_1	t_2	…	t_i	…	t_n	**NOTES**
$B(t)$	$B(t_1)$	$B(t_2)$	…	$B(t_i)$	…	$B(t_n)$	
$\sqrt{A(t)}$	$\sqrt{A(t_1)}$	$\sqrt{A(t_2)}$	…	$\sqrt{A(t_i)}$	…	$\sqrt{A(t_n)}$	
$\frac{l_d - B(t)}{\sqrt{A(t)}}$	$\frac{l_d - B(t_1)}{\sqrt{A(t_1)}}$	$\frac{l_d - B(t_2)}{\sqrt{A(t_2)}}$	…	$\frac{l_d - B(t_i)}{\sqrt{A(t_i)}}$	…	$\frac{l_d - B(t_n)}{\sqrt{A(t_n)}}$	
$R^*(t)$	$R^*(t_1)$	$R^*(t_2)$	…	$R^*(t_i)$	…	$R^*(t_n)$	

On the basis of the results of calculations included in Table 1, a reliability curve can be plotted (Figure 1). While calculating the reliability on the basis of the dependence (37) for the aircraft's flying time under consideration we use the table of standard normal distribution.

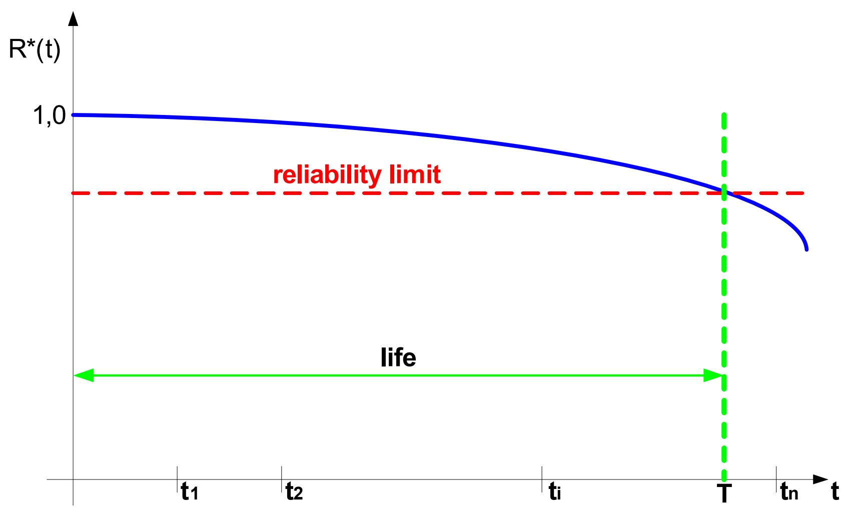

Fig. 1 - The reliability function

RESULTS AND CONCLUSIONS

To illustrate the developed method, results of calculations of the mechanical component's reliability have been show in Table 2 and Figure 2. The mechanical component in question comes from the spar-fixing mechanism of a wing from an aircraft of some selected type.

Table 2 - Results of calculations of mechanical component's reliability

AIRCRAFT's FLYING TIME (h)	1000	1400	1700	2000	2300	2600	2900
$B(t)$	1.89	2.75	3.42	4.12	4.81	5.51	6.23
$\sqrt{A(t)}$	1.04	1.29	1.45	1.61	1.75	1.89	2.02
$\frac{l_d - B(t)}{\sqrt{A(t)}}$	4.90	3.30	2.46	1.80	1.25	0.79	0.38
$R^*(t)$	1.0	1.0	0.993	0.964	0.895	0.784	0.648

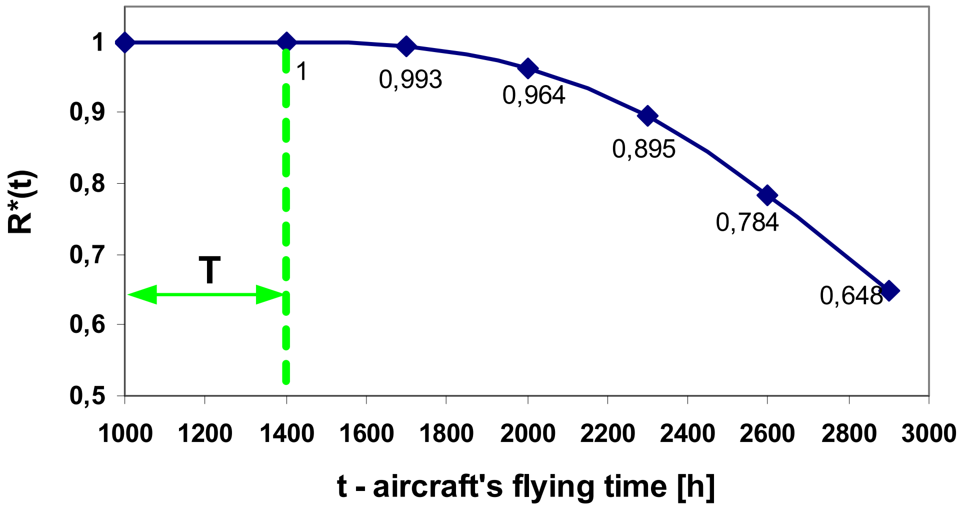

Fig. 2 - The reliability function for a mechanical component

Having assumed that, on the account of the component under consideration, the aircraft's reliability cannot be lower than R*=0.999, the component's life will be hours of flying time.

CONCLUSIONS

The present paper outlines a method to find fatigue reliability of a structural component against the aircraft's flying time. Practical use of the presented method needs what follows:

- data on the spectrum of load onto the aircraft structure's component,
- knowledge of material data referring to the structural component under consideration,
- data on structural component's dimensions and crack location,
- data on structural component's fatigue strength,
- knowledge of assumptions on structure safety requirements.

Constant values used in the presented method may be found using the collected material data. Some of them can also be found using data on crack propagation against the aircraft's flying time, ones collected in the course of aircraft service. Methods helpful in determining dependences for some constants include: a method of moments or a method that makes use of the likelihood function.

Issues of the effect of the sequence of occurrence of load cycles of various values on the crack increments in the component in question have been neglected in the above presented technique of finding the structural component's reliability. The suggested method will be improved in further works.

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