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STRENGTH AND LIFE ANALYSIS IN PLASTIC STRAIN RANGE. NEUBER VERSUS STRAIN ENERGY CONSERVATION PRINCIPLE

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ABSTRACT

The Science of Construction had a remarkable improvement with the mathematical theory of elasticity. A safety criterion was defined, in order to guarantee the integrity of the structure under the operating load: "Stress at each point of the structure to be <= of yield stress(residual strain 0.002) ". This type of analysis can be carried out simply assuming the linear correlation between stress and strain (Hooke´s law).

The next step was to calculate the structure collapse mechanism and load (=max load supported by structure up to catastrophic collapse). Analysis becomes more complex since it requires the knowledge of the plastic flow segment of the material stress-strain curves up to the strain failure. Collapse analysis must comply with the following requirements: applied and reaction load equilibrium in each point/subcomponent of the structure, compatibility of element deformations, and in addition plastic strain energy balance.

A tragic event occurred at Versailles on May 8, 1842. Two locomotives and seventeen cars were involved due to the rupture of an axle of the first locomotive. Almost one hundred people died. The enquiry draws the conclusion that failure was due to "fatigue", for the first time. Fatigue failure due to load cycling, became a major issue for safety and for structure life estimation

Keywords: energy, fatigue, Neuber, plasticity, static.

INTRODUCTION

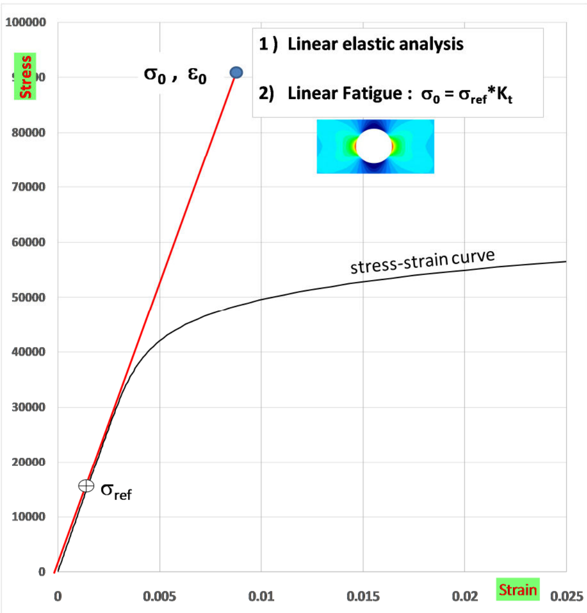
Plasticity has a robust influence on life and crack onset. Neuber hyperbola [1], is currently used as a "qualitative" approach for fatigue and ultimate load static analysis. When the calculated stress *via Hooke´s law,* is violating the material stress-strain curve, a local relaxation occurs relocating the unrealistic linear elastic stress-strain on the material plastic flow range.

The methodology presented on this paper, assumes that "local relaxation" is driven by the Energy Conservation Principle: plastic energy = linear elastic energy. The rationale behind this approach is that, for peaky stress gradient due to notches, the relaxation remains confined on the notches itself, without modifying the overall structure equilibrium.

The validation of the Energy Conservation approach, is done by comparing the analytical results with FEM non linear analysis, Neuber hyperbola and strain gage from test data.

PAPER PURPOSE

This paper compares the accuracy of Neuber Strain Energy methodology on calculation of plastic stress, from “peaky” stress gradient due to a linear analysis. Both methods are then qualified against notched test specimen data, equipped with strain gages [2].

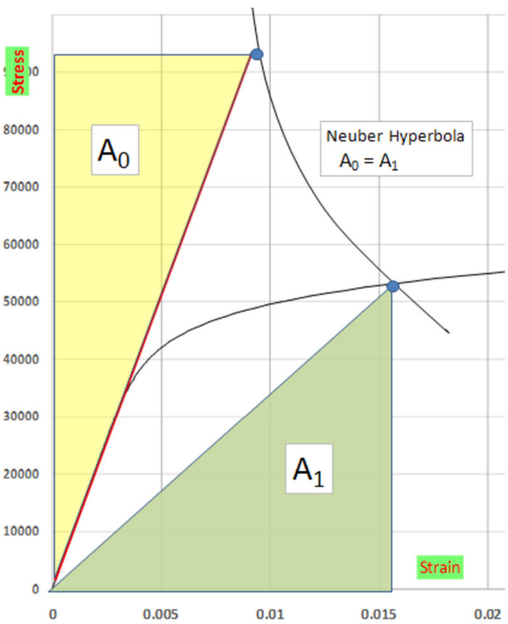


“Peaky” stress relaxation applies to following structural conditions:

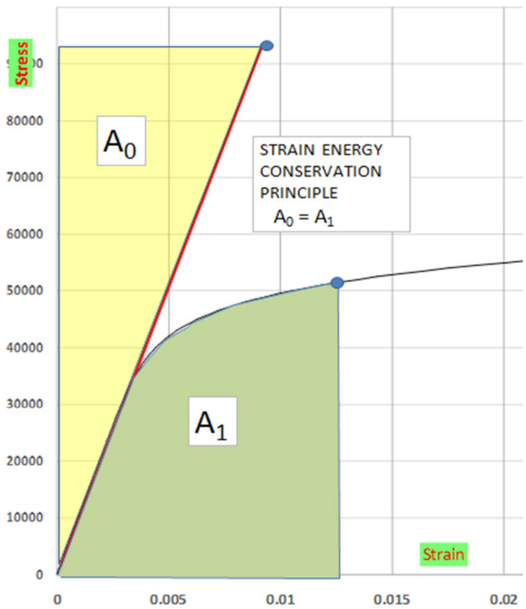
1. Structure static analysis, as long as the stress relaxation doesn’t modify local and/or overall force balance (= Free Body Diagram). (Methods not viable for structure collapse analysis).
2. Fatigue stress as product of stress concentration factor Kt and reference stress. The Kt [4] is defined assuming a local linear stress field based on notched geometry and material infinitely elastic.

Neuber Hyperbola criterium

Strain Energy criterium



$$\sigma_0 * \varepsilon_0 = \sigma_1 * \varepsilon_1 = \text{Constant}$$

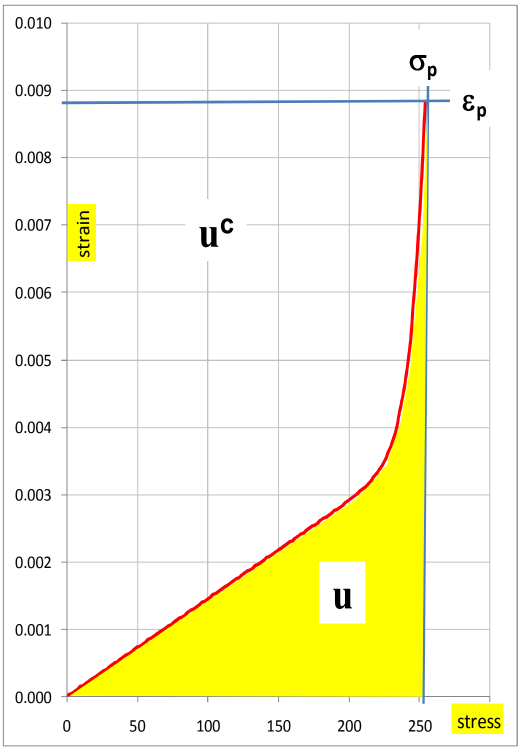


$$A_0 = \frac{1}{2}\sigma_0 * \varepsilon_0 = \int_0^{\bar{\varepsilon}} \sigma(\varepsilon) * d\varepsilon$$

Fig. 1 - Neuber Hyperbola and Strain Energy approach

STRAIN ENERGY FORMULATION

Strain energy formulation is based on complementary energy as shown on figure below:



$$U_{tot} = \sigma_p * \varepsilon_p$$

$$U = \int_0^{\sigma_p} \varepsilon(\sigma)\, d\sigma$$

$$U^c = \int_0^{\sigma_p} \sigma(\varepsilon)\, d\varepsilon = U_{tot} - U$$

Fig. 2 - Complementary Energy Calculation

Ramberg-Osgood Ref. 0 $$\varepsilon_p = \frac{\sigma_p}{E} + 0.002 * \left(\frac{\sigma_p}{\sigma_y}\right)^n$$

$$U_{tot} = \sigma_p * \varepsilon_p = \sigma_p * \left[\frac{\sigma_p}{E} + 0.002 * \left(\frac{\sigma_p}{\sigma_y}\right)^n\right] = \frac{\sigma_p^2}{E} + \frac{0.002}{\sigma_Y^n} * \sigma_p^{n+1}$$

$$U = \int_0^{\sigma_p} \varepsilon(\sigma)\, d\sigma = \frac{\sigma_p^2}{2E} + \frac{0.002}{(n+1)\sigma_Y^n} * \sigma_p^{n+1}$$

$$U^C(\sigma) = U_{tot} - U = \frac{\sigma_p^2}{2E} + \frac{0.002 * n}{(n+1)\sigma_Y^n} * \sigma_p^{n+1}$$

ALGORITHM

The stress and strain σ_p, ε_p (in plastic flow range) is calculated by assuming the equivalence of linear elastic strain energy (U_{LE}) and the complementary energy (U^c):

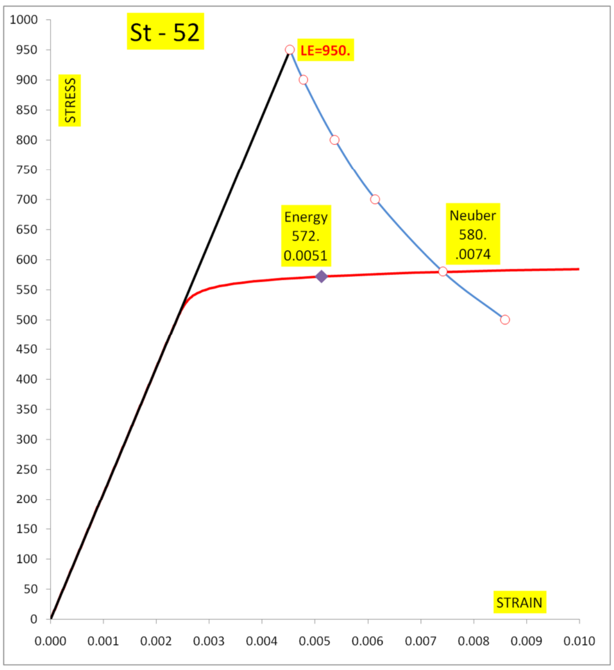
$$U_{LE} = \frac{1}{2}\sigma_{LE} * \varepsilon_{LE} = \frac{\sigma_{LE}^2}{2E} \qquad , \qquad U^c = \frac{\sigma_P^2}{2E} + \frac{0.002 * n}{(n+1) * \sigma_y^n} * \sigma_p^{n+1}$$

System equation

$$1) \quad \frac{\sigma_P^2}{2E} + \frac{0.002 * n}{(n+1) * \sigma_y^n} * \sigma_p^{n+1} = \frac{\sigma_{LE}^2}{2E}$$

$$2) \quad \varepsilon_p = \frac{\sigma_p}{E} + 0.002 * \left(\frac{\sigma_p}{\sigma_Y}\right)^n$$

HYPERBOLA - ENERGY : MAJOR DIFFERENCES



The following remarks apply:

1. Stress variation (572., 580.) in plastic range between Neuber and Energy methods is negligible, if compared with the strain variation (0.0051, 0.0074)
2. Neuber method is estimating higher strain values , extremely conservative in term of margin of safety, neglecting a consistent strain plastic energy reservoir. Higher estimation of plastic strain is also questionable for F&DT analysis.

Fig. 3 - Neuber hyperbola and Energy - major differences.

NEUBER and ENERGY VERSUS TEST DATA: COMPARISON

Energy Conservation Principle results and Neuber approach, are compared with the notched test results taken from Ref. [2].

Test specimen data:

1. Material: St-52 and AlMgSi 1
2. Cross section : 8 x 40 mm
3. Hole in the center as a notch: 10.0 mm ($K_t = 2.42$)
4. Inside the hole two strain gages were applied as shown on Figure 4 below

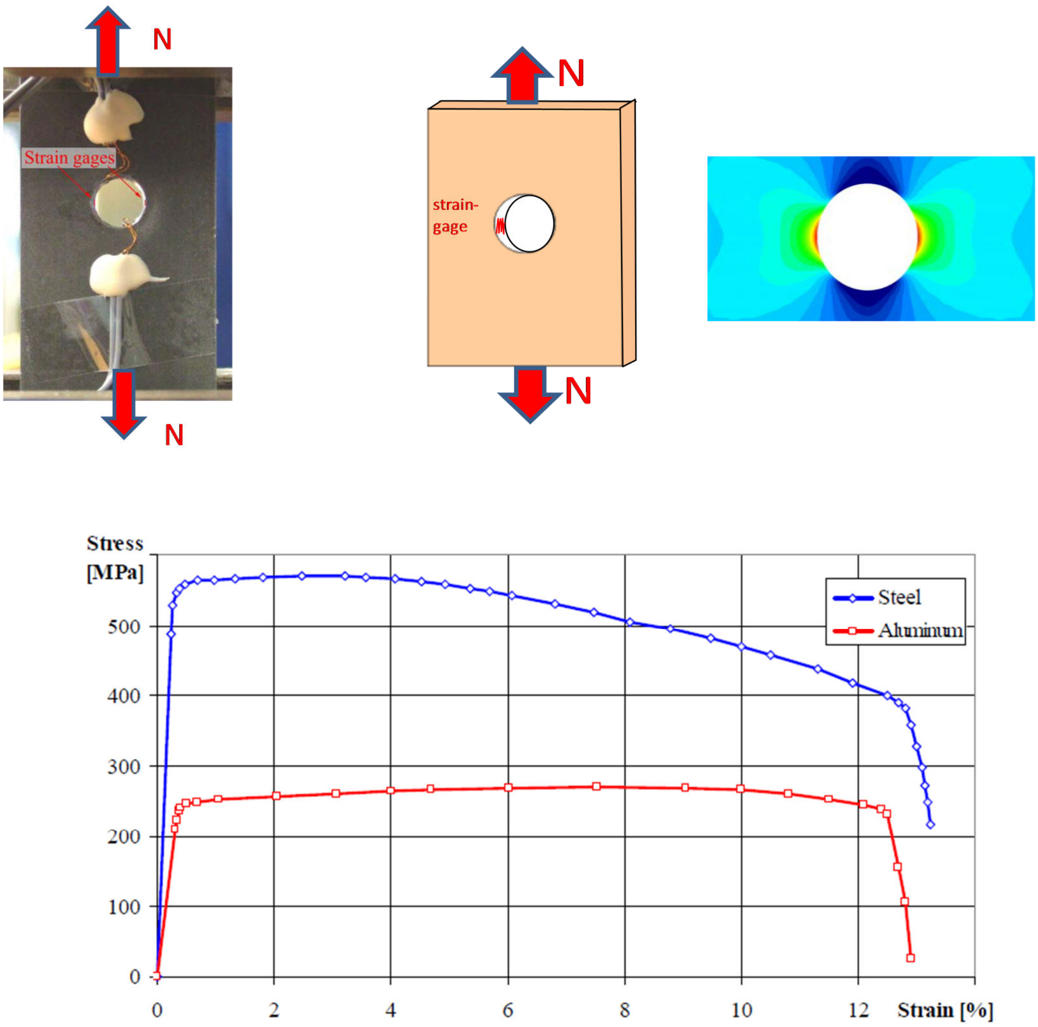
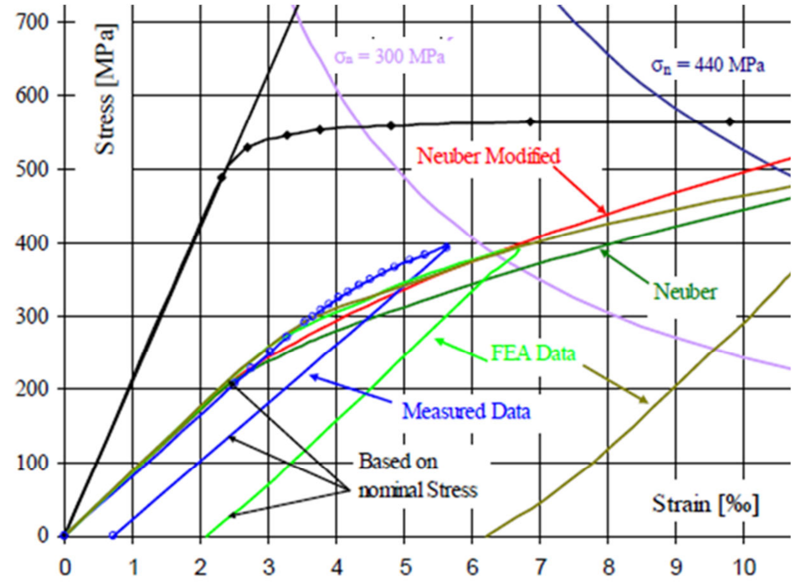
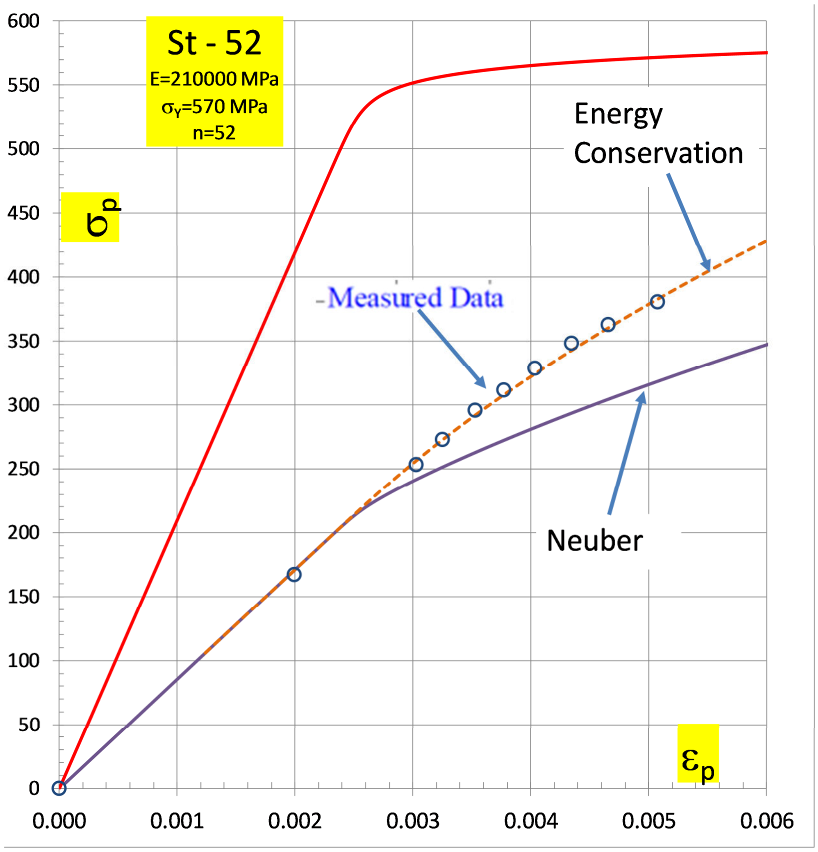


Fig. 4 - Test setting and Stress-Strain material curve

St-52 Test Data versus Analysis: Comparison



Ref. [2]



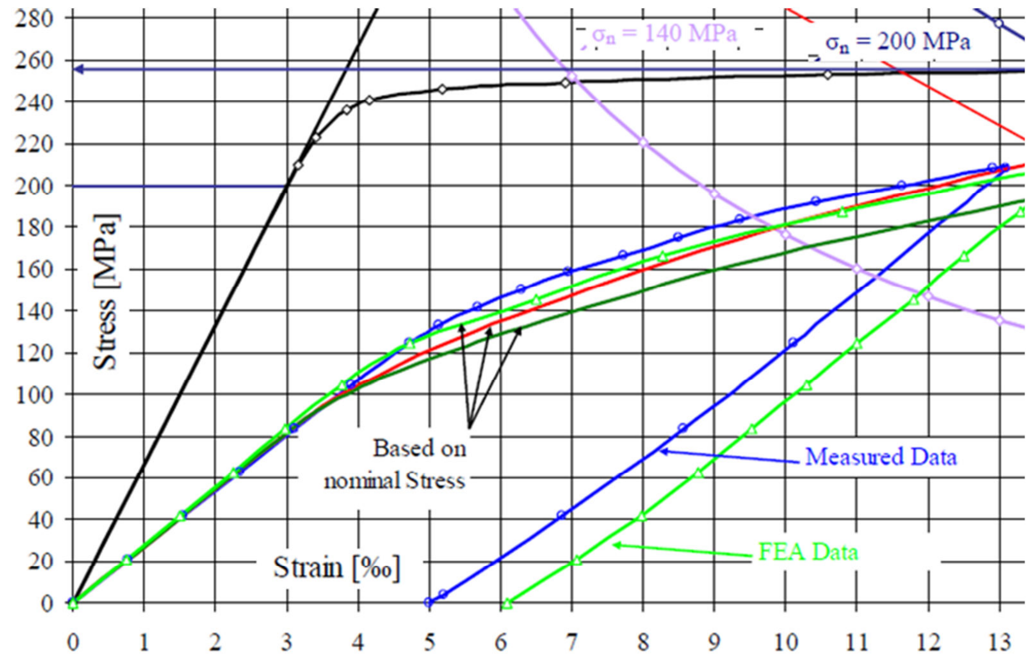
Calculation results (ALGORITHM)

Measured Test Data	
strain	stress [Mpa]
0	0
0.001992	166.7
0.003024	253.4
0.003252	272.8
0.003528	295.7
0.003768	311.6
0.004032	328.8
0.004344	348.2
0.004656	363.0
0.005076	380.1

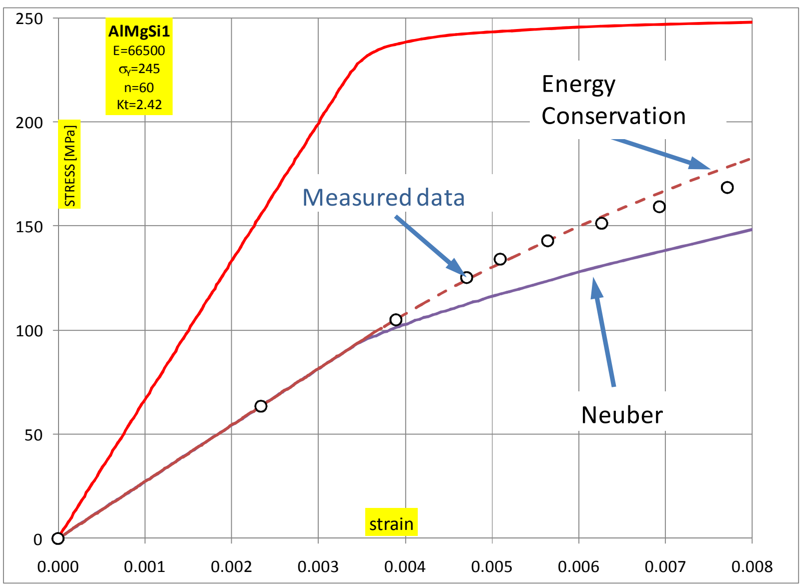
Note that calculated Neuber curve is matching the curve reported on Ref. 0.

Fig. 1 - St 52 Test Data and Analysis - comparison

AlMgSi 1 Test Data versus Analysis: Comparison



Ref. [2]



Calculation results (see ALGORITHM)

Measured Test Data	
strain	stress [MPa]
0.00000	0
0.00234	63.3
0.00390	105.1
0.00471	125.3
0.00510	134.1
0.00564	142.9
0.00627	151.2
0.00693	159.6
0.00771	168.8
0.00847	176.7
0.00933	184.6
0.01041	193.8
0.01158	201.3
0.01284	209.7

Note that calculated Neuber curve is matching the curve reported on Ref. [2].

Fig. 6 - AlMgSi 1 Test Data and analysis comparison

ALGORITHM

The calculation can be performed by using a spreadsheet system (Excel).
Goal-Seek plug in to calculate the σ_p by solving the energy equation.

MATERIAL DATA : St 52

E (Mpa)	σ_Y (Mpa)	n (-)	Kt	σ_{dp} (Mpa)	ε_{dp} (-)
210000	570.00	52.00	2.42	514.8	0.002451

1		2	3	4	5			6	7
Linear Elastic		$U^c = U_{LE}$	σ_p stress	ε_p strain	Neuber hyperbola			σ_{neuber}	σ_{energy}
σ_{LE}	ε_{LE}				$H_{hyper} = 2U^c$	$\sigma_{hyperbola}$	$\varepsilon_{hyperbola}$		
0.00	0.00000	0.000000	0	0.00	0.00000	19.38	0.000092	8.01	0.00
40.00	0.00019	0.003810	40	0.00019	0.00762	53.77	0.000256	22.22	16.53
42.00	0.00020	0.004200	42	0.00020	0.00840	59.66	0.000284	24.65	17.36
44.00	0.00021	0.004610	44	0.00021	0.00922	59.66	0.000284	24.65	18.18
547.68	0.00261	0.714176	534	0.00261	1.42835	564.51	0.003897	280.87	226.31
552.57	0.00263	0.726992	536	0.00263	1.45398	564.89	0.003943	282.60	228.34
659.45	0.00314	1.035422	556	0.00320	2.07084	571.56	0.005028	321.00	272.50
680.78	0.00324	1.103484	558	0.00332	2.20697	572.62	0.005266	328.81	281.31

1) Linear elastic stress and strain curve
2) Elastic strain energy = complementary energy : $U_{LE} = U^c$
3) Goal Seek plug in and macro , to solve the energy balance equation σ_p
4) Ramberg-Osgood equation to calculate the relevant strain ε_p
5) Calculate $\sigma_{hyperbola}$ and $\varepsilon_{hyperbola}$
6) $$\sigma_{neuber} = \frac{\sqrt{E * \sigma_h * \varepsilon_h}}{Kt}$$
7) $$\sigma_{energy} = \frac{\sqrt{2 * E * U^c}}{Kt}$$

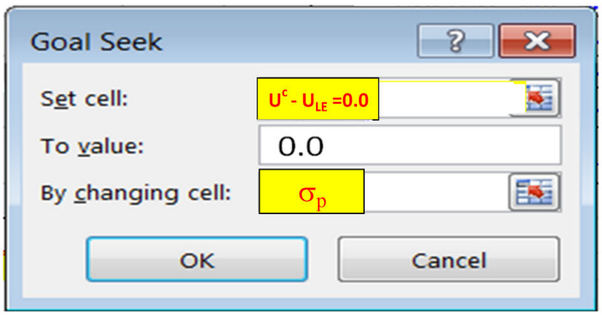


Fig. 7 - Algorithm

MATERIAL CURVE REFERENCED TO DIRECT PROPORTIONAL STRESS

The formulation of the stress -strain curve is based on Ramberg-Osgood equation (Ref. [3]):

$$\varepsilon = \frac{\sigma}{E} + .002 * \left(\frac{\sigma}{\sigma_Y}\right)^n \quad , \quad \sigma_y = \text{yield stress, } E = \text{Elastic modulus}$$

To simplify the numerical calculation, and split the curve in two segment (elastic and Elastic-Plastic), the stress- strain curve is rearranged in term by referring to Direct Proportional stress σ_{dp} (residual strain = 0.00001)

$$\varepsilon = \frac{\sigma}{E} + 0.00001 * \left(\frac{\sigma}{\sigma_{dp}}\right)^n \qquad \sigma_{dp} = \sigma_y * \left(\frac{0.00001}{0.002}\right)^{\frac{1}{n}}$$

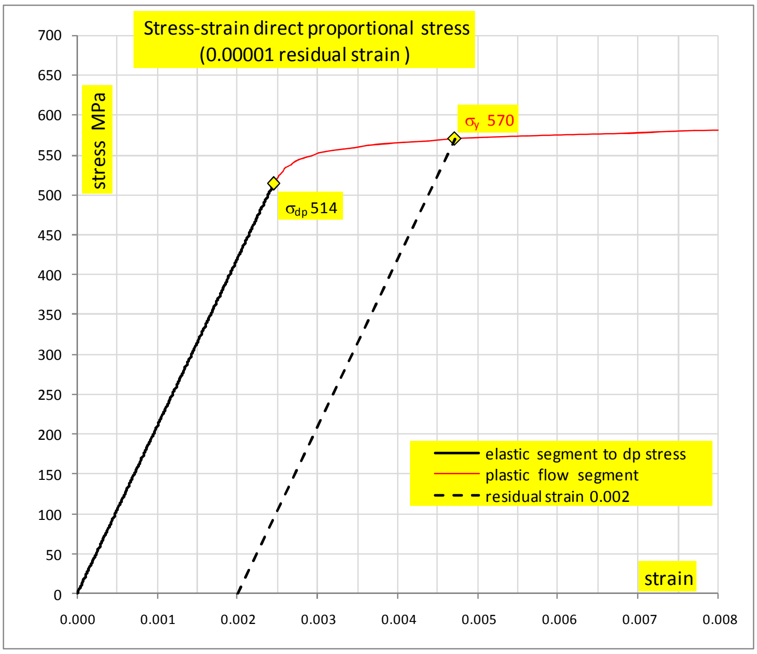


Fig. 8 - Ramberg-Osgood versus Direct Proportional Stress

The results of the energy approach is graphically shown on the diagram below for St-52 material. Linear elastic stress $\sigma_{LE} = Kt * \sigma_{ref}$ is the initial data. The values (σ_p , ε_p) on the stress-strain material curve, indicates the Strain Energy Equivalence point ($U_{LE} = U^c$).

Example

$$\sigma_{ref} = 413. \quad [MPa]$$

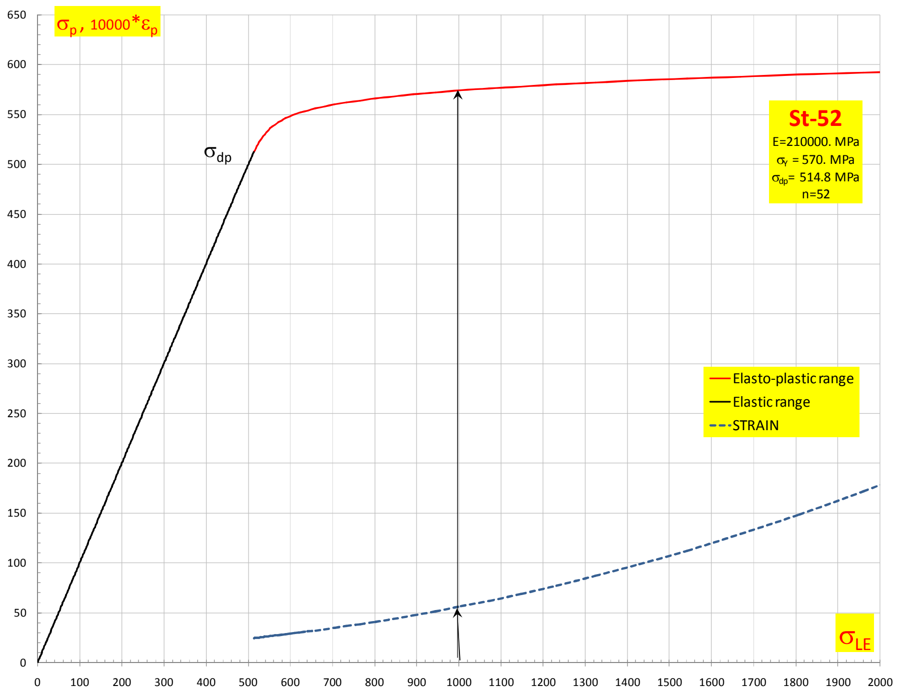
$$\sigma_{LE} = \sigma_{ref} * Kt = 413. * 2.42 = 1000. \quad [MPa]$$

$$U_{LE} = \frac{\sigma_{LE}^2}{2 * E} = \frac{1000^2}{2 * 210000} = 2.3809$$

\- -

$$\sigma_p = 574. \quad [MPa]$$

$$\varepsilon_p = 0.00561$$



σ_{LE}	U^c	σ_p	ε_p
514	0.6279	512	0.0024
516	0.6337	514	0.0025
518	0.6397	516	0.0025
521	0.6459	518	0.0025
523	0.6524	520	0.0025
526	0.6593	522	0.0025
529	0.6667	524	0.0025
532	0.6746	526	0.0025
536	0.6831	528	0.0026
539	0.6925	530	0.0026
543	0.7027	532	0.0026
548	0.7142	534	0.0026
553	0.7270	536	0.0026
558	0.7415	538	0.0027
564	0.7580	540	0.0027
571	0.7769	542	0.0027
579	0.7988	544	0.0028
588	0.8242	546	0.0028
599	0.8539	548	0.0029
611	0.8887	550	0.0029
625	0.9297	552	0.0030
641	0.9781	554	0.0031
659	1.0354	556	0.0032
681	1.1035	558	0.0033
705	1.1844	560	0.0035
733	1.2808	562	0.0036
766	1.3957	564	0.0038
802	1.5328	566	0.0041
844	1.6965	568	0.0044
891	1.8921	570	0.0047
945	2.1257	572	0.0051
1005	2.4048	574	0.0056
1072	2.7382	576	0.0062
1148	3.1366	578	0.0069
1232	3.6125	580	0.0077
1325	4.1808	582	0.0087
1429	4.8592	584	0.0098
1543	5.6687	586	0.0112
1669	6.6344	588	0.0129
1808	7.7858	590	0.0148
1961	9.1581	592	0.0171
2129	10.7929	594	0.0199
2313	12.7394	596	0.0232
2515	15.0560	598	0.0271
2735	17.8115	600	0.0317

Fig. 9 - Plastic flow Stress-Strain versus Linear Elastic stress

CONCLUSION

Neuber hyperbola has been widely used over the past decades as a method to predict the effect of stress concentration on static strength and life.

The approach based on Strain Energy Conservation Principle, object of this paper, is meant to refine the calculation.

The outcomes of this approach are:

1. Accurate ultimate static Margin of Safety (based on strain rather than stress)
2. Reliable fatigue life estimation for strain cycling in plastic segment.

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