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ASYMPTOTIC STOKES FLOWS USING AXIAL GREEN FUNCTION METHOD WITH REFINEMENT

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ABSTRACT

The axial Green function methods (AGMs) have been developed for efficient numerical computations of the solutions of partial differential equations that come from physical phenomena, for instance, the electric potential, the heat, the convection-diffusion, and the flows. Recent progress in AGM enables us to calculate problems caused by the geometric singularity and/or multi-scale of a domain. The axial lines should be refined enough to resolve the solution near the geometric singular/extreme region of interest, and elsewhere the axial lines are coarsely distributed. This often gives rise to the non-matching axial lines along the interface between the extreme and the other regions. At the non-matching nodes on the interface, we develop how to glue the solutions together across the interface numerically. On the other hand, the stress/drag blow-up in Stokes flow is one of extreme phenomena of interest. Here, the stress blow-ups take place between two adjacent circular bodies that are asymptotically close. Since it is fundamental to understand singular behavior of the solution in this case, we accurately calculate the blow-up stress between close bodies to predict how serious it is. To do this efficiently, the AGM with refinement has to be employed. According to our calculation, the stress blow-up becomes the reciprocal of square distance between bodies, which is quite interesting result particularly in collision model.

Keywords: axial Green function, refinement, stress blow-up, asymptotic Stokes flow.

INTRODUCTION

By the axial Green's function, we mean that it is one-dimensional Green's function of an ordinary differential operator defined on lines parallel to axis, belonging to the multi-dimensional domain. In general, the finite difference method uses this kind of lines, called the grids, but the admissible grids in this method are so restrictive that the method cannot work unless the domain is simple or the grids are gradually changing in space. The axial Green's function methods (AGMs) that we have developed, work fine in arbitrary domains without deterioration of accuracy, and furthermore they do even in randomly spacing axial lines. The use of Green's function takes place in the boundary element (BEM) method, which can reduce the dimension of the problem by discretizing the boundary of the domain. This is possible only when finding the fundamental solution or Green's function of the multi-dimensional differential operator, called partial differential operator. The BEM has been successful in Laplace operator, Lame operator in linear elasticity, Stokes operator in fluid mechanics, Helmholtz operator, and so on. However, if the material coefficients are functions of space variable, then the BEM suffers from finding the multi-dimensional Green's function in the domain or even a fundamental solution in entire space. The advantages of AGMs are obvious in two points: (1) Arbitrarily distributed axial lines are available, which is inconvenient in FDMs, and (2) It is much easier than BEMs to find one-dimensional Green's functions. Based on these facts, we are able to implant these advantages to the refinements of axial lines in some regions of interest. The refined regions can be independently handled by using the representation formula for the solution in terms of axial Green's functions. In this talk, using the AGM for Stokes flow with refinement, we delve deeply into the asymptotic Stokes flow: the stress blow-ups and the drag change in distance between two adjacent bodies.

RESULTS AND CONCLUSIONS

The two circular bodies are placed close enough. The left circular cylinder is staying while the right is moving in the right way with unit speed as shown in Figure 1.

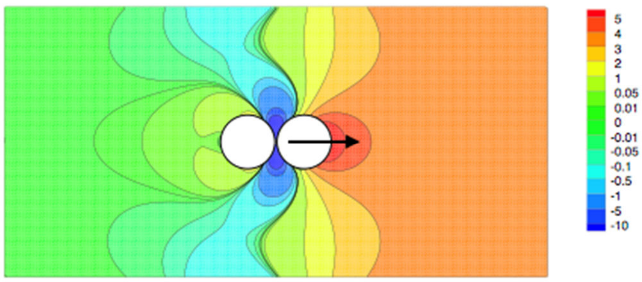

Fig. 1 - Pressure between asymptotically adjacent bodies

As the distance between bodies gets asymptotically smaller, the flow computation is non-trivial without refinements. The refinement works fine in a non-matching manner in association with AGM. More on computations in asymptotic regions are preformed using this refinement.

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REFERENCES

[1] W. Lee, H.-K. Kim and D. W. Kim (2017). Axial Green Function Method for Axisymmetric Electromagnetic Field Computation. IEEE Transactions on Magnetics, 53(6), pp. 7206504.

[2] W. Lee and D. W. Kim (2014). Localized Axial Green's Function Method for the Convection-Diffusion Equations in Arbitrary Domains. Journal of Computational Physics, 275, pp. 390-414.

[3] S. Jun, and D. W. Kim (2011). Axial Green's Function Method for Steady Stokes Flow in Geometrically Complex Domains. Journal of Computational Physics, 230, pp. 2095-2124.

[4] D. W. Kim, S. K. Park, and S. Jun (2008). Axial Green's function method for multi-dimensional elliptic boundary value problems. International Journal for Numerical Methods in Engineering, 76, pp. 697-726.