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TIME-HARMONIC ANALYSIS OF LINEAR ANISOTROPIC ELASTIC SOLIDS WITH A BOUNDARY ELEMENT METHOD

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ABSTRACT

In this paper, a time-harmonic analysis of three-dimensional linear elastic anisotropic solids is performed using a boundary element formulation which based on the regularized displacement boundary integral equations. Time-harmonic anisotropic elastic displacement and traction fundamental solutions are expressed in integral form. Mixed representation of the boundary functions and standard collocation procedure are employed for spatial discretization. Results of the numerical experiment are provided to demonstrate the accuracy of the presented boundary element approach.

Keywords: BEM, anisotropy, elastic solids, time-harmonic analysis, forced vibrations.

INTRODUCTION

For the three-dimensional transient dynamic or time-harmonic analysis of the practical problems involving anisotropic materials numerical simulation is the only way to obtain reliable results. Boundary Element Method (BEM) is a powerful and accurate numerical method particularly well suited for the wave propagation problems. Key difficulty associated with the implementation of the BEM to the problems of anisotropic elasticity is the lack of closed-form Green's functions (fundamental solutions). Time-harmonic anisotropic elastic displacement and traction fundamental solutions are expressed as a sum of static and dynamic terms:

$$\bar{g}_{ij}(\mathbf{y}-\mathbf{x},\omega) = g_{ij}^{S}(\mathbf{y}-\mathbf{x}) + \bar{g}_{ij}^{D}(\mathbf{y}-\mathbf{x},\omega),$$
$$\bar{h}_{ij}(\mathbf{y}-\mathbf{x},\omega) = h_{ij}^{S}(\mathbf{y}-\mathbf{x}) + \bar{h}_{ij}^{D}(\mathbf{y}-\mathbf{x},\omega),$$

where ω is circular frequency, $\bar{g}_{ij}(\mathbf{y}-\mathbf{x},\omega)$ and $\bar{h}_{ij}(\mathbf{y}-\mathbf{x},\omega)$ are the time-harmonic displacement and traction fundamental solution and superscripts "*S*" and "*D*" denote corresponding static and frequency dependent terms.

For general anisotropic materials static (singular) and dynamic (regular) terms are expressed as an integral over a unit circumference and an integral over a unit sphere (Wang, 1994), respectively. Using the static part of the traction fundamental solution the conventional displacement boundary integral equations are regularized as follows

$$\int_{\Gamma}\left[\bar{u}_k(\mathbf{y},\omega)\bar{h}_{jk}(\mathbf{y}-\mathbf{x},\omega) - \bar{u}_k(\mathbf{x},\omega)h_{jk}^{S}(\mathbf{y}-\mathbf{x})\right]\mathrm{d}\Gamma(\mathbf{y}) - \int_{\Gamma}\bar{t}_k(\mathbf{y},\omega)\bar{g}_{jk}(\mathbf{y}-\mathbf{x},\omega)\mathrm{d}\Gamma(\mathbf{y}) = 0,\ \ \mathbf{x}\in\Gamma,$$

where $\bar{u}_k$ and $\bar{t}_k$ are the displacement and traction vectors, Γ is the surface of the solid under consideration. Standard collocation procedure, with the mixed representation of the surface Γ

and the boundary functions $\overline{u}_k$ and $\overline{t}_k$, leads to a resulting system of linear algebraic equations.

NUMERICAL EXAMPLE

Consider an anisotropic elastic solid shown in Figure 1 (Gaul, 2003). The solid is clamped at one face and subjected to a time-harmonic traction with the amplitude $t_0 = -1 \cdot 10^8$ Pa at another face. The displacement $u_1(0.05, 0, x_3)$ for the circular frequency $\omega = 50\text{kHz}$ is shown in Figure 2.

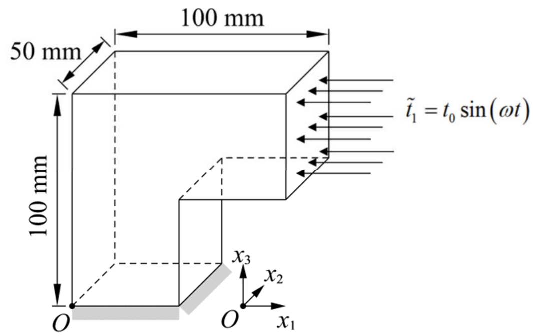

Fig. 1 - Anisotropic elastic solid under a time-harmonic loading

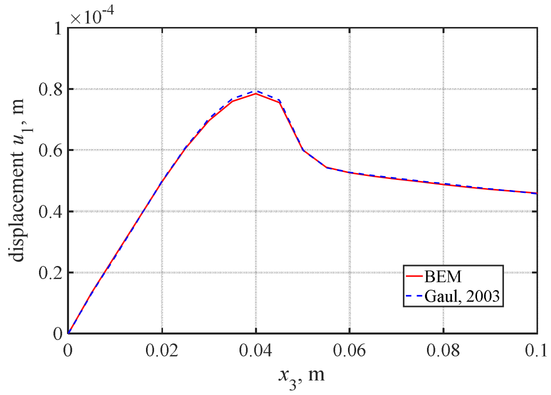

Fig. 2 - The displacement $u_1(0.05, 0, x_3)$ for the circular frequency

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