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THE RELIABILITY INDEX APPROACH WITH EVOLUTIONARY ALGORITHMS: APPLICATION TO THE RBRDO PROBLEM OF COMPOSITE STRUCTURES

Gonçalo das Neves Carneiro, Carlos Conceição António(*)
INEGI/LAETA, Faculty of Engineering, University of Porto, Porto, Portugal
(*)*Email:* cantonio@fe.up.pt

ABSTRACT

A new methodology to solve the Reliability Index Approach (RIA) minimization problem with high levels of accuracy and still with practical efficiency is presented. This method is based on the exclusive use of genetic algorithms with elitist strategy, to make possible to find the global Most Probable Failure Point, in the uncertainty space. To overcome the computational costs inherent to these class of algorithms, the RIA minimization problem is reformulated and a new heuristic is proposed. The method is then applied to the Reliability-based Robust Design Optimization problem of composite structures, where minimum weight and maximum system robustness are objectives and the target reliability index is a constraint.

Keywords: uncertainty, robustness, reliability, RBRDO, optimization, composites.

INTRODUCTION

Reliability assessment is a fundamental step to be taken over optimized structural solutions, because optimization itself usually evolves in the opposite direction of safety. In the industry, many of the sought objectives are related with cost reduction, which will often imply a reduction in structural strength. Therefore, constraining the design optimization of structures to meet certain levels of safety is a common procedure. However, reliability assessment is an expensive task to be executed with precision and is often overlooked. Most of the solutions found in the literature are based in the so called Reliability Index Approach (RIA) (Melchers, 1999). This method is based on the concepts of *reliability index*, β, and *Most Probable Failure Point* (MPP) and is formulated as a minimization problem. To solve it, many heuristics are based on gradient and local methods, which can only guarantee to find local MPP solutions and will overestimate the actual reliability level of the structure (Valdebenito, 2010).

On this paper, a new methodology to solve the RIA problem with high levels of accuracy and still with practical efficiency is presented. This method is based on the exclusive use of genetic algorithms (GA) with elitist strategy, to make possible to find the global MPP, in the uncertainty space. To overcome the computational costs inherent to these class of algorithms, the RIA minimization problem was reformulated and a new heuristic was developed. The method is then applied to the Reliability-based Robust Design Optimization (RBRDO) problem of composite structures, where minimum weight and maximum system robustness are objectives and the target reliability index is a constraint.

RBRDO OF COMPOSITE STRUCTURES BASED ON THE RIA

With the RIA, the reliability constraints are written in terms of β. During the optimization process, it is required the computation of β, for each updated design. Each reliability constraint is matched with a target reliability index, β_{tar}. To obtain β by the RIA, the following optimization problem is solved:

$$\beta = \min \ \|\boldsymbol{\pi}\|$$
$$\text{subject to } G_i(\mathbf{x}, \boldsymbol{\pi}) = 0 \tag{1}$$

where $G_i, i = 1{,}2$ is the limit-state function of critical displacements and stresses, respectively. On this paper, reliability assessment is only evaluated for the stresses. $\boldsymbol{\pi} \sim \mathrm{N}(0{,}1)$ is the set of uncertainty variables and $\mathbf{x}$ the set of design variables. The solution of this problem $\boldsymbol{\pi}^*_{\mathrm{RIA}}$ is called the MPP. The reliability index is its norm with respect to the origin of the standard random space and is compared with the target value β_{tar}. Thus, in the outer design optimization cycle, each reliability constrain is written as $\beta - \beta_{tar} \geq 0$.

In this work it is considered the RBRDO problem to the design of a composite structure, which may represent a component of a larger assembly. The reliability constraint is solved by the RIA. As mentioned, to achieve both an affordable computational time and precision to find the global MPP, with GA, a new heuristic is proposed to solve the RIA. The RBRDO problem is written as:

$$\begin{aligned} &\min && \mathbf{F}(\mathbf{x}, \boldsymbol{\pi}) = \{\mathrm{E}(W(\mathbf{x}, \boldsymbol{\pi})); \mathrm{Var}(G_i(\mathbf{x}, \boldsymbol{\pi}))\} \\ &\text{subject to} && G_1(\mathbf{x}, \boldsymbol{\pi}) \equiv \frac{\bar{u}}{u_a} - 1 \leq 0 \\ &&& g_2(\mathbf{x}, \boldsymbol{\pi}) \equiv \beta - \beta_{tar} \geq 0 \\ &&& x_j^l \leq x_j \leq x_j^u \qquad j = 1, \dots, m \end{aligned} \tag{2}$$

where $W(\mathbf{x}, \boldsymbol{\pi})$ is the weight of the composite structure. In the above formulation the variability of the weight is neglected and only the feasibility robustness is considered. Here, $\mathrm{E}(W(\mathbf{x}, \boldsymbol{\pi}))$ is the expected value of the structural weight and $\mathrm{Var}(G_i(\mathbf{x}, \boldsymbol{\pi}))$ corresponds to the determinant of the Variance-Covariance matrix of both deterministic limit-state functions G_1 and G_2 (António and Hoffbauer, 2016). Also, note that in eq. (2), G_2 transformed into g_2, the probabilistic stress limit-state function. Thus, G_2 is only implicitly considered in the reliability constraint.

The results obtained from the application of the proposed approach to composite structures show that the developed RBRDO model is appropriated to consider uncertainty in the structural parameters and that the new heuristic for reliability assessment produces optimal results with and efficient computational costs.

APPLICATION

The optimization process of the proposed RBRDO problem comprises two nested optimization cycles: an exterior cycle of Robust Design Optimization (RDO) complemented with an inner cycle for the reliability assessment. To solve the RDO problem, a previously developed algorithm, suited for this class of problems, is implemented, named the MOGA_2D (António and Hoffbauer, 2016). The solution of the RIA inner cycle is obtained by an improved micro-Genetic Algorithm, appropriate to tackle the difficult reliability assessment problem, here named as mGA_RIA. Both algorithms are discussed below.

The MOGA_2D algorithm

The MOGA_2D is a dominance-based multi-objective Genetic Algorithm that searches the design space with the purpose to discover and store Pareto-optimal solutions. The evolutionary procedure is developed in two parallel populations: namely short population (SP) and enlarged population (EP). The first one is developed in a local sense and is used to find local Pareto-optimal solutions. These solutions are then stored in the EP and ranked according to a global criterion of dominance, defining at the end of the evolutionary process the set of global Pareto-optimal solutions.

The ranking procedures of both populations are based on the concept of constrained-dominance (Deb, 2001). With this concept it is possible to compare and create a measure of fitness for solutions of multiple objectives and constraints. At SP, it is fundamental for the so called *fitness assignment* procedure (local dominance). Now, the fitness measure of the solutions no longer depends only on the absolute value of a fitness function, but also on the concept of dominance. Individual fitness is now called *shared fitness* and is calculated according to the niche occupied by the solution and the number of individuals with the same level of dominance in its neighborhood. As one sees, although the elitist strategy adopted in SP is based on fitness, it is also based on dominance, implicitly. Solutions that are not dominated by other solutions are said to be of *rank 1*. These solutions are then sent to the EP, where their global dominance status is measured, directly by the concept of constrained-dominance. Global rank 1 solutions, at the end of the evolutionary process, will defined the Pareto-optimal front. Four genetic operators are executed, namely, implicit mutation, crossover, similarity control and selection.

The mGA_RIA algorithm

On these section the term “solution” refers to the solutions found during the reliability assessment inner cycle and “design solution” refers to the solutions found in the outer RDO cycle.

The mGA_RIA is here classified as a micro-genetic algorithm, since it is developed with a small population and a small number of genes per variable. It is applied to each design solution with the purpose of evaluating the corresponding reliability level, measured in the uncertainty space. While the algorithm itself is built over the traditional genetic operators, new heuristics had to be developed and incorporated in the evolutionary process to tackle the difficulties inherent to the RIA problem, defined in eq. (1).

The goal is to find the point on the limit-state function, which has the minimum distance to the origin of the standard-normal uncertainty space. The first setback, when dealing with structural optimization, is the fact that the limit-state function is not known explicitly (only through model evaluation) and so it is impossible to know the relative positon of the origin in relation to the limit-state function. Also, genetic algorithms *per se* are not capable of verifying equality constraints. The combination of both these difficulties makes the RIA problem very difficult to handle.

To overcome these challenges, the following developments were introduced. Given the scope of structural optimization under probabilistic integrity constraints, it is known *a posteriori* that the limit-state function has the shape of a (hyper) ellipsoid and only its size and position in the uncertainty space are unknown. That said, and given the affinity between the ellipsoid and the sphere, the vector of uncertainty variables is decomposed in directional and magnitude components, being the random variables considered in the search process of the mGA_RIA the N direction cosines, $\mathbf{a}$, plus the norm of the vector, β, on a total of $(N+1)$ variables. Now, to find the MPP two heuristics are applied to make the search process more efficient and controlled. First, to make sure the equality constraint is verified, an operator to correct the magnitude of each solution is introduced. This heuristic will increase, or decrease, the value of β of each solution, in an iterative fashion, while keeping the direction constant, until the equality constraint $G_2(\beta|\mathbf{a},\mathbf{x})=0$ is verified. The magnitude of the increments is defined by means of an exponential function, asymptotically tending to zero when the equality constraint is verified. The second heuristic applied gives both the ability to increase the resolution of the GA and to focus the search process on a preferential zone of the uncertainty space. At the first stages of the search procedure, the population is freely distributed through all the domain. However, and given the huge size of the search space compared to the value β_{MPP}, it is not of practical interest to keep searching on directions one already knows are not pointing towards the MPP. Therefore, after the elite of the population finds points on the limit-state function that are "close enough", the search space is reduced to the (hyper) volume defined by the elite, the elite solutions are recoded in the new space and the rest of the population is randomly generated. This process, is then repeated after a pre-determined number of generations, to define a new search space around the new elite, while a minimum level of diversity in the population is preserved, after which there are no advantages in defining a new search space. Here, "close enough" means that the value β of the solutions of the elite is under a certain value that is accepted to be a coarse estimate of β_{MPP}. That means the mGA_RIA will need some information of the problem. A prior estimation of the actual reliability index is calculated by (António 1995)

$$\beta^0 = \left(\frac{G_2}{\sqrt{\sum_{i=1}^{N_\pi} \left(\frac{\partial G_2}{\partial \pi_i} \sigma_{\pi_i} \right)^2}} \right)_{(\mathbf{x}|\boldsymbol{\mu}_\pi)} \tag{3}$$

where, σ_{π_i} is the standard deviation of random variable π_i. Every time this operator is run two phenomena might occur: the search space is reduced and/or translated. This happens because the reduced space must have a prescribed minimum size and will be centered around a new elite. The described heuristic also has the advantage of allowing the GA to start with a coarse resolution, evaluating the uncertainty space at a large scale and then to refine its search, augmenting the resolution. Because of this, low numbers of solutions in the population and genes per variable are enough to find the MPP.

RESULTS

A clamped cylindrical shell laminated structure is considered as show in Figure 1. Nine vertical loads of mean value P_k=11.5 KN are applied along the free linear side (AB) of the structure. This side is constrained in the y-axis direction. The structure is divided into four macro-elements, grouping all elements, and there is one laminate for each macro-element. The laminates' distribution is as shown also in Figure 1. The balanced angle-ply laminates with five layers and with the stacking sequence $[+a/-a/0/-a/+a]$ are considered in a symmetric construction. Ply angle a is a design variable and is referenced to the x-axis of the reference axis. The design variable h_i denotes the laminate thickness and four laminates are considered. A composite material built with the carbon/epoxy system denoted T300/N5208 (Tsai, 1987) is used in the presented analysis. This is a unidirectional composite of long carbon fibers aggregated in an epoxy matrix. The macro mechanics' mean values of the elastic and strength properties of the ply material used in the laminates of the structure is presented in Table 1. The elastic constants of the orthotropic ply are the longitudinal elastic modulus E_1, the transversal elastic modulus E_2, the in-plane shear modulus G_{12} and the in-plain Poisson's ration ν_{12}. The ply strength properties are the longitudinal tensile strength X, the longitudinal compression strength X', the transversal tensile strength Y and the transversal compression strength Y' and the shear strength S. In the RBRDO procedure, the allowable value u_a of the constraint of displacement is $u_a = 8.0 \times 10^{-2}$ $[m]$ and $\beta_{tar} = 3$.

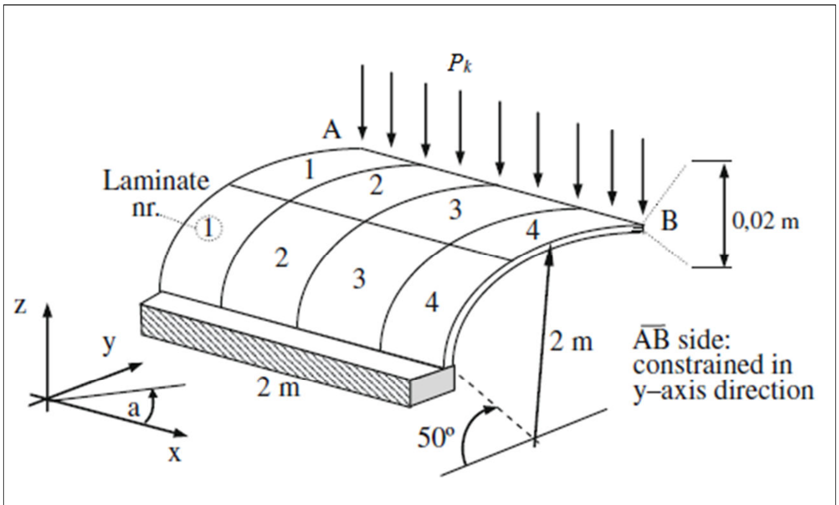

Fig. 1 - Geometric definition of the cylindrical shell structure and composite laminates distribution

Table 1 - Mean values of mechanical properties of composite layers

Material	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}
T300/N5208	181.00	10.30	7.17	0.28
	$X; X'$	$Y; Y'$	S (MPa)	ρ (kg/m^3)
T300/N5208	1500; 1500	40; 246	68	1600

The values of the standard deviations, σ_{π_i}, are considered to 6% of the mean values of the uncertainty variables. Results are shown in the next figures: the Pareto-optimal front and the of both distribution of the reliability index and critical Tsai number are displayed. To highlight the effects of the RIA over the design optimization, the same structure was also optimized by two alternative approaches: RBRDO with reliability assessment executed by the Performance Measure Approach (PMA), which is mathematically defined as the inverse problem of the RIA; and only RDO, without reliability assessment. These developments can be found in the following references (das Neves Carneiro and António, 2017) and (António and Hoffbauer, 2016), respectively. Figure 2 shows the Pareto-optimal fronts, obtained by the three methods. It is seen that optimal solutions have similar distributions on the solution space, albeit the reliability constraint is responsible to make the solutions slightly heavier, when the system variance increases.

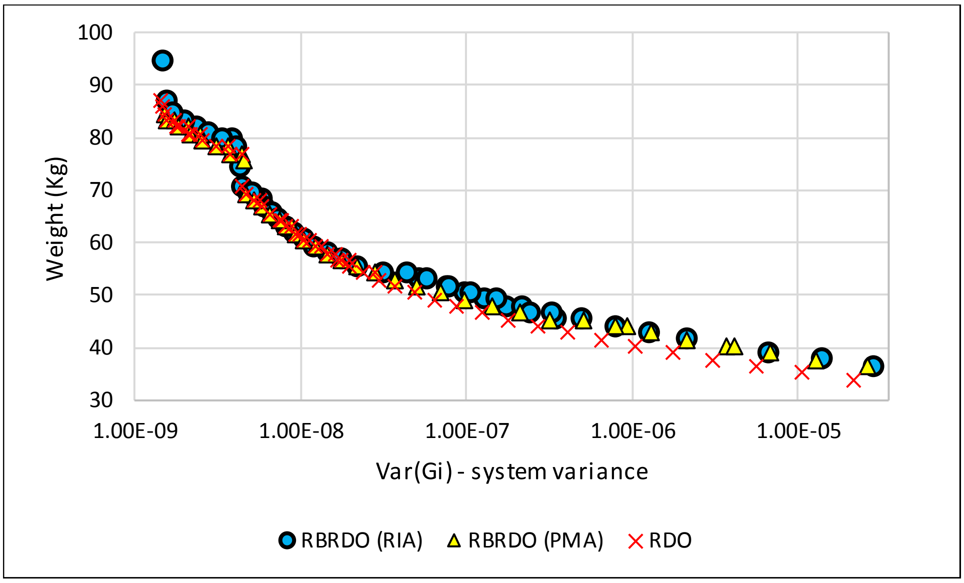

Fig. 2 - Pareto-optimal fronts obtained with RBRDO (RIA), RBRDO (PMA) and RDO

Figure 3 shows the distribution of the critical Tsai numbers, for the three cases. While the Pareto fronts are similar, these distributions expose the differences between the optimization processes. For RDO, it is seen that the critical Tsai numbers decrease consistently almost until the value 1.00, which is only acceptable from a deterministic point-of-view. With both the RIA and the PMA, the reliability constraint appears to inhibit solutions to have critical Tsai number values under 1.20. Between these two, some similarities are seen, even if some solutions have different levels of structural integrity. The differences may be related to the different paradigms of each method. From deterministic point-of-view the RIA seems to be a more conservative approach.

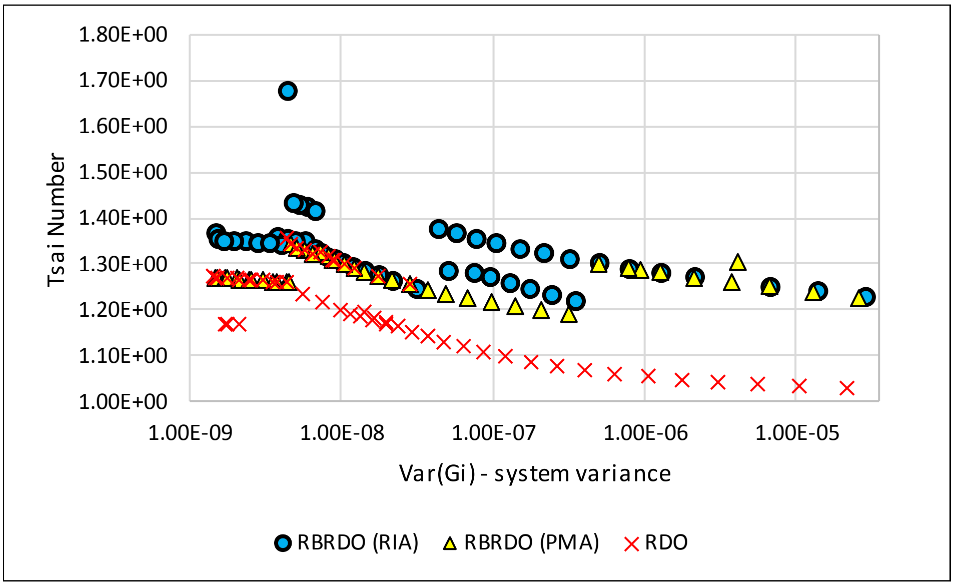

Fig. 3 - Distribution of the Tsai number obtained with RBRDO (RIA), RBRDO (PMA) and RDO

However, from a probabilistic point-of-view, the RIA appears to be less conservative than the PMA. As seen in Figure 4, the reliability indexes obtained by the first method are closer to the target value $\beta_{tar} = 3$, than with the second. This occurrence may be justified by the anisotropy of the composite laminate structure, which leads to solution of similar weight, variability and Tsai number having different fracture envelops (ellipsoids) and, thus, a different reliability index.

Despite this phenomenon, in both cases, it is also seen that the distributions of β_{MPP} and $\bar{R}$ have similar shapes, which allows to speculate about a functional relationship between critical the Tsai number and the reliability index.

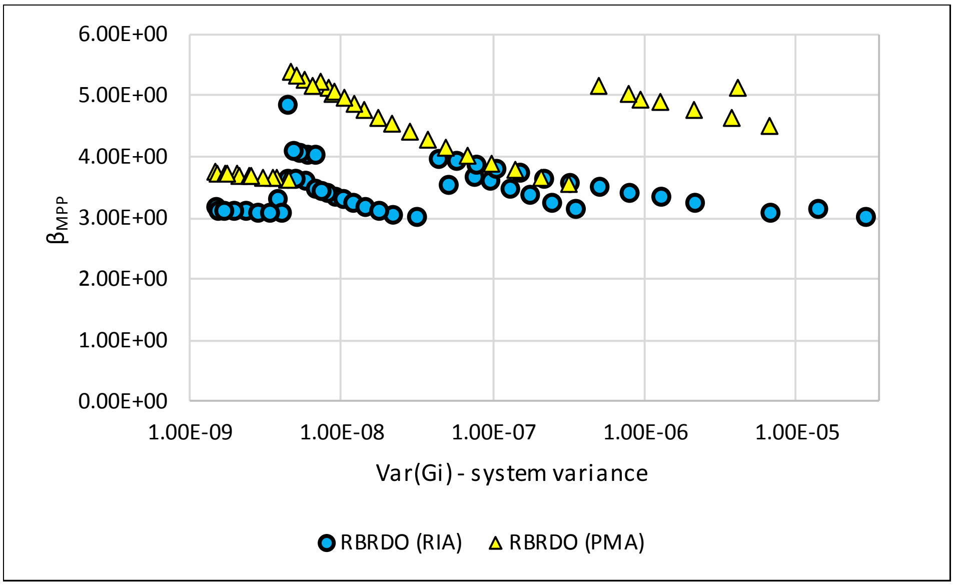

Fig. 4 - Distribution of the reliability index obtained with RBRDO (RIA), RBRDO (PMA) and RDO

CONCLUSIONS

In this paper, a new approach to the multi-objective RBRDO of ply-angle composite laminate shell structures is proposed. RBRDO procedures are known to be very inefficient, given the several different methods involved and the large number of solution evaluations needed to converge. The proposed methodology attempts to combine efficiency with higher levels of accuracy, by the exclusive use of GA, avoiding premature convergence in local minima as in gradient-methods. Design optimization is considered as the bi-objective minimization problem of the weight (optimality) and the determinant of the variance-covariance matrix (robustness). Reliability assessment is made by the RIA, modified to make it more efficient and accurate, as GAs tend to compensate their elevated accuracy with computational effort. An example of a balanced angle-ply laminate composite shell was presented. The results obtained from the application of the proposed approach to composite structures show that the developed RBRDO model is appropriated to consider uncertainty in the structural parameters and that the new heuristic for reliability assessment produces optimal results with and efficient computational costs.

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