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UNCERTAINTY QUANTIFICATION USING A NEW NON INTRUSIVE STOCHASTIC APPROACH: APPLICATION TO THE ESTABLISHMENT OF INUNDATION MAPS DUE TO DAM BREAK FLOWS

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ABSTRACT

This work deals with uncertainty propagation analysis using B-splines Basis Functions Expansion (BSBF) combined with an element-based sampling approach. The coefficients of the approximation on this basis are obtained through a regression technique. The convergence of the method with respect to the computational effort is investigated and compared to the classical Point collocation polynomial chaos expansion (Pcol) and the Monte Carlo (MC) methods. The propagation of input parameters uncertainties through the shallow-water equations hydraulic model is studied with the proposed approach. Several numerical tests related to dam break flows are established with the goal of obtaining the inundations maps with their associated uncertainties.

Keywords: uncertainty quantification, B-splines basis functions, dam break.

INTRODUCTION

The increase of computational power has contributed to the development of advanced numerical models in several fields of engineering sciences. The numerical results and designed products derived from these models are subjected to numerous uncertainties that must be taken into account. Uncertainty quantification (UQ) analysis is considerably widespread these last decades as an attracting research area owing to its various engineering applications such as structural design, aerospace engineering, nuclear safety analysis, thermal engineering and hydraulic-environmental studies. The sampling methods (Monte Carlo, LHS...) are among the most common and straightforward methods for uncertainty propagation. The computational cost of these methods becomes, very often, prohibitive.

As an alternative, the non-intrusive polynomial chaos expansion which belongs to the stochastic expansion methods is widely used to quantify uncertainties propagation in numerical models (Ghanem, 1991). The coefficients of the expansion are evaluated either with projection technique by using the quadrature approach (Hosder, 2010) or regression technique by solving an oversized linear system of equations by using least square method (Blatman, 2010). In this paper, a new non-intrusive regressive stochastic method is proposed where the expansion is defined on the cumulative probabilities domain which is decomposed into elements (called Bézier elements) and where the B-splines functions (with arbitrary order of interpolation) are used as polynomial interpolations. Also, the sampling of the input parameters is established locally in the Bezier-elements. The use of such interpolations allows obtaining efficiently smooth statistics of the outputs (mean and variance) in the case of discontinuous wave flows (Cottrell, 2009). The multi-element aspect of the method and the

compact support feature of the BSBF are explored to increase the efficiency of the sampling procedure and the smoothness of the approximation.

RESULTS AND CONCLUSIONS

The performance of the proposed method is assessed first by considering a benchmark test where an analytical function output is known $[f(X_1, X_2) = e^{1.5(X_1+X_2)}]$ as a deterministic model with two input uncertain parameters (X_1, X_2) following a uniform distribution with a given values of the mean and standard deviation (Hosder, 2007). The evolution of the relative error with the number of samples of the statistical moments (mean and standard deviation) of the output response, which is evaluated in comparison with the exact solution, is shown in Figure 1.

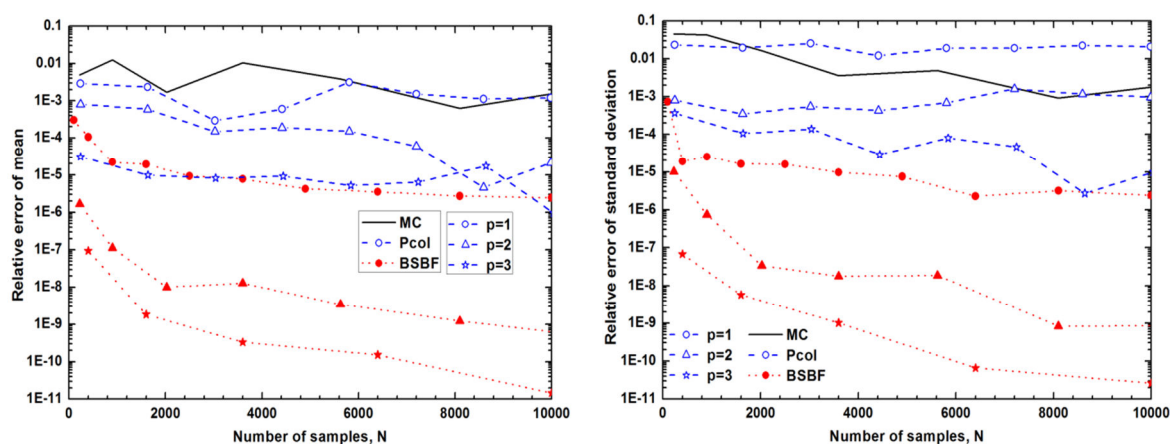


Fig. 1 - Relative error of the mean and the standard deviation (Std) obtained with BSBF, Pcol and MC methods.

It is observed that by increasing the number of samples, the BSBF technique presents a better convergence rate compared to the Point collocation expansion and the Monte Carlo methods. The effect of the polynomial order (p in the figure) of the base functions on the convergence rate can also be analyzed by examining Figure 1. The increase of the polynomial order leads to a significant improvement of the convergence rate of the BSBF expansion compared to the point collocation method. To further illustrate the performance of the proposed method, the computational efficiency of the BSBF technique is evaluated on a more realistic case, consisting of a dam break flow on a real river subjected to various uncertain input parameters.

REFERENCES

- [1] Blatman G, Sudret B. An adaptive algorithm to build up sparse polynomial chaos expansions for stochastic finite element analysis. Probabilistic Engineering Mechanics, 2010, 25, pp. 183-197.
- [2] Cottrell J. A, Thomas J. R. H, Yuri B. Isogeometric analysis toward integration of cad and FEA. A John Wiley and Sons, Ltd, 2009.
- [3] Hosder S, Walters R.W, Balch M. Efficient sampling for non-intrusive polynomial chaos applications with multiple uncertain input variables. 48th AIAA/ASME/ASCE/AMS/ASC structure, structural dynamics and materials conference, 23-26 April 2007, Hawaii, pp. 1-16.
- [4] Ghanem, R., Spanos, P. D. Stochastic Finite Element: A Spectral Approach. Springer, New York. 1991.