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USING DESIGN S-N CURVES AND DESIGN STRESS SPECTRA FOR PROBABILISTIC FATIGUE LIFE ASSESSMENT OF VEHICLE COMPONENTS

Miloslav Kepka^(*), Miloslav Kepka Jr.

Regional Technological Institute, research center of Faculty of Mechanical Engineering, University of West Bohemia, Pilsen, Czech Republic

^(*)*Email: kepkam@rti.zcu.cz*

ABSTRACT

The contribution explains a possibility of using design S-N curves and design stress spectra for probabilistic fatigue life assessment of vehicle components. The design S-N curves can be considered on the basis of either experiments or using some standards. The design stress spectra can be generated theoretically, but it is more appropriate to derive them from the results of representative stress measurement during the characteristic operation of the vehicle. The resulting fatigue life distribution function is then a probabilistic interpretation of the service fatigue life of the vehicle component under consideration.

Keywords: S-N curve, stress spectrum, fatigue life, vehicle component, probabilistic approach.

INTRODUCTION

Typical input information for evaluating the fatigue life of critical sections in structures under cyclic loading with respect to high-cycle fatigue includes the S-N curve and stress spectra for the key operating modes. The S-N curves can be constructed using fatigue data from a sufficient number of test pieces representing the structural detail under examination. Statistical evaluation of fatigue tests can provide confidence intervals and tolerance limits for the chosen probability of a particular curve and the corresponding coefficients. However, it can also be determined by estimations or obtained from standards for design of structures (BS 7608:1993).

The stress-spectra are most often evaluated by the application of the "rain flow" method to measured stress-time histories. However, the resulting histogram of load cycles often has a characteristic shape and design stress spectra representing the required life can therefore be derived theoretically from experience (Neugebauer, 1989).

In order to convert the stress data into fatigue damage levels by means of calculation, cumulative damage hypothesis is employed e.g. Miner's rule. Based on relevant stress spectra and S-N curve parameters, the fatigue damage is calculated and the service life estimate is obtained and compared with the requirement for the part's life. Authors in last IRF conference (Kepka, 2016) performed parametric calculations of allowable service stresses in vehicle components under fatigue loading.

The fatigue properties of the construction nodes, however, show scattering and their service load is often random. For this reason, the probability interpretation of fatigue life calculations

is more correct than the deterministic interpretation. An acceptable form of such a result is the fatigue life distribution function (Figure 1). The principle of its derivation has already been presented in the literature by way an example of a structural node of an articulated bus (Kepka, 2016).

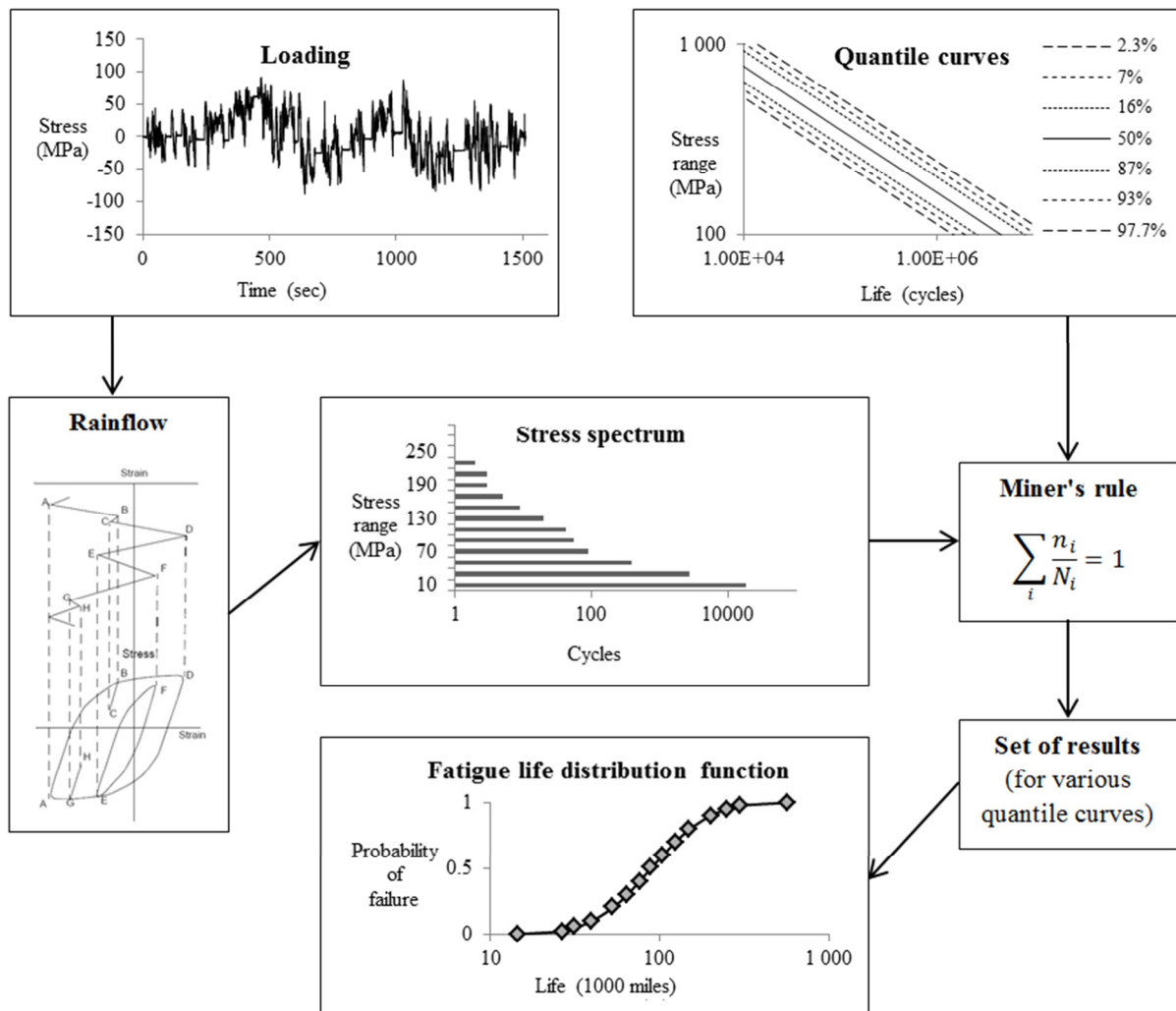


Fig. 1 - Fatigue life distribution function

Over the last twenty years, Research and Testing Institute Plzen has been developing a methodology of computational and experimental investigation of strength and fatigue life of bodies of road vehicles for mass passenger transport. A summary of this methodology - which was used for designing many Skoda trolleybuses and buses - has already been presented to the public (Kepka, 2009). It involves computational estimation of fatigue service life of structural details of the vehicle body. It continues to be developed by the Regional Technological Institute, which is a research center of the Faculty of Mechanical Engineering of University of West Bohemia (Kepka, 2015).

The detail of interest was a severely stressed beam joint in the top corner of the door opening in the bus body shown in Figure 2. The critical cross section was monitored by strain gauge T6. The desired (design) fatigue service life of the virtual vehicle in question was defined via

vehicle mileage: $L_d = 1\,000\,000$ km. This study is a virtual investigation because the specific values used in commercial contracts for various manufacturers are confidential. However, the values and data employed in this study are realistic and can be encountered in real life of a bus or a similar vehicle (trolleybus, battery bus or other vehicle).

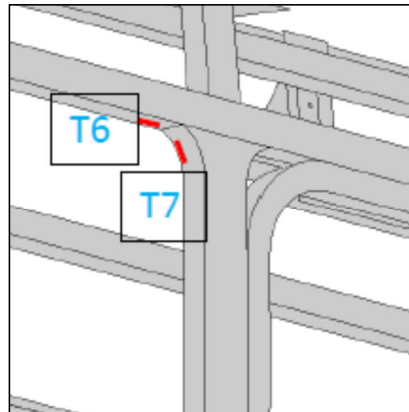


Fig. 2 - Schematic illustration of the detail of interest - important strain gauge T6

In order to calculate fatigue service life of structures and their parts operating under cyclic loads, the following data are necessary:

- Information on their fatigue strength;
- Information on their service loads.

In high-cycle fatigue scenarios, the input data includes:

- The S-N curve;
- Stress spectra for major operating modes.

This input information must apply to the same (critical) cross-section of the component. Stress characteristics are converted to fatigue damage using cumulative damage rules, which have been proposed by various authors (Palmgren-Miner, Corten-Dolan, Haibach and others).

S-N CURVE: THEORETICAL BACKGROUND

A durability of a material or a component against high cycle fatigue damage is usually characterized by an S-N curve, which describes a relationship between stress amplitude σ_a (or stress range $\Delta\sigma$) and cycles to fatigue failure N_f (or occurrence of a macroscopic fatigue crack). Some standards (for example British Standard BS 7608) are suitable for taking into account the scatter of material properties (or fatigue characteristics of an assessed construction nodes). In this standard, the fatigue curve is defined as follows:

$$\log(N_f) = \log C_0 - s \cdot d - m \cdot \log(\sigma_a), \quad (1)$$

where

N_f is the number of cycles to limit fatigue stage,

σ_a stands for the stress amplitude,

m is the inverse slope of the σ_a versus $\log N_f$,

s is the standard deviation of $\log N_f$,

d is the number of standard deviations s below the mean fatigue life curve,

C_0 is parameter defining the mean line S-N relationship.

Standard deviation of $\log N_f$ can be calculated on basis of experimental data, the value of standard deviation can be also found in the literature (standards and guidelines). Fatigue curves for a various certainty of survival can be described by selecting standard deviation. For instance, at $d = 0$, equation (1) describes a mid-range fatigue curve (failure probability of 50%). Fatigue curves shifted by two standard deviations ($d = 2$) below the mean curve, provided that log-normal distribution applies, represent a failure probability of 2.3% (this means a probability of survival of 97.7%). The certainty of survival is converted to a standard deviation using the following values. Linear interpolation is used for values not in the Table 1.

Table 1 - Certainty of survival conversion to standard deviation

Certainty of survival (%)	d - Number of standard deviations
99.9	-3
99.4	-2.5
97.7	-2
93	-1.5
84	-1
69	-0.5
50	0
31	0.5
16	1
7	1.5
2.3	2
0.6	2.5
0.1	3

The following procedures can be used for determination of S-N curve:

- a) The most reliable method of determination of S-N curve parameters is based on statistical evaluation of a sufficiently large set of laboratory fatigue tests of identical test specimens. The conditions of fatigue test are described in international standards.
- b) For some typical joints, S-N curves can be considered according to various design standards, industry regulations and recommendations. The word "typical" in this case means (similarity) in the geometry, material and technological design.
- c) The S-N curve can also be derived using open access publications, catalogs and test reports summarizing the results of fatigue tests of typical structural nodes or components.
- d) The S-N curve of the construction node can also be derived from known fatigue properties of the material. It is most often the derivation of the fatigue limit and slope of S-N curve of the critical cross-section of the real component from the known material fatigue curve or even from the static strength characteristics of the material (tensile test diagram). It should be noted that all of the approaches described above are more accurate than this one.
- e)

S-N CURVE: CASE STUDY

The S-N curve of the examined node was obtained by a combination of the above approaches. We have identified it for two variants of the bodywork profiles: made of low carbon steel S235JR and also for stainless steel version X2CrNi12.

In order to determine the fatigue strength of the evaluated structural detail, laboratory fatigue testing was carried out. Test pieces were made from thin-walled welded closed sections which had 70×50 mm cross-section and 2 mm wall thickness and were made of S235JR.

The critical cross-section of the joint was subjected to reverse bending load (the cycle stress ratio was $R = -1$). During testing, the stresses acting on the critical cross-section were measured by strain gauges attached approximately 5 mm from the toe of the fillet weld. The measured values by strain gauges T6 can therefore be referred to as the equivalent structural stress. The limit state was defined by the instant at which a macroscopic fatigue crack forms (1 to 2 mm). In all cases, fatigue cracks initiated in the transition zone of the fillet weld. Figure 3 shows a photograph of the test stand. Table 2 summarises test results.

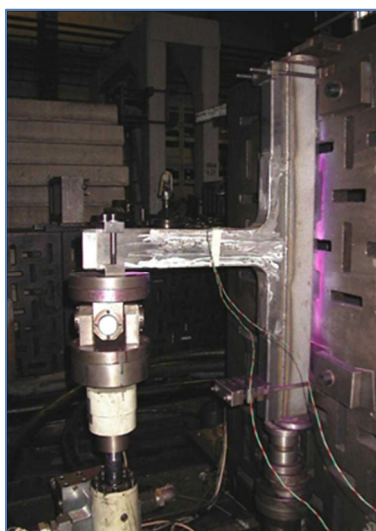


Fig. 3 - Test stand

Table 2 - Results of laboratory fatigue tests

Number of test specimen	Testing stress amplitude σ_a (Mpa)	N° of cycles to limit fatigue stage N_f	Remark
1	140	50 000	
2	120	140 000	
3	110	170 000	
4	100	500 000	
5	80	1 250 000	
6	70	900 000	
7	60	2 000 000	runouts
8	50	2 000 000	runouts

Statistical evaluation of the fatigue test data yielded the parameters of the S-N curve for the structural detail made from S235JR in the form (1):

$$\log(N_f) = 14.54 - 0.19 \cdot d + 4.53 \cdot \log(\sigma_a); \sigma_c = 60 \text{ MPa.}$$

For the stainless steel version, fatigue test results were not available. In the material database (WIAM METALLINFO, 2018) basic material characteristics were found for both materials. We considered the geometric and technological consensus of both designs, and we only took into account the effect of higher fatigue of X2CrNi12 material ($\sigma_c = 180 \text{ MPa}$) compared to S235JR ($\sigma_c = 160 \text{ MPa}$). In the ratio of both values $180/160 = 1.125$, we moved the mean S-N curve of the construction node made of X2CrNi12 toward higher fatigue strength and fatigue life. All other parameters (inclined branch slope, break point, and standard deviation of $\log N_f$) have been retained. The estimated parameters of the S-N curve for the structural detail made from X2CrNi12 in the form (1) are:

$$\log(N_f) = 14.78 - 0.19 \cdot d + 4.53 \cdot \log(\sigma_a); \sigma_c = 67.5 \text{ MPa.}$$

The results of laboratory fatigue tests and both mean S-N curves are shown in Figure 4.

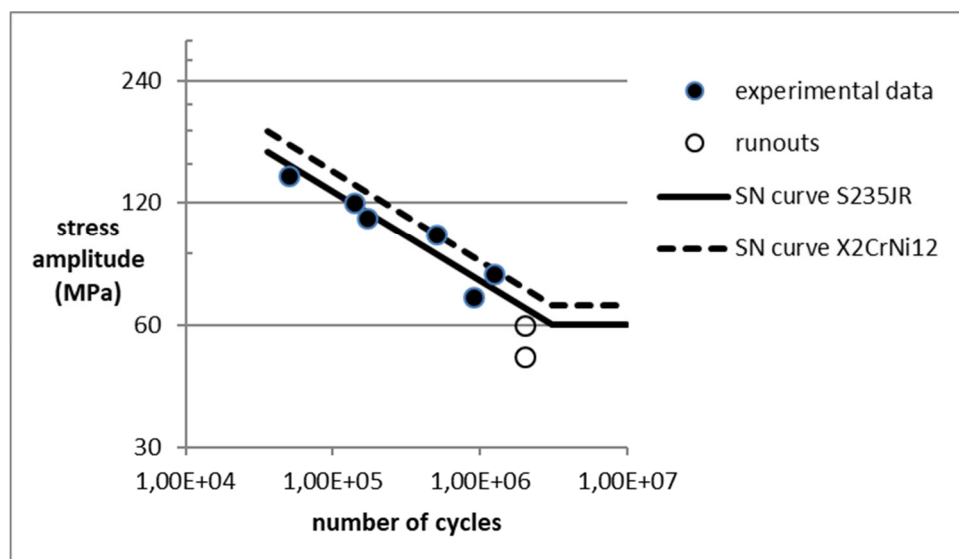


Fig. 4 - Results of laboratory fatigue tests and mean S-N curves

STRESS SPECTRA: THEORETICAL BACKGROUND

If accurate data from a measurement is not available, the stress spectra should be estimated. The design stress spectra for this parametric study was generated using the relative coordinates $\sigma_{ai} / \sigma_{amax}$ (Ruzicka et al, 1987).

$$h_i = H_{tot} \cdot \left(\frac{H_{max}}{H_{tot}} \right) \left(\frac{\sigma_{ai}}{\sigma_{amax}} \right)^s \quad (2)$$

- σ_{amax} - maximum stress amplitude in the spectrum,
- H_{max} - number of cycles with σ_{amax} amplitude in the spectrum,
- H_{tot} - total number of cycles in the spectrum,
- s - shape parameter of the spectrum,
- h_i - cumulative frequency of cycles with an amplitude of σ_{ai} .

At a constant width of the classification interval $d\sigma_a$, one can derive the discrete stress spectrum $\sigma_{ai} - n_i$:

- absolute class frequency is calculated: $n_i = h_i - h_{i+1}$,
- this frequency is assigned to the mid-point of the class (or, safely, to its upper limit) σ_{ai} .

With this interpretation of various oscillation processes, the cumulative frequency distribution of cycles h_i is plotted in semi-log graphs using various shape parameters s (Figure 5):

- $s = \infty$ Rectangular distribution for a constant-amplitude harmonic process,
- $s = 1$ Normal distribution for a steady-state random Gaussian process with a standard deviation of σ_{RMS} ,
- $s = 2$ Linear distribution for a process consisting of a number of steady-state random Gaussian sub-processes with various values of σ_{RMS} .

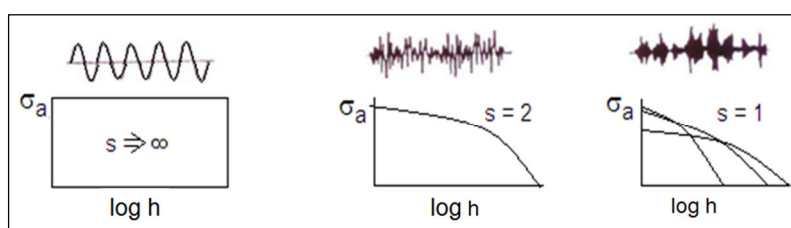


Fig. 5 - Characteristic shapes of one-parameter design stress spectra

Measurements of service loads in road vehicles were interpreted several times in literature (Neugebauer, 1989) with the following observations: long-term monitoring of rides on an irregular road surface yields linear-distribution stress spectra ($s = 1$) and manoeuvres (curve riding, braking, etc.) lead to stress spectra with normal distributions ($s = 2$).

An estimate of the total number of cycles H_{tot} in the design stress spectrum can be obtained from the equation:

$$H_{tot} = \frac{L_d}{v} \cdot 3600 \cdot f \quad (3)$$

L_d - design life of the body (mileage in km),

v - average speed (kph),

f - dominant frequency of vibration (Hz).

The frequency H_{max} of maximum service stress amplitudes σ_{amax} during the design life of the body L_d depends on operating conditions of the vehicle given by the user.

For the parametric studies, operating condition indicators P and L_1 , were established to indirectly express the severity of the conditions:

$$P = \frac{H_{max}}{H_{tot}} \quad \text{and} \quad L_1 = \frac{L_d}{H_{max}} \quad (4)$$

P - probability of occurrence of the maximum service stress amplitude σ_{amax} ,

L_1 - travelled distance (in kilometres), in which the maximum service stress amplitude σ_{amax} occurs once.

Between both indicators the following relationship is valid:

$$P = \frac{L_d}{L_1 \cdot H_{tot}} \quad (5)$$

If there is a sufficiently representative record of the stress time history, it is more accurate to evaluate the stress spectra from these data. The random processes are converted with the "rain flow" method into one-parameter or two-parameter histograms of stress cycles depending on the magnitude of their amplitudes and mean values.

STRESS SPECTRA: CASE STUDY

Therefore, the design stress spectrum parameters (s , H_{tot} , S_{max} and H_{max}) for designing and sizing the beam joints of the body were selected on the basis of the following considerations.

The major cause of the service loads that act on the structural detail is the vibration of the body due to the vehicle's ride on irregular road surfaces of various qualities. For this reason, the shape parameter of the design stress spectrum was chosen as $s = 1$.

The design life of the body was set as $L_d = 1\,000\,000$ km. The dominant vibration frequency was assumed to be $f = 10$ Hz. The average travel speed was set as $v = 50$ kph. After substituting the values into equation (3), the estimation of total number of cycles in the specified body life was found as $H_{tot} = 7.2 \cdot 10^8$ cycles.

Maximum stress amplitudes σ_{amax} were estimated by strain gauge measurements on model testing track. The model testing track was made from artificial obstacles in a form of cylinder segment having the basis of 500 mm and height of 60 mm. The artificial obstacles were laid on an even asphalt road in such composition, that overruns of them occurred gradually by right-hand wheels, by both wheels simultaneously and by left-hand wheels with distance of 15 m. Two loading states were measured (run with empty vehicle and run with fully loaded vehicle) and results of the measurements are collected in Table 3.

Table 3 - Measurement on model testing track

Maximum stress amplitudes σ_{amax} (MPa) measured on model testing track						
Strain gauge	Empty vehicle			Loaded vehicle		
	Both	Left	Right	Both	Left	Right
T6	84	75	45	108	73	44

As safe values for generation of design stress spectra were finally used the following values: $\sigma_{amax} = 85$ MPa for empty vehicle and $\sigma_{amax} = 110$ MPa for fully-loaded vehicle.

Travelled distance L_1 , in which the maximum stress amplitude σ_{amax} occurs once, was chosen in three alternative variants, $L_1 = 10, 100$ and 1000 km, it means with three different values $H_{max} = 10^5, 10^4$ and 10^3 cycles. The parameters of all alternative design stress spectra are summarised in Table 4.

Table 4 - Parameters of alternative design stress spectra

Parameters of design stress spectra						
	Design stress spectra - Empty vehicle			Design stress spectra - Loaded vehicle		
	DSS_10_E	DSS_100_E	DSS_1000_E	DSS_10_L	DSS_100_L	DSS_1000_L
σ_{amax}	85	85	85	110	110	110
H_{max}	1,0E+06	1,0E+05	1,0E+04	1,0E+06	1,0E+05	1,0E+04
H_{tot}	7,2E+08					
s	1					

These alternative design stress spectra were representations of the potential service at various operating condition indicators P and L₁. In this particular case, the relationship between the operating condition indicators is as follows:

$$P = \frac{L_d}{L_1 \cdot H_{tot}} = \frac{1.3889 \cdot 10^{-3}}{L_1} \quad (6)$$

FATIGUE LIFE CALCULATION: THEORETICAL BACKGROUND

The mostly the fatigue damage D is calculated using the linear cumulative damage rule. According to this rule, the limit state with respect to fatigue is reached (i.e. the fatigue life of the structural part is exhausted) when the following condition is met:

$$D = \sum_i \frac{n_i}{N_i} = D_{lim} \quad (7)$$

- D - fatigue damage caused by the stress spectrum imposed,
- n_i - number of cycles applied at the i-th level of stress with the amplitude σ_{ai},
- N_i - limit life under identical loading (the number of cycles derived from the S-N curve for the part in question at the amplitude σ_{ai}),
- D_{lim} - limit value of fatigue damage.

Various rules apply various boundary conditions to fatigue damage calculation. A schematic representation of these boundary conditions is shown in Figure 6.

Account is taken of the damage caused by cycles with small amplitudes (σ_{ath} < σ_{ai} < σ_c), which occur very frequently. A threshold value σ_{ath} is applied to the conversion of stress to damage, and therefore the damage caused by cycles with amplitudes of σ_{ai} < σ_{ath} < σ_c is neglected.

A limit value is set for the fatigue damage. According to Miner, D_{lim} = 1, but it has been shown experimentally that this value may decline towards hazardous levels D_{lim} < 1.

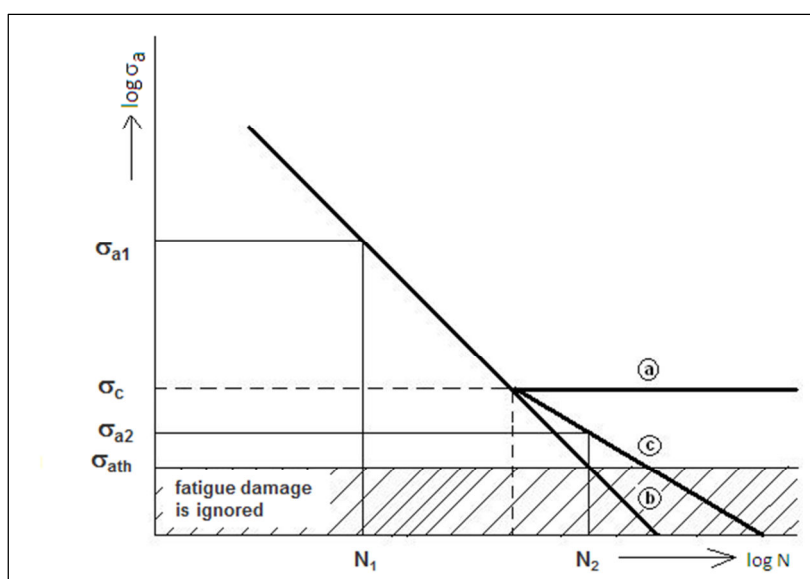


Fig. 6 - Boundary conditions for calculating cumulative fatigue damage

FATIGUE LIFE CALCULATION: CASE STUDY

In the present case, the Haibach-modified version of the Palmgren-Miner rule was chosen for calculating fatigue damage. The limit number of cycles N_i was determined as follows:

- for $\sigma_{ai} \geq \sigma_c$:
$$N_i = N_c \cdot \left(\frac{\sigma_c}{\sigma_{ai}}\right)^w \quad (8)$$

- for $\sigma_c > \sigma_{ai} \geq \sigma_{ath}$:
$$N_i = N_c \cdot \left(\frac{\sigma_c}{\sigma_{ai}}\right)^{wd} \quad (9)$$

Haibach recommends the exponent for the lower part of the S-N curve to be set as $wd = 2w - 1$. In this study, the value chosen was $w_d = 8$. The threshold stress amplitude for taking the resulting fatigue damage into account was given as $\sigma_{ath} = 0.5\sigma_c$ in the present case. In order to safely provide for calculation inaccuracy associated with the use of the cumulative fatigue damage rule, the limit value of fatigue damage was taken as $D_{lim} = 0.5$. The computational estimate of fatigue life (in km run) is then obtained from equation:

$$L = \frac{D}{D_{lim}} \cdot L_m \quad (10)$$

Predicted service life are collected in Table 5. The parametric calculations showed that profiles from X2CrNi12 are more suitable for construction of bodywork.

Table 5 - Calculated service life for design stress spectra

Calculated service fatigue life (km)						
Material	Design stress spectra - Empty vehicle			Design stress spectra - Loaded vehicle		
	DSS_10_E	DSS_100_E	DSS_1000_E	DSS_10_L	DSS_100_L	DSS_1000_L
SG235JR	252 000	909 000	3 010 000	60 000	192 000	562 000
X2CrNi12	543 000	2 106 000	7 522 000	118 000	399 000	1 248 000

CONCLUSIONS: PROBABILISTIC INTERPRETATION OF SOME RESULTS

The real service stresses are measured by means of strain gauges. In this case, the service stress- time histories were measured for a municipal public transport vehicle riding on an irregular surface along a route whose total length was $L_m \approx 40$ km. The signals were analysed in the frequency domain, and insignificant high frequencies were eliminated. Finally, they were analysed using the rainflow technique in order to calculate fatigue damage and predict the fatigue service life of the structural detail. As part of this rainflow step, small cycles with amplitudes below 2 MPa were filtered out.

In high-cycle fatigue of welded structures, the mean stress of the cycle does not play a major role. Therefore, only the one-parameter stress spectra $\sigma_{ai}-n_i$ were used for subsequent calculations.

Figure 7 shows procedure of extrapolation of measured stress-time histories into so called measured stress spectra.

The measured stress spectra were considered sufficiently representative for the assessment of operational reliability (service fatigue life). When considering the scatter of fatigue properties of the evaluated structural node, it was possible to calculate fatigue life distribution function (FLDF, see Figure 1) for both materials considered (low carbon steel S235JR and stainless steel X2CrNi12) and for both load cases (empty and fully loaded vehicle).

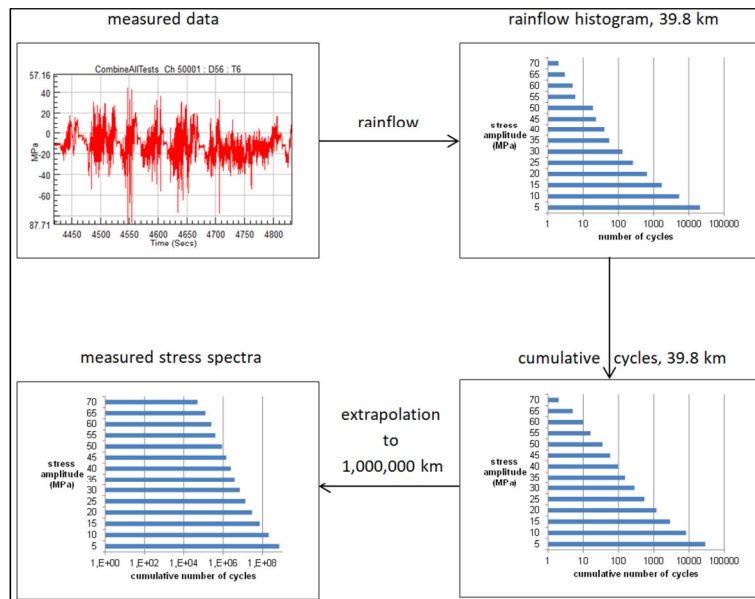


Fig. 7 - Procedure of extrapolation of measured stress spectra from the measured stress-time histories (empty vehicle)

The parts of calculated fatigue life distribution functions are shown in Figure 8. It can be seen that when S235JR steel is used, there is still a considerable risk (probability) of service failures. The use of X2CrNi12 steel, on the other hand, already guarantees the required service reliability (required service fatigue life).

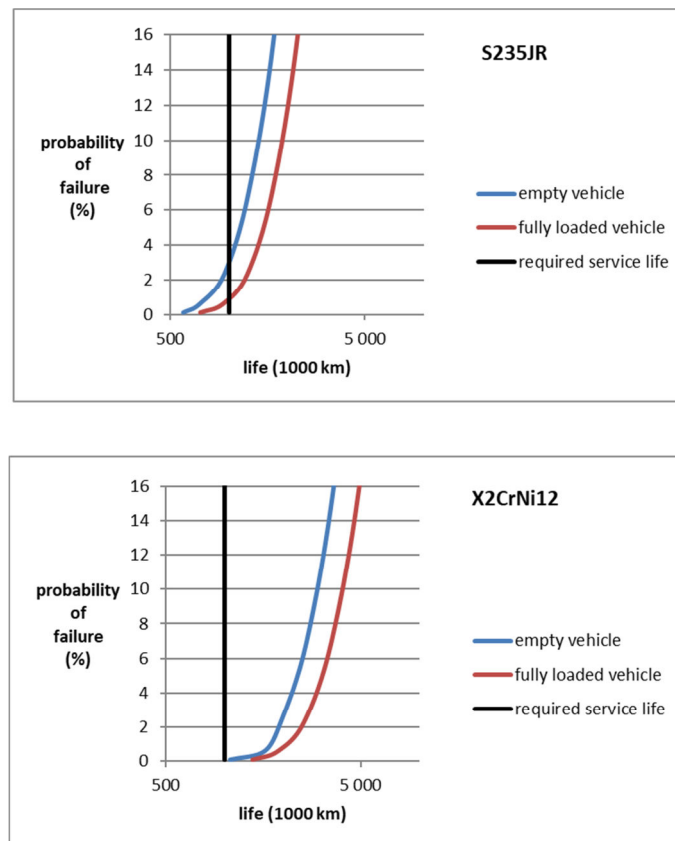


Fig. 8 - Fatigue life distribution functions

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