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# RATIONAL DESIGN OF CONDITIONS FOR RELIABILITY TEST

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## ABSTRACT

One of the basic tasks when preparing reliability tests is to assign properly accumulated test time, it means the sample size of tested items and test duration, with respect to the presumed reliability level of a tested item and a determined confidence level at which the results of the test are to be evaluated. The accumulated test time is to be assigned so that the aims of the test could be actually attainable. In the paper there are introduced basic parameters describing the course of a reliability test and the way of evaluating it. In doing so, main attention is paid to the test evaluation with the use of one-side confidence limits. In the article the effect of single test parameters on the course and evaluation of the test is analysed. Further to the results of this analysis, the author suggests possible methods of optimal test time assignment.

**Keywords:** reliability test, confidence level, time of test, reliability measure.

## INTRODUCTION

The common aim of reliability tests is to estimate a reliability measure which characterizes the reliability level of a tested sample. That measure might be Mean Time between Failures, Mean Time to Failure or failure rate. The fact that on the basis of test results we cannot determine the true value of a relevant measure which represents all population, but only estimate it poses a certain problem. The reason is that during the test only a certain sample of all population is always used and test data are usually censored (by the number of failures or time). If the result of a test is qualified with the use of point estimation as the average value of an observed variable of a relevant sample, then it has only a limited informative value. The thing is that the performed estimation of a relevant reliability measure does not provide any information about the relation between an estimated measure value and the true (unknown to us) value, so in fact we do not know how accurate this performed estimation is.

This lack of precision might be compensated by using confidence limits which enable us to determine the value intervals of a relevant measure in which there is its true value with probability selected in advance. The unquestioned advantage of applying this approach is that a test might be evaluated at the confidence level set in advance (Wasserman, 2003).

However, the application of confidence levels always brings certain difficulties while preparing reliability tests, namely when assigning accumulated test time, i.e. test duration and a sample size. These parameters affect significantly the course of the test and its evaluation, and if set incorrectly, the expected aims of the test might not be achieved. In the article there is an analysis of how test time influences the course of the reliability test and its evaluation, and possible procedures for optimal setting this test parameter are introduced.

It is presumed in the introduced analysis that a required reliability measure which is supposed to be evaluated during a reliability test is a Mean Time to Failure $m$ and that the evaluation of lower one-side confidence limit $m_{L1}$ is required. Similarly, the suggested procedures might be used also for evaluating other reliability measures and other types of confidence limits.

Because of a limited range of the article, a random variable – time to failure which is observed during the test, is presumed to have an exponential distribution. However, the procedures introduced below can be similarly applied even if a random variable will follow another distribution.

## LOWER ONE-SIDE CONFIDENCE LIMIT

The lower one-side confidence limit on mean time to failure $m_{L1}$ can be calculated using the following equation (IEC 60605-4)

$$m_{L1} = \frac{2T^*}{\chi^2_{1-\alpha}(\nu)}, \quad (1)$$

where $T^*$ is accumulated test time and $\chi^2_{1-\alpha}(\nu)$ denotes (1-$\alpha$) fractile of the cumulative $\chi^2$ distribution with $v$ degrees of freedom. And the number of the degrees of freedom $v$ depends on the number of failures $r$ recorded during the test, the way of terminating the test, and whether tested items are replaced after the failure or not. The number of the degrees of freedom is in single cases expressed by the following equations (IEC 60605-4),

for time terminated tests with replacement

$$\nu = 2r + 2, \quad (2)$$

for time terminated tests without replacement

$$\nu = 2r + 1, \quad (3)$$

and for failure terminated tests

$$\nu = 2r. \quad (4)$$

It is assumed in further analysis that the test is performed as a time terminated test with replacement. It means that the test time $t_d$ (the time clocked from the beginning of the test to the termination of the test) is assigned before the test starts, and when it elapses, the test is terminated. It also means that if the failure of an element occurs, the failed element is replaced by another one. The procedures introduced below might be applied in a similar way even if the test is carried out without replacement, or is terminated by a failure.

The course of the test is characterized mainly by the data which directly go in the evaluation of the test. It results from the equation (1) that the accumulated test time $T^*$ along with the number of observed failures $r$ is primarily involved. Both these variables are interconnected.

Accumulated test time $T^*$ is given by the number of elements used in the test $N$ and test duration $t_d$. In the case of time terminated test with replacement for accumulated test time $T^*$ it applies

$$T^* = N\, t_d. \quad (5)$$

The number of failures observed during the test $r$ is affected by the inherent level of tested item reliability, i.e. by the true value of mean time to failure $m$ and accumulated test time $T^*$. The formula stated below applies for the expected number of failures during the test (Kececioglu, 2002)

$$\bar{r} = \frac{T^*}{m} . \tag{6}$$

It results from the equation (5) that the accumulated test time $T^*$ is given in advance by basic test parameters, i.e. the number of tested items $N$ and test duration $t_d$, but the extent of the test is a lot better characterized by the expected number of failures which makes $T^*$relative in relation to the inherent reliability of a tested item described by the parameter $m$.

With the use of equations (1), (2) and (6) it might be derived the relation which describes the ratio of a lower one-side confidence limit $m_{L1}$ to the true value of mean time to failure $m$

$$\frac{m_{L1}}{m} = \frac{2\bar{r}}{\chi^2_{1-\alpha}(2r+2)} . \tag{7}$$

In Fig. 1 there is a graph representing the dependence of this ratio on the number of failures $r$ during the test for different confidence levels. The figure shows that for the small number of failures $r$ the value $m_{L1}$ is significantly lower than the value $m$, and the higher the confidence level, the higher the difference between both values. With the growing number of failures the value of the ratio has been asymptotically getting to 1.

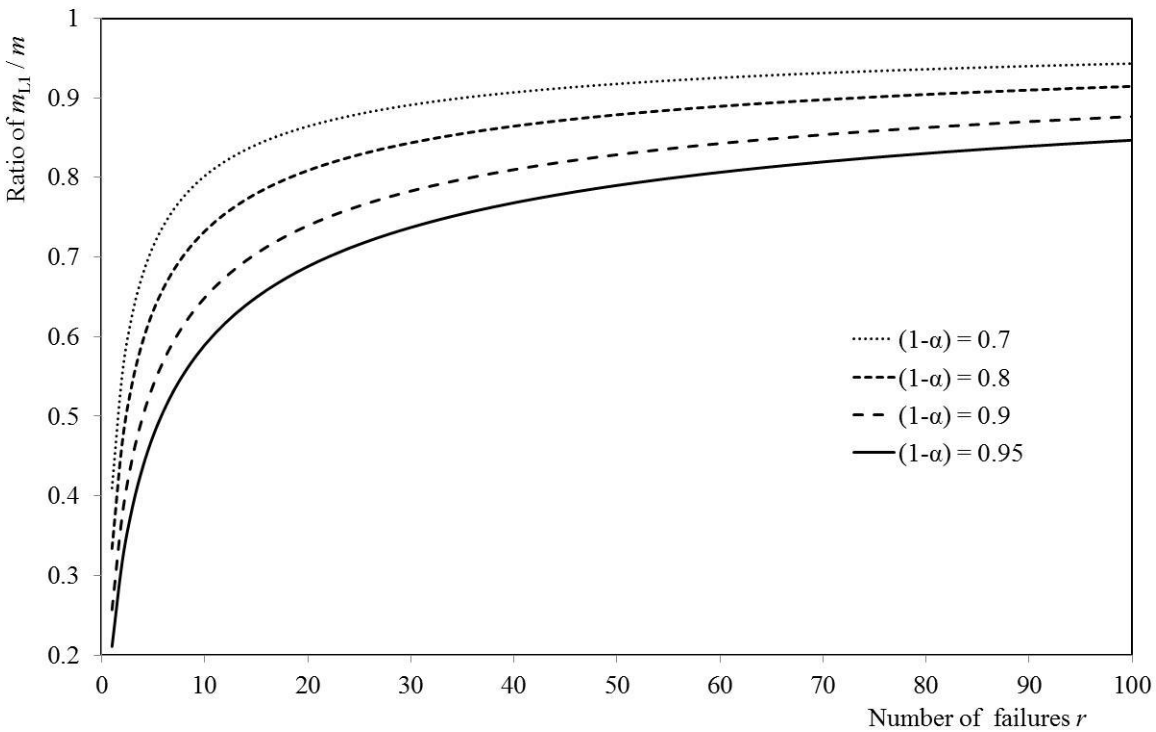


Fig. 1 Influence of number of failures on a value of the lower one-side confidence limit

## VERIFICATION OF RELIABILITY LEVEL USING CONFIDENCE LIMIT

The evaluation of reliability test results with the use of one-side confidence limit is often applied in practice as a tool for assessing whether a tested item has reached a determined reliability level (MIL-HDBK-781A). In this case, as part of the test, the evaluated parameter is required to prove with the determined confidence that it has reached a required value $m_R$. If the requirement is specified this manner, it is not the true value of a relevant parameter $m$

which is verified, but its lower one-side confidence limit. The aim of the test then is to verify whether the following condition applies

$$m_{L1} \geq m_R \,. \tag{8}$$

It results from this requirement and the dependencies put in Fig. 1 that if achieving a required value $m_R$ with certain confidence is required to be proved, the true value of mean time to failure $m$ should be significantly higher than this required value.

If the conditions under which the test is to be carried out have been determined, i.e. a required level $m_R$ the achievement of which is to be proved during the test with the specified confidence (1-$\alpha$) and the accumulated test time $T^*$ (or test time $t_d$ and the number of tested items $N$), the allowed maximum number of failures $a$ which can occur during the test might be determined. The requirement is met if the following equation applies (Kececioglu, 2002)

$$r \leq a \,. \tag{9}$$

The allowed maximum number of failures can be determined using the equations (1), (2) and (8) according to the following formula

$$a = \max_{r \in \mathrm{N}_0} \left\{ r \;\middle|\; m_R \leq \frac{2T^*}{\chi^2_{1-\alpha}(2r+2)} \right\}, \tag{10}$$

where $\mathrm{N}_0$ is a set of natural numbers.

For the needs of further analysis there will be used a relative test time $T_{REL}$ which expresses the ratio of accumulated test time $T^*$ to the required value of mean time to failure $m_R$:

$$T_{REL} = \frac{T^*}{m_R} \,. \tag{11}$$

With the use of formula (8), the equation (9) might be then adapted this way:

$$a = \max_{r \in \mathrm{N}_0} \left\{ r \;\middle|\; T_{REL} \geq \frac{\chi^2_{1-\alpha}(2r+2)}{2} \right\}. \tag{12}$$

In the graph in Fig. 2 there has been put the dependence of the allowed number of failures $a$ on relative test time $T_{REL}$ for different confidence levels (1-$\alpha$).

## SETTING TEST CONDITIONS

The graph in Fig. 2 shows that there is always certain minimum relative test time for each confidence level, and fulfilling the value $m_R$ can be proved during this time only on condition that no failure occurs during the test. The length of relative test time $T_{REL}$ in this specific case ($a$ = 0) for different confidence levels is put in the table below.

Table 1 Minimum relative test times

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **(1-$\alpha$)** | 0.6 | 0.7 | 0.8 | 0.9 | 0.95 | 0.975 | 0.99 |
| $\chi^2_{1-\alpha}(2)$ | 1.8 | 2.4 | 3.2 | 4.6 | 6.0 | 7.4 | 9.2 |
| $T_{REL}$ | 0.9 | 1.2 | 1.6 | 2.3 | 3.0 | 3.7 | 4.6 |

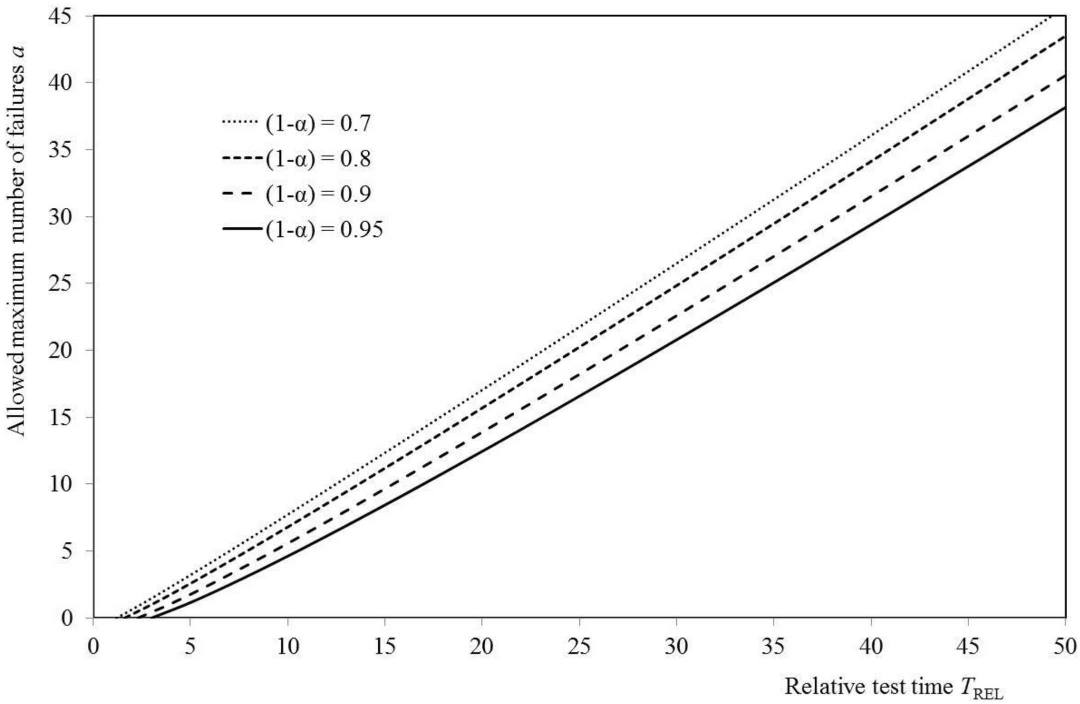


Fig. 2 Dependence of the maximum allowed number of failures on relative test time

It results from the information above that the relative test time has to be always longer than minimum time stated for a given confidence level in Table 1. The fact that during the test this condition will be met creates only theoretical presumption to meet the requirement on reliability (9), but in fact the procedure itself will not guarantee that this aim will be really achieved. For these reasons when planning the test time, it is also necessary to assess a real chance of a tested item under given conditions to succeed during the test and meet the requirement (9).

For this purpose it is necessary to determine a true value of mean time to failure *m*. For instance a design value of a parameter or information about the reliability of similar items from operation or tests can be adopted. The probability of item acceptance P(A) during the test might be set then by applying the Poinsson distribution (Tobias, 2012; Kececioglu, 2002)

$$\mathrm{P(A)} = \mathrm{P}(r \le a \mid m) = \sum_{k=0}^{k=a} \frac{\bar{r}^k \mathrm{e}^{-\bar{r}}}{k!} \,. \tag{13}$$

The expected number of failures can be expressed with the use of equations (6) and (11) in the formula below

$$\bar{r} = T_{\mathrm{REL}} \frac{m_R}{m} \,. \tag{14}$$

The formula (13) enables us to determine what the probability of meeting requirement (9) by an item can be, provided that its true value of mean time to failure equals actually *m.* In Fig. 3 there is the dependence of this probability P(A) on a relative test time $T_{\mathrm{REL}}$, providing the allowed maximum number of failures *a* is for each considered value $T_{\mathrm{REL}}$ determined according to the formula (12).

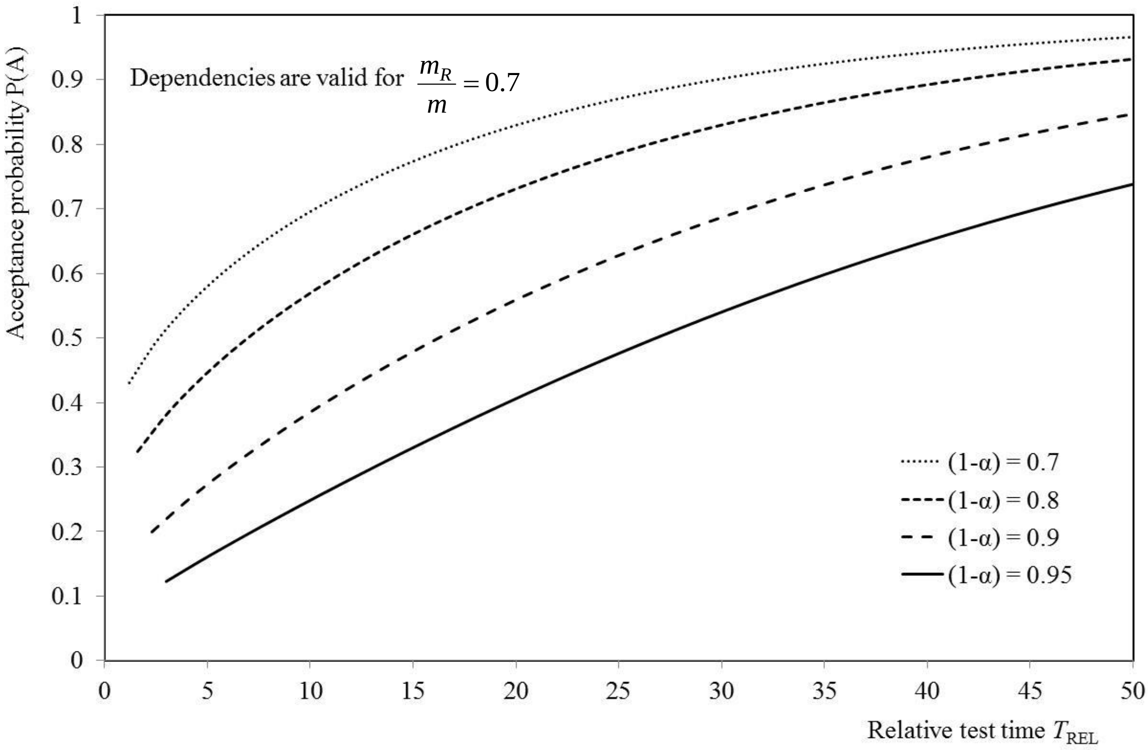


Fig. 3 Dependence of acceptance probability a relative test time

Similarly, when applying equation (13), it is possible to calculate the dependency of acceptance probability on the ratio of a required value $m_R$ to a true value $m$. The Fig. 4 can serve as an example of this dependence for $T_{REL} = 10$ and for different values of the allowed maximal number of failures $a$.

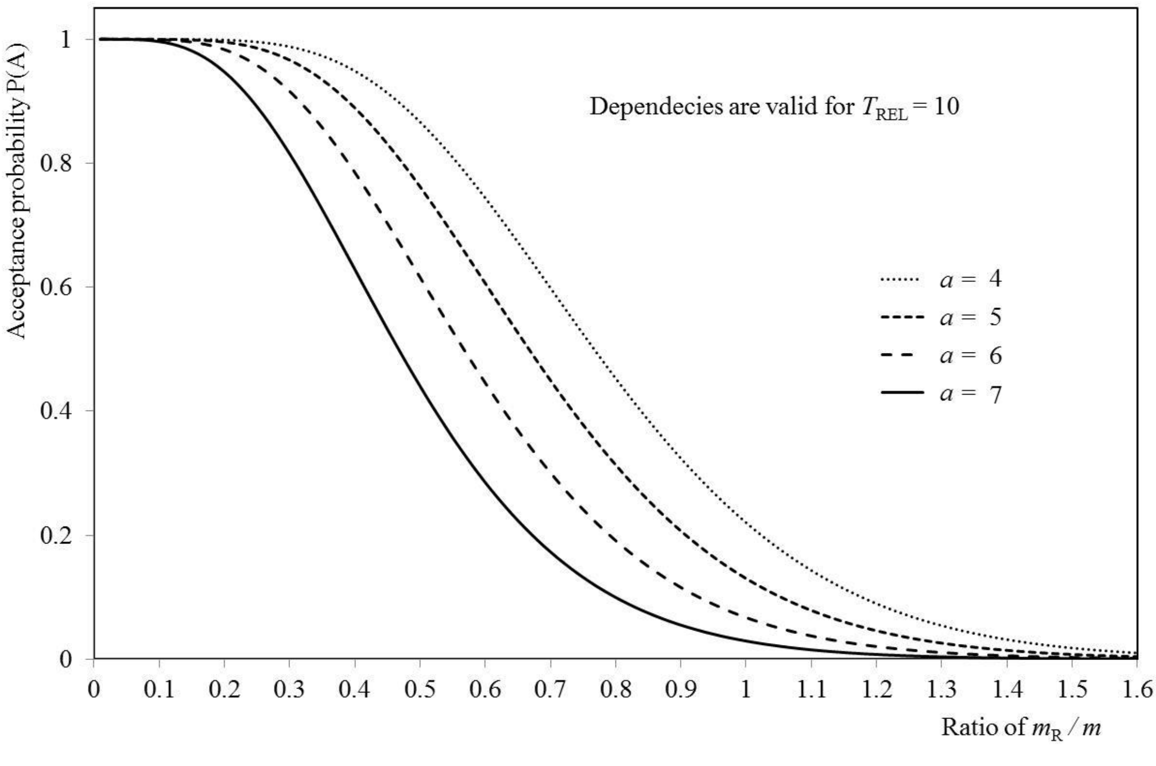


Fig. 4 Dependence of acceptance probability on a true value of reliability measure.

Following the information introduced above, an optimal procedure for setting test conditions might be formulated. The procedure is based on the assumption that a required reliability level $m_R$ as well as a confidence level (1-$\alpha$) which is to be fulfilled is determined. There are three procedures that might be used.

The first procedure results from the assumption that the true level of item reliability $m$ is known and the aim of the procedure is to assign such test time $T^*$ during which meeting determined requirements can be proved with the probability P(A) set in advance. In this case, with the help of equations (12) and (13), a graph showing the dependence of probability P(A) on a relative test time $T_{REL}$ (see Fig. 3) will be plotted, and there will be determined for what relative test time a required probability P(A) has been achieved.

The second procedure is based on the presumption that there is a fixed test time $T^*$, and the aim of the procedure is to assign what the true reliability level $m$ of a tested item should be in order to succeed in a test with the probability P(A) set in advance. In this case, with the use of formula (12), it is possible to determine the value of the allowed number of failures $a$ for a relevant test time $T^*$. When using these values along with equation (13), the dependence of acceptance probability P(A) on value $m$ (see Fig. 4) is plotted and relevant value $m$ identified with respect to a required probability P(A).

The last procedure follows the assumption that there is a fixed test time $T^*$, and the true level of item reliability $m$ is known. The aim of the procedure is to specify what the probability P(A) that the item succeeds during the test might be. This probability can be determined by direct calculation with the use of equations (12) and (13).

## CONCLUSION

It results from the analysis developed above that the course of the test and its evaluation is affected mainly by the following parameters: a required reliability level $m_R$, a true reliability level $m$, confidence (1-$\alpha$) which is to prove that an item meets the requirement, and an accumulated test time $T^*$. These parameters predetermine to some extent whether the expected aim will be achieved, that is to say whether an item will succeed during the test.

Therefore it is necessary to pay particular attention to the question whether the true level of item reliability is high enough for an item to succeed during the test with acceptable probability. The actual course of the test and its evaluation is affected significantly by a determined accumulated test time, that is to say the size of a tested sample and test duration. Extending test time always increases the cost of the test performance (a demand for greater extent of a tested sample or prolonging test duration). On the other hand, however, this increases the probability of achieving the test aim. For that reason it is necessary to determine a test time so that, with respect to other test parameters, prerequisites for proving that a tested item meets set requirements could be made.

The condition that a true reliability level $m$ is always supposed to be higher than a required value $m_R$ is a basic prerequisite for performing a successful test (Vintr, 2009).

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## REFERENCES

IEC 60605-4 Equipment reliability testing - Part 4: Statistical procedures for exponential distribution - Point estimates, confidence intervals, prediction intervals and tolerance intervals. IEC, Geneva, 2001.

MIL-HDBK-781A Reliability Test Methods, Plans and Environments for Engineering Development, Qualification and Production. Department of Defense, Washington, 1996.

Kececioglu DB. Reliability & Life Testing Handbook. DEStech Publications, Lancaster, 2002.

Wasserman GS. Reliability Verification, Testing, and Analysis in Engineering Design. Marcel Dekker, New York, 2003.

Vintr Z. Rational Design of Conditions for Reliability Test. In: The Proceedings of 8th International Conference on Reliability, Maintainability and Safety – ICMRS 2009. IEEE, Piscataway, 2009.

Tobias PA, Trindade DC. Applied Reliability. CRC Press, Boca Raton, 2012.