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A DISPLACEMENT FORMULATION FOR CYLINDRICAL SHELLS UNDER RADIAL LOADS

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ABSTRACT

A displacement formulation for cylindrical shells subjected to radial loads is presented. Static loads are applied to straight thin pipes using a variational method and results are compared both to finite element analysis and a shell element from Ansys program. The formulation includes a combination of trigonometric functions in two variables, longitudinal and circumferential direction. Then a system of differential equations is solved for two different load cases and the vertical displacement is obtained in the points where the load is applied.

Keywords: cylindrical shell, trigonometric functions, point load.

INTRODUCTION

The basic assumptions in the present analysis of the deformation of straight pipes are that the shell is thin and the transversal section is inextensible, not including pressure effects.

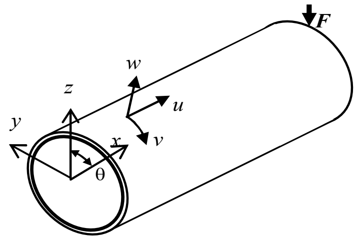

Fig. 1 - Shell displacements

Considering the parameter x as longitudinal and θ as circumferential, u the longitudinal displacement, v the circumferential displacement and w the transversal displacement, kinematics expressions relating the mid-surface strains to the displacements are:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = 0$$

$$\varepsilon_{\theta\theta} = \frac{1}{r}\left(\frac{\partial v}{\partial \theta} + w\right) = 0 \tag{1}$$

$$\gamma_{x\theta} = \frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x} = 0$$

while the curvatures and twist are expressed by:

$$\chi_{\theta\theta} = \frac{1}{r^2}\left(-\frac{\partial v}{\partial \theta} + \frac{\partial^2 w}{\partial \theta^2}\right)$$

$$\chi_{x\theta} = \frac{2}{r}\left(-\frac{\partial v}{\partial x} + \frac{\partial^2 w}{\partial x \partial \theta}\right) \tag{2}$$

The total energy is defined by the following expression:

$$\frac{1}{2}\int_0^{2\pi}\int_0^{L} Dk^2_{\theta\theta} + \frac{D(1-\nu)}{2}k^2_{x\theta}\, r dx d\theta \tag{3}$$

The displacement is formulated using odd functions in θ combined with different functions in x. Several cases for x-functions are considered. The geometric parameters are the pipe length L, the thickness h and the radius r. The material is steel and load cases are:

Case 1: a point load in the pipe end,

Case 2: pinched forces applied in the middle of the pipe.

RESULTS AND CONCLUSIONS

A total of 2 different approaches were tested for both load cases. The pipe dimensions are represented in table 1.

Table 1 - Pipe dimensions

Length, m	Radius, m	Thickness, m
0.40 0.2	0.159945	0.00335 0.00396 0.00457

The results show good agreement with other published solutions using multi-nodal or finite element analysis, as represented in table 2. The deformation is larger when the pipe length increases and the ring defection is smaller when the pipe thickness increases.

Table 2 - Vertical displacement for h=0.00335m (Case 2)

Different formulations	L=0.2m	L=0.4m
Finite element	0.0151	0.0242
Multi-nodal	0.0140	0.0230
Variational	0.0142	0.0233

This study shows that the functional approach with minimization of the total energy leads to the same results as compared with finite elements, as published by (Fonseca et al, 2011).

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