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A NONLINEAR SOIL CONSTITUTIVE MODEL BASED ON HYPERBOLIC TANGENT FUNCTION

Ding YuQin[1(*)], Zhang YongXing[2]
[1] Department of Civil Engineering, ZheJiang University City College, HangZhou, China
[2] College of Civil Engineering, ChongQing University, ChongQing, China
[(*)]*Email:* dingyq@zucc.edu.cn

ABSTRACT

The nonlinear soil characteristics have received much attention since the soil nonlinearity under strong motions was found. A new nonlinear soil constitutive model based on hyperbolic tangent function is presented. In which the shape of proposed backbone curve is the same as that of the hyperbolic tangent function curve shifted and enlarged by two factors. Such new model is expressed in a simple form and has a straightforward physical meaning. The only thing need to do for obtaining hysteresis loop is determination of original shear strain and shear stress values at the beginning of unloading and reloading. Results comparison with that carried out by the standard hyperbolic model show that it is feasible to use this new constitutive model in seismic ground response analysis. Practical significance is of interest to develop such nonlinear soil constitutive model.

Keywords: Nonlinear；soil constitutive model；hyperbolic tangent function

INTRODUCTION

Stable numerical integration techniques and realistic constitutive models of soil are the two important and necessary components of nonlinear seismic ground response analyses. Construction of soil constitutive models is basically involved in two ways: elasto-plastic constitutive modes and the nonlinear cyclic stress-strain models. The empirical rules that describe the loading and unloading paths in the stress-strain space are the Masing rules (Masing, 1926). Kondner (1963), Hardin and Drnevich (1972) developed the nonlinear hyperbolic model to model the stress strain behaviour of soil subjected to constant rate of loading. To avoid the limitation of shear stress increasing infinitely with increase in strain, Chen Guoxing and Zhuang haiyang (2005) modified the Davidenkov model.

Nonlinear cyclic stress-strain models were used to improve the numerical simulation by the equivalent linear method. Commonly, the hyperbolic or Ramberg-Osgood backbone curves with the Masing rule were used to construct loading, unloading and reloading portions of the stress-strain curves. However，some parameters in these models are difficult to determine and the computation is expensive. As a result, attempts have been made to derive a simple, straightforward and feasible new model to describe the nonlinear stress strain relationship.

This work is focused on the research on nonlinear constitutive models of soil. A new nonlinear stress-strain model of soil was created in which the backbone curve was constructed based on hyperbolic tangential function. This method is simple, straightforward. What most important is that the parameters are easily to determine and no parameter need to be recalculated for the hysteresis loop.

ABOUT THE STANDARD HYPERBOLIC TANGENT FUNCTION

The standard hyperbolic tangent function, y=tanh(x), is a famous hyperbolic trigonometric function. Fig. 1 shows the curve of hyperbolic tangent function. Obvious natures of the curve can be found as:

1) It has a similar shape with the common stress-strain backbone curve;
2) It is symmetric with respect to the origin of coordinates;
3) It approaches to two asymptotes of y=1 and y=-1;
4) It is continuous in (-∞, ∞) and thus can yields the continuous differential coefficient.

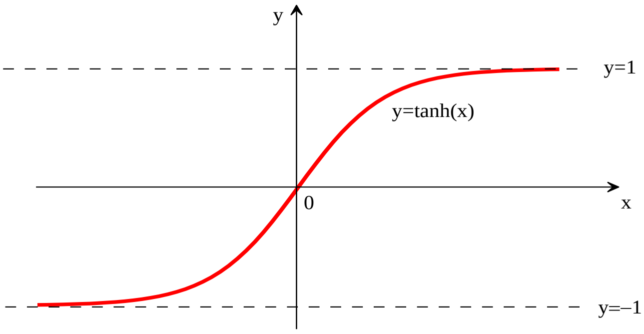

Fig. 1 Hyperbolic tangent function

The hyperbolic tangent function is perfectly satisfied with the basic features of soil stress-strain relationship. It is, therefore, possible to find a new type of backbone curve for the soil constitutive model based on hyperbolic tangent function.

CONSTRUCTION OF BACKBONE CURVE

Since the hyperbolic tangent function contains a good requirement of soil constitutive model, it yields a modified function which can better match the backbone curve for a special purpose. The shape of proposed backbone curve is the same as that of the hyperbolic tangent function curve shifted and enlarged by two factors:

$$y(x) = a\tanh(bx) \tag{1}$$

Based on the supposition above, a new form of backbone curve can be written according to curve-fitting method as:

$$\tau(\gamma) = a\tanh(b\gamma) \tag{2}$$

where *a* and *b* are the two coefficients of curve-fitting.

The sketch of backbone curve creation is illustrated in Fig. 2. The two coefficients *a* and *b* can be determined in accordance with the original shear modulus G_{max} and the maximum shear stress τ_{max} .

The original shear modulus, G_{max}, is the slope of backbone curve at origin point (0,0) of coordinates:

$$G_{max} = \frac{d\tau}{d\gamma}\Big|_{\gamma=0} = \frac{d}{d\gamma}\left[a\tanh(b\gamma)\right]\Big|_{\gamma=0} = ab \tag{3}$$

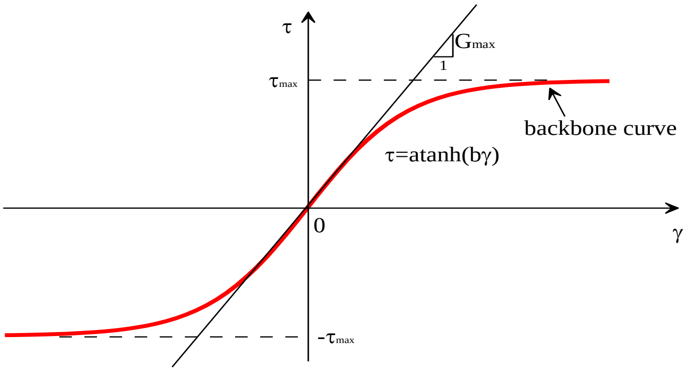

Fig. 2 Sketch of backbone curve

Maximum shear stress is the extremum of asymptote, it is defined as:

$$\tau_{\max} = \lim_{x \to +\infty} \tau(\gamma) = \lim_{x \to +\infty} a \tanh(b\gamma) = a \tag{4}$$

Substituting (4) into (3), it results in:

$$b = \frac{G_{\max}}{a} = \frac{G_{\max}}{\tau_{\max}} \tag{5}$$

Then the new expression of backbone curve is carried out by substituting equations (4) and (5) into Eq.(2):

$$\tau(\gamma) = \tau_{\max} \tanh\left(\frac{G_{\max}}{\tau_{\max}} \gamma \right) \tag{6}$$

CONSTRUCTION OF NONLINEAR SOIL CONSTITUTIVE MODEL

The general principle of constructing the hysteresis loop function is related to that of the backbone curve. The calculation of the unloading and reloading curve is basically dependent on the shift rule of backbone curve.

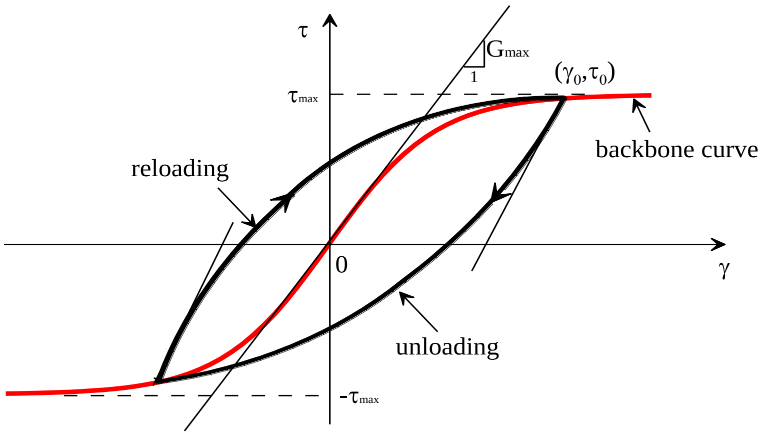

Fig. 3 Sketch of hysteresis loop

To construct the tress-strain curve of unloading portion, as shown in Fig.3, given point (γ_0, τ_0) is the origin point of unloading portion of hysteresis loop, then the unloading curve approaches the asymptote of $\tau = -\tau_{\max}$. The expression of unloading curve is:

$$\tau(\gamma) = \tau_0 + a_1 \tanh\left[b_1 (\gamma - \gamma_0) \right] \tag{7}$$

Maximum shear stress is defined as:

$$\tau = \lim_{x \to -\infty} \tau(\gamma) = -\tau_{\max} \tag{8}$$

Then

$$-\tau_{\max} = \lim_{x \to -\infty} \left\{ \tau_0 + a_1 \tanh\left[b_1 \left(\gamma - \gamma_0 \right) \right] \right\} = \tau_0 - a_1 \tag{9}$$

So the coefficient a_1 can be written as:

$$a_1 = \tau_{\max} + \tau_0 \tag{10}$$

The shear modulus of unloading portion at point (γ_0, τ_0) is the maximum value, which has the definition as:

$$G_{\max} = \frac{d\tau}{d\gamma}\Big|_{\gamma=\gamma_0} = \frac{d}{d\gamma}\left\{ \tau_0 + a_1 \tanh\left[b_1 \left(\gamma - \gamma_0 \right) \right] \right\}\Big|_{\gamma=\gamma_0} = a_1 b_1 \tag{11}$$

From equations (10) and (11), the coefficient b_1 can be obtained:

$$b_1 = \frac{G_{\max}}{\tau_{\max} + \tau_0} \tag{12}$$

Substituting equations (10) and (12) into Eq. (7), yields the expression of unloading curve:

$$\tau(\gamma) = \tau_0 + \left(\tau_{\max} + \tau_0 \right) \tanh\left[\frac{G_{\max}}{\tau_{\max} + \tau_0} \left(\gamma - \gamma_0 \right) \right] \tag{13}$$

The same procedure for computing reloading curve is undertaken as that for unloading curve. yields the expression of reloading curve:

$$\tau(\gamma) = \tau_0 + \left(\tau_{\max} - \tau_0 \right) \tanh\left[\frac{G_{\max}}{\tau_{\max} - \tau_0} \left(\gamma - \gamma_0 \right) \right] \tag{14}$$

So far, the backbone curve, unloading and reloading curves are built based on the hyperbolic tangent function. This new nonlinear stress-strain model of soil is so called NHT model (nonlinear hyperbolic tangent model).

COMPARISON OF NHT MODEL WITH STANDARD HYPERBOLIC MODEL

From the afore mentioned NHT model, it’s normalized secant shear modulus can be obtained in the form of Eq.(15) according to the upper bound of backbone curve.

$$\frac{G}{G_{\max}} = \frac{\gamma_r}{\gamma} \tanh\left(\frac{\gamma}{\gamma_r} \right) \tag{15}$$

where γ_r is the reference shear strain.

As we know before, the initial loading curve of soil constitutive model is given by the hyperbolic relation, the corresponding normalized secant shear modulus of such standard hyperbolic model can be written as:

$$\frac{G}{G_{\max}} = \frac{1}{1 + \dfrac{\gamma}{\gamma_r}} \tag{16}$$

In order to check the accuracy and validity of NHT model, two types of soil with different shear velocity (V_s=360, 500m/s, respectively) is selected. The maximum shear modulus can, however, be assumed to compute by $G_{max} = \rho V_s^2$, where ρ is mass density, and V_s is the shear velocity of soil.

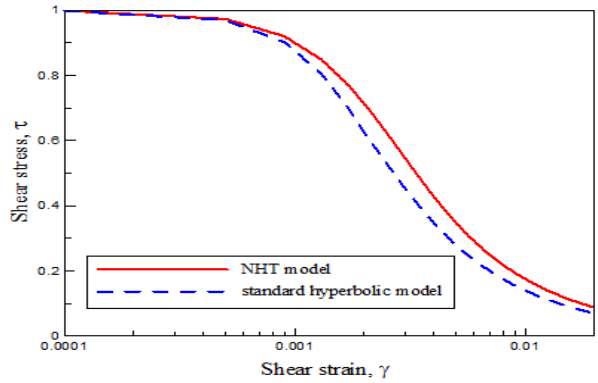

Fig. 4 Comparison of normalized secant shear modulus for soft soil

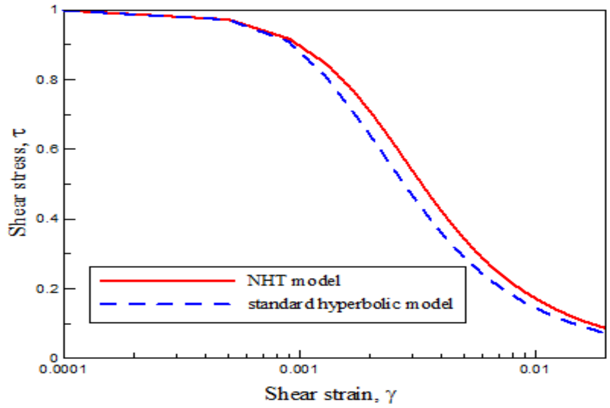

Fig. 5 Comparison of normalized secant shear modulus for stiff soil

Results are presented for the two selected soil, as shown in Fig.4 and Fig.5. In which the normalized secant shear modulus for the two selected soil obtained from NHT model (red bold line) and from the standard hyperbolic model (blue dashed line) is compared.

Significant features are visible in the figures between the two models: First, the shape of modulus reduction curves obtained by the two models is similar. Both of red bold line and the blue dashed line reduce with the increasing of shear strain. When the shear strain γ is less than 0.001, the reduction changes slightly, while γ ranges from 0.001 to 0.01, the curves reduce sharply. Second, the modulus reduction curves for NHT model and standard hyperbolic model are consistent with each other when γ is less than 0.001. That is to say using NHT model can have same normalized secant shear modulus results as those by standard hyperbolic model at small to moderate shear strain values. In addition, it is obvious that at large shear strain, the NHT results are higher than those calculated by standard hyperbolic model. The NHT model overestimates the results.

RESULTS AND CONCLUSIONS

This paper presents a new nonlinear soil constitutive model based on hyperbolic tangent function. In which the shape of proposed backbone curve is the same as that of the hyperbolic tangent function curve shifted and enlarged by two factors. Such new model is expressed in a simple form and has a straightforward physical meaning. The only thing need to do for obtaining hysteresis loop is determination of original shear strain and shear stress values at the beginning of unloading and reloading. Results comparison with that carried out by the standard hyperbolic model show that using NHT model can have same normalized secant shear modulus results as those by standard hyperbolic model at small to moderate shear strain values. In addition, it is obvious that at large shear strain, the NHT results are higher than those calculated by standard hyperbolic model. The NHT model overestimates the results.

It is, therefore, concluded that in comparison with the standard hyperbolic model, NHT model is feasible and valid at small to moderate shear strain. For the large shear strain, NHT model will overestimate the results. High accuracy to use NHT model at small to medium shear strain is validated.

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