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RELIABILITY ESTIMATION OF AIRCRAFT STRUCTURAL COMPONENTS WITH SELECTED FAILURE MODELS

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ABSTRACT

In the paper the way of assessing the reliability of aircraft structural components in the case of a failure attributable to the surface wear and a fracture of this component has been presented. Some elements of the random-walk theory have been used to formulate a model to assess reliability of the structural component. The paper covers some specific density functions for the component's surface wear-out. In this part the model has been based on a difference equation that, after rearrangement, results in a partial differential equation of the Fokker-Planck type. A density function of the component's surface wear-out is a solution to this equation. In the second part of the paper the model has been generalised and enriched with the probability of a catastrophic failure to the component. What has resulted is a generalised equation of the Fokker-Planck type. With the solution to this equation applied, a dependence to estimate the reliability of the aircraft structural component has been arrived at for the case the failure results from the surface wear, with a catastrophic failure, i.e. the component's fracture, also possible. To conclude, a numerical example has been provided with a structural component in the form of an aircraft tyre used to illustrate the case.

Keywords: Fokker-Planck equation, wear and tear, catastrophic failure, failure mode

INTRODUCTION

The reliability assessment of structural components throughout the process of system's operation is closely related to the predicted health/maintenance status of these components. Most often, this assessment is described with diagnostic parameters. Any change in the item's health/maintenance status results from changes in values of the diagnostic parameters. The nature of loads, i.e. of destructive processes affecting the structural components, provokes these changes. Many and various processes play a decisive role in the changes of values of the diagnostic parameters of machine components, just to mention the wear-and-tear, fatigue corrosion, etc. All these processes may produce changes in dimensions, strength, etc. of the components. Recognition of the physics of all these processes and the analytical description thereof both prove very difficult because of the complexity of the issues arising. To make the description of effects of destructive processes easier, simplifications are a natural and quite often used technique (Risken, 1984), (Tomaszek, 2001). What is intended to present in the paper, is the reliability estimation of a structural component affected by the wear-and-tear processes resulting in the loss of weight and changes in dimensions, and what is more, the component's strength being affected as well. It has also been assumed that an aircraft tyre would be a good example meeting the above-listed requirements. Furthermore, some elements of the random-walk theory have been used to assess reliability of the structural component in question.

The following assumptions have been formulated:

- The structural component's health/maintenance status has been determined with one predominant diagnostic parameter „z" in the form of deviation from the nominal (rated) value:

$$z = | X - X^{nom} |, \tag{1}$$

where: X - current value of the diagnostic parameter,
X^{nom} - nominal (rated) value of the diagnostic parameter.

- The load affecting the component is discrete in its nature. Good examples of such a load are loading pulses that affect the aircraft landing gear during the landing.
- The change in value of deviation of the diagnostic parameter occurs mainly during the aircraft landing.
- Parameter „z" is a non-decreasing one.
- Value of deviation of the diagnostic parameter „z" determines the reliability status of the structure. If it remains within the interval:

$$z \in [0, z_d], \tag{2}$$

  then the component will be acknowledged as a fit-for-use (serviceable) one. Otherwise, the component will be recognised as an unfit-for-use (unserviceable) one.
- The measurement of deviation of the diagnostic parameter is taken in the discrete-time with a step h, where:

$$h = \frac{X_{nom}}{\hat{k}}, \tag{3}$$

  with $\hat{k}$ appropriately matched.
- Deviation of the diagnostic parameter takes discrete values in the form:

$$z_k = k \cdot h, \tag{4}$$

  where: $k = 0, 1, 2, 3 \ldots$
- The diagram showing changes with time in values of the diagnostic-parameter deviation has been presented in Figure 1.

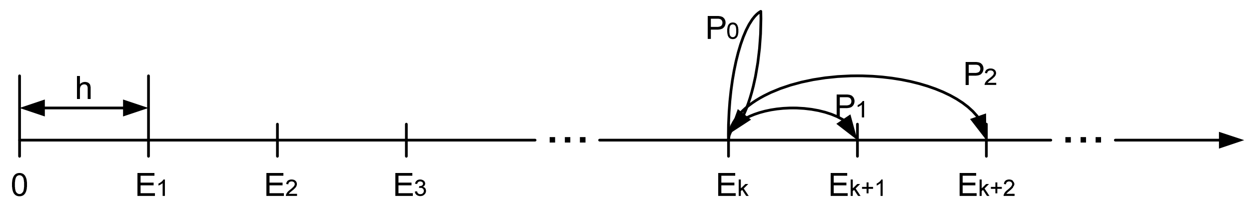


Figure 1. The diagram of changes in the deviation of the diagnostic parameter

Where: $P_0 + P_1 + P_2 \cong 1$;
E_k - the component's status ($k = 0, 1, 2, \ldots,$);
P_i - probability of the component's status.

DETERMINATION OF THE DENSITY FUNCTION OF STRUCTURAL COMPONENT'S WEAR-OUT WITH ELEMENTS OF THE RANDOM-WALK THEORY APPLIED

Therefore, what has been assumed in the paper is that, the wearing-out of a structural component manifests itself both in the loss of the component's weight and changes in its linear dimensions. Another assumption is that the load affecting the component in the course of its operation consists of randomly generated short-time pulses, the number of which is described with a process that proceeds at the rate λ:

$$\lambda = \frac{P}{\Delta t}, \tag{5}$$

where: P - probability of using the component in time Δt; Δt - time interval when the load pulse occurs.

The pulse rate may be identified with the rate of aircraft flights. It can be assumed that:

$$(1 - \lambda\Delta t) + \lambda\Delta t \approx 1\,. \tag{6}$$

Due to the load pulses affecting the component some discrete increment in the wear-and-tear value occurs (there is a random process after states $E_0, E_1, E_2,\ldots, E_k,\ldots$). It has also been assumed that a single pulse may cause some increment in the wear-and-tear, with some suitable probability $P_0, P_1, P_2, P_3,\ldots$.Other possibilities have been assumed to be only slightly probable. Hence,

$$P_0 + P_1 + P_2 + \ldots + P_k \cong 1\,. \tag{7}$$

Let $U_{k,t}$ denote the probability that the deviation of the diagnostic parameter at time instance t has reached the state k. For the above listed assumptions one can write down a difference equation that describes the way the deviation of the diagnostic parameter increases:

$$U_{k,t+\Delta t} = (1 - \lambda\Delta t)U_{k,t} + \lambda\Delta t P_0 U_{k,t} + \lambda\Delta t P_1\, U_{k-1,t} + \lambda\Delta t P_2\, U_{k-2,t}\,, \tag{8}$$

where:

$$(1 - \lambda\Delta t) + \lambda\Delta t P_0 + \lambda\Delta t P_1 + \lambda\Delta t P_2 \approx 1\,. \tag{9}$$

Equation (8) in the functional notation takes the following form:

$$u(z,t+\Delta t) = (1 - \lambda\Delta t)u(z,t) + \lambda\Delta t P_0\, u(z,t) + \lambda\Delta t P_1\, u(z-h,t) + \lambda\Delta t P_2\, u(z-2h,t)\,. \tag{10}$$

The difference equation (10) can be rearranged in a partial differential equation, with the following approximations applied (Risken, 1984):

$$\left.\begin{aligned} &u(z,t+\Delta t) \cong u(z,t) + \frac{\partial u(z,t)}{\partial t}\Delta t\,;\\ &u(z-h,t) \cong u(z,t) - \frac{\partial u(z,t)}{\partial z}h + \frac{1}{2}\frac{\partial^2 u(z,t)}{\partial z^2}h^2\,;\\ &u(z-2h,t) = u(z,t) - \frac{\partial u(z,t)}{\partial z}2h + \frac{1}{2}(2h)^2\frac{\partial^2 u(z,t)}{\partial z^2}\,; \end{aligned}\right\}. \tag{11}$$

Having made substitution in equation (10) with approximations (11) we arrive at what follows:

$$\frac{\partial u(z,t)}{\partial t} = -\lambda(P_1 h + P_2 2h)\frac{\partial u(z,t)}{\partial z} + \frac{1}{2}\lambda(P_1 h^2 + P_2 (2h)^2)\frac{\partial^2 u(z,t)}{\partial z^2}. \tag{12}$$

Notation:

$$b = P_1 h + P_2 2h, \tag{13}$$

$$a = P_1 h^2 + P_2 (2h)^2. \tag{14}$$

Hence,

$$\frac{\partial u(z,t)}{\partial t} = -\lambda b \frac{\partial u(z,t)}{\partial z} + \frac{1}{2}\lambda a \frac{\partial^2 u(z,t)}{\partial z^2}. \tag{15}$$

Equation (15) takes the form of the Fokker-Planck equation (Risken, 1984).

Let us find a particular solution to equation (15), such as to satisfy the following conditions: when $t \to 0$, the solution is convergent with the Dirac delta function, i.e. $u(z,t) \to 0$ for $z \neq 0$ and $u(0,t) \to +\infty$, in such a way, however, that the function integral u equals unity for all $t > 0$. The solution to equation (15) with the above-formulated conditions takes the following form (Risken, 1984), (Tomaszek, 2008):

$$u(z,t) = \frac{1}{\sqrt{2\pi a \lambda t}} e^{-\frac{(z-b\lambda t)^2}{2a\lambda t}}. \tag{16}$$

Dependence (16) is the searched for probability density function of the structural component's wear-out due to randomly occurring destructive pulses. Since the number of pulses (aircraft landings) is described with the following relationship:

$$N = \lambda t, \tag{17}$$

the density function of the structural component's wear-out can be written down in the following form:

$$u(z,N) = \frac{1}{\sqrt{2\pi a N}} e^{-\frac{(z-bN)^2}{2aN}}, \tag{18}$$

where: bN - the expected value of deviation of the diagnostic parameter at time instance t when the number of load pulses is N; aN - value of variance in deviation of the diagnostic parameter up to the time instance t when the number of load pulses is N.

To use the density function (18), one should estimate parameters a and b. Data needed to use formulas (13, 14) are not always available. Another way to estimate these parameters is to use the likelihood function. The following product is called the likelihood function:

$$L = u(N_0, z_0; b, a) \prod_{k=0}^{n-1} u(N_k, z_k, N_{k+1}, z_{k+1}; b, a), \tag{19}$$

where: $(z_0, z_1, z_2, \ldots, z_n)$ will be observed values of deviation of the diagnostic parameter from the nominal value for the number of load cycles $(N_0, N_1, N_2, \ldots, N_n)$.

Estimation of unknown parameters b and a is arrived at by means of solving the following system of equations:

$$\left.\begin{aligned}\frac{\partial \log L}{\partial b}=0\\ \frac{\partial \log L}{\partial a}=0\end{aligned}\right\}. \tag{20}$$

Calculation formulae take the following forms:

$$b^*=\frac{z_n}{N_n}, \tag{21}$$

$$a^*=\frac{1}{n}\sum_{k=0}^{n-1}\frac{[(z_{k+1}-z_k)-b*(N_{k+1}-N_k)]^2}{(N_{k+1}-N_k)}. \tag{22}$$

With b^* and a* parameters estimated one can write down a formula for the reliability of the structural component (Tomaszek, 2008):

$$R(N)=\int_{-\infty}^{z_d}u(z,N)dz, \tag{23}$$

where: $u(z,N)$ is given with equation (18) and z_d - the permitted value of deviation of the diagnostic parameter.

A MODEL OF THE STRUCTURAL COMPONENT'S WEARING-OUT WITH A POSSIBILITY OF ITS ABRUPT DESTRUCTION

It has been assumed that the load affecting the structural component is such as to cause the surface abrasive wear, and at the same time the value of load is such as to eventually result in an abrupt failure (destruction) of the component (i.e. in the component's fracture). Operation of an aircraft tyre in the course of a hard landing is a good example of such a case.

Let the following be (Figure 2):
Σ - a random variable of temporary strength of the structural component of density function $g_1(\sigma)$,
W - a random variable of stress due to load pulse of density function $g_2(w)$.

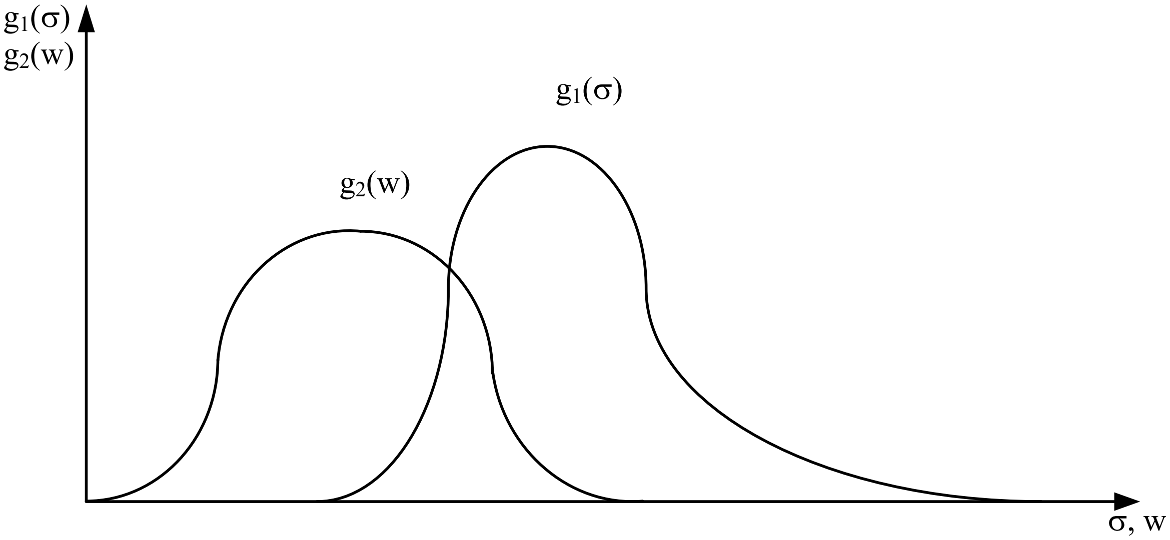


Figure 2. The component-loading diagram

An abrupt failure to the component will occur if:

$$w - \sigma > 0\,, \tag{24}$$

where: w and σ are realisations of random variables W and Σ.
Hence,

$$\Psi = W - \Sigma\,. \tag{25}$$

Density function of the random variable Ψ is to be found from the relationship:

$$f(\psi) = \int_{C_1}^{C_2} g_1(w-\psi)\, g_2(w)\, dw\,, \tag{26}$$

where: C_1, C_2 - boundary values of stresses resulting from the load pulse.

The probability of a failure to the component will be:

$$P_w = \lambda\, \Delta t\, Q\,, \tag{27}$$

where:

$$Q = P\{W - \Sigma > 0\} = \int_{0}^{\infty} f(\psi)\, d\psi\,. \tag{28}$$

Events that occur in the course of the component's wearing-out and abrupt failure sum up to unity. Hence,

$$(1-\lambda\Delta t) + \lambda\Delta t\, P_0(1-Q) + \lambda\Delta t\, P_1(1-Q) + \lambda\Delta t\, P_2(1-Q) + \lambda\Delta t Q \cong 1\,. \tag{29}$$

Let $U_{z,t}$ denote the probability that at time instance t the deviation of the component's wear-and-tear is z. With the relationship (29) applied one can write down the following difference equation (Risken, 1984), (Żurek, 2012):

$$U_{z,t+\Delta t} = (1-\lambda\Delta t)U_{z,t} + \lambda\Delta t\, P_0(1-Q)U_{z,t} + \lambda\Delta t\, P_1(1-Q)U_{z-h,t} + \lambda\Delta t\, P_2(1-Q)U_{z-2h,t}\,. \tag{30}$$

Equation (30) takes the following form in the functional notation:

$$u(z,t+\Delta t) = (1-\lambda\Delta t)u(z,t) + \lambda\Delta t P_0(1-Q)u(z,t) + \lambda\Delta t P_1(1-Q)u(z-h,t) + \\ + \lambda\Delta t\, P_2\,(1-Q)u(z-2h,t)\,. \tag{31}$$

With account taken of dependences (11), equation (31) can be written down in the form:

$$u(z,t) + \frac{\partial u(z,t)}{\partial t}\Delta t = (1-\lambda\Delta t)u(z,t) + \lambda\Delta t P_0(1-Q)u(z,t) + \\ + \lambda\Delta t P_1(1-Q)\Big(u(z,t) - h\frac{\partial u(z,t)}{\partial z} + \frac{1}{2}h^2\frac{\partial^2 u(z,t)}{\partial z^2}\Big) + \\ + \lambda\Delta t P_2(1-Q)\Big(u(z,t) - 2h\frac{\partial u(z,t)}{\partial z} + \frac{1}{2}(2h)^2\frac{\partial^2 u(z,t)}{\partial z^2}\Big)\,. \tag{32}$$

Hence,

$$u(z,t)+\frac{\partial u(z,t)}{\partial t}\Delta t=\left[(1-\lambda\Delta t)+\lambda\Delta tP_0(1-Q)+\lambda\Delta tP_1(1-Q)+\lambda\Delta tP_2(1-Q)\right]u(z,t)+$$

$$+[\lambda\Delta t\,P_1(1-Q)\,h+\lambda\Delta t\,P_2(1-Q)\,2h]\frac{\partial u(z,t)}{\partial z}+$$

$$+\frac{1}{2}\left[\lambda\Delta t\,P_1(1-Q)\,h^2+\lambda\Delta t\,P_2(1-Q)(2h)^2\right]\frac{\partial^2 u(z,t)}{\partial z^2}\,. \tag{33}$$

After simplification and the division by Δt the following is arrived at:

$$\frac{\partial u(z,t)}{\partial t}=-\lambda\,Q\,u\,(z,t)-(\lambda\,P_1(1-Q)\,h+\lambda P_2\,(1-Q)\,2h)\,\frac{\partial u(z,t)}{\partial z}+$$

$$+\frac{1}{2}(\lambda P_1(1-Q)h^2+\lambda P_2(1-Q)(2h)^2)\frac{\partial^2 u(z,t)}{\partial z^2}\,. \tag{34}$$

Notation:

$$c=\lambda Q\,,$$

$$\hat{b}=\lambda(1-Q)(P_1h+P_2\cdot 2h)=\lambda(1-Q)\cdot b\,,$$

$$\hat{a}=\lambda(1-Q)(P_1h^2+P_2(2h)^2)=\lambda(1-Q)\cdot a\,.$$

Finally, we arrive at the following equation:

$$\frac{\partial u(z,t)}{\partial z}=-cu(z,t)-\hat{b}\,\frac{\partial u(z,t)}{\partial z}+\frac{1}{2}\,\hat{a}\,\frac{\partial^2 u(z,t)}{\partial z^2}\,. \tag{35}$$

Equation (35) is more general that the Fokker-Planck one. Solution to the Fokker-Planck equation (i.e. without *"-cu(z,t)"* term) for the above-mentioned condition takes the following form (Tomaszek, 2008):

$$\bar{u}(z,t)=\frac{1}{\sqrt{2\pi\lambda\,(1-Q)\,at}}\;e^{-\frac{\left(z-\lambda\,(1-Q)\,bt\right)^2}{2\lambda\,(1-Q)\,a\,t}} \tag{36a}$$

or

$$\bar{u}(z,N)=\frac{1}{\sqrt{2\pi(1-Q)\,aN}}\;e^{-\frac{\left(z-(1-Q)\,bN\right)^2}{2\,(1-Q)\,a\,N}}\,. \tag{36b}$$

Therefore, solution to equation (35) will take the following form (Tomaszek, 2001):

$$u(z,t)=e^{-ct}\,\bar{u}\,(z,t)\,. \tag{37}$$

Also, the following density function is the solution to equation (35):

$$u(z,t)=\lambda\,e^{-\lambda\,t}\,\bar{u}(z,t)\,, \tag{38}$$

where:

$$\int_{0}^{\infty}\int_{-\infty}^{\infty} \lambda\, e^{-\lambda t}\, \bar{u}(z,t)\, dz\, dt = 1$$

ESTIMATION OF THE STRUCTURAL COMPONENT'S RELIABILITY WITH ITS WEAR AND-TEAR AND POSSIBILITY OF ITS ABRUPT DESTRUCTION

Causes for failures to the structural component in the form of both the wear-and-tear and a catastrophic failure form, in the sense of reliability, a series structure. It can be assumed that any structural component has been composed out of 'two elements', one of them being subjected to the wearing-out processes, whereas the other one responding to the loading processes with its strength.

Hence, reliability of any structural component is described with the following relationship:

$$R(t) = R_1(t)R_2(t)\ , \tag{39}$$

where: $R_1(t) = e^{-ct}$; $R_2(t) = \int_{-\infty}^{z_d} \bar{u}(z,t)dz$; z_d - permissible, for safety reasons, value of deviation of the diagnostic parameter; $\bar{u}(z,t)$ - density function of the deviation value determined with relationship (36).

Equation (39) can be written down in the form:

$$R(t) = e^{-ct}\int_{-\infty}^{z_d} \bar{u}(z,t)dz\,, \tag{40}$$

where:

$$\bar{u}(z,t) = \frac{1}{\sqrt{2\pi\,\lambda(1-Q)\Delta t}}\, e^{-\frac{(z-(1-Q)\,b\,\lambda t)^2}{2(1-Q)\,a\lambda t}}\ .$$

Unreliability of the component will be:

$$\hat{Q}(t) = 1 - e^{-ct}\int_{-\infty}^{z_d} \bar{u}(z,t)dz\ . \tag{41}$$

The failure density function can be defined in the following way:

$$f(t) = \frac{\partial}{\partial t}\hat{Q}(t)\ . \tag{42}$$

Hence:

$$f(t) = c\, e^{-ct}\int_{-\infty}^{z_d} \bar{u}(z,t)\,dz + e^{-ct}\frac{d}{dt}\int_{z_d}^{\infty} \bar{u}(z,t)\,dz\ . \tag{43}$$

Using what has been discussed in (Tomaszek, 2008), (Żurek, 2012) one can write:

$$\frac{d}{dt}\int_{z_d}^{\infty} \bar{u}(z,t)\,dz = \frac{z_d + (1-Q)b\lambda t}{2t} \cdot \frac{1}{\sqrt{2\pi(1-Q)a\lambda t}}\, e^{-\frac{(z_d - (1-Q)b\lambda t)^2}{2(1-Q)a\lambda t}} . \tag{44}$$

Hence,

$$f(t) = c\, e^{-ct} \int_{-\infty}^{z_d} \bar{u}(z,t)\,dz + e^{-ct} \cdot \frac{z_d + (1-Q)b\lambda t}{2t} \cdot \frac{1}{\sqrt{2\pi(1-Q)a\lambda t}}\, e^{-\frac{(z_d - (1-Q)b\lambda t)}{2(1-Q)a\lambda t}} . \tag{45}$$

With the reliability and density function defined, one can determine the failure rate:

$$\chi(t) = \frac{f(t)}{R(t)} , \tag{46}$$

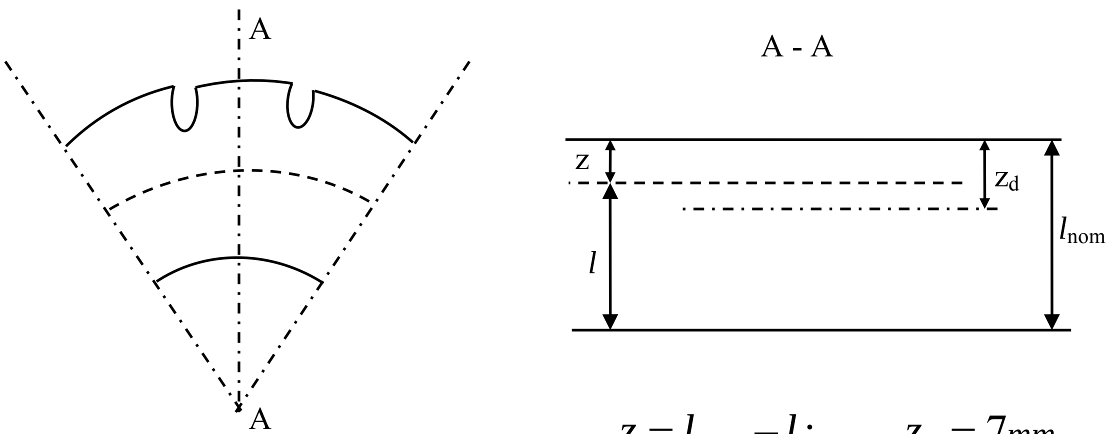
where: *f(t)* has been defined with equation (45), and reliability *R(t)* with relationship (40).

Applying the failure rate defined with formula (46), one can put down the reliability of a structural component as affected by the forming failure in the following way:

$$R(t) = e^{-\int_0^t \chi(w)\,dw} . \tag{47}$$

RESULTS

An aircraft tyre presented in the form of the tyre's section in Figure 3 has been given consideration in the below-given example. The tyre tread keeps wearing out while the tyre is operated, in particular in the course of aircraft take-offs and landings. The tyre's strength is affected by heavy loads and hence, the tyre may suffer an abrupt failure during a hard landing (Żurek, 2012).



$$z = l_{nom} - l; \qquad z_d = 7mm$$

Figure 3. The cross-section of the tyre

Where: l_{nom} - nominal (rated) thickness of the tread; l - current thickness of the tread; z - current wear of the tread; z_d - permissible wear of the tread.

To estimate reliability of the aircraft tyre, relationship (40) is to be used:

$$R(N) \cong e^{-QN} \int_{-\infty}^{z_d} \frac{1}{\sqrt{2\pi(1-Q)aN}} e^{-\frac{(z-(1-Q)bN)^2}{2(1-Q)aN}} dz \,. \tag{48}$$

Hence, after normalization:

$$R(N) = e^{-QN} \cdot \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{(z_d-(1-Q)bN}{\sqrt{(1-Q)aN}}} e^{-\frac{\chi^2}{2}} dx \,. \tag{49}$$

The following data are assumed to estimate reliability according to formula (49) (Żurek, 2012): b = 0,0166; a = 0,00051; Q = 0,00001; z_d = 7 mm. Computational results have been published in Table 1 and Figure 4.

Table 1 Reliability of an aircraft tyre

N	20	50	100	150	200	250	300	350	400	450	500
$\beta(N)$	66,023	38,638	23,646	16,306	11,523	7,982	5,164	2,817	0,797	- 0,981	- 2,574
$R_2(N)$	1	1	1	1	1	1	1	0,998	0,787	0,163	0,005
$R_1(N)$	1	1	0,999	0,999	0,998	0,998	0,997	0,997	0,996	0,996	0,995
$R(N)$	1	1	0,999	0,999	0,998	0,998	0,997	0,995	0,784	0,162	0,005

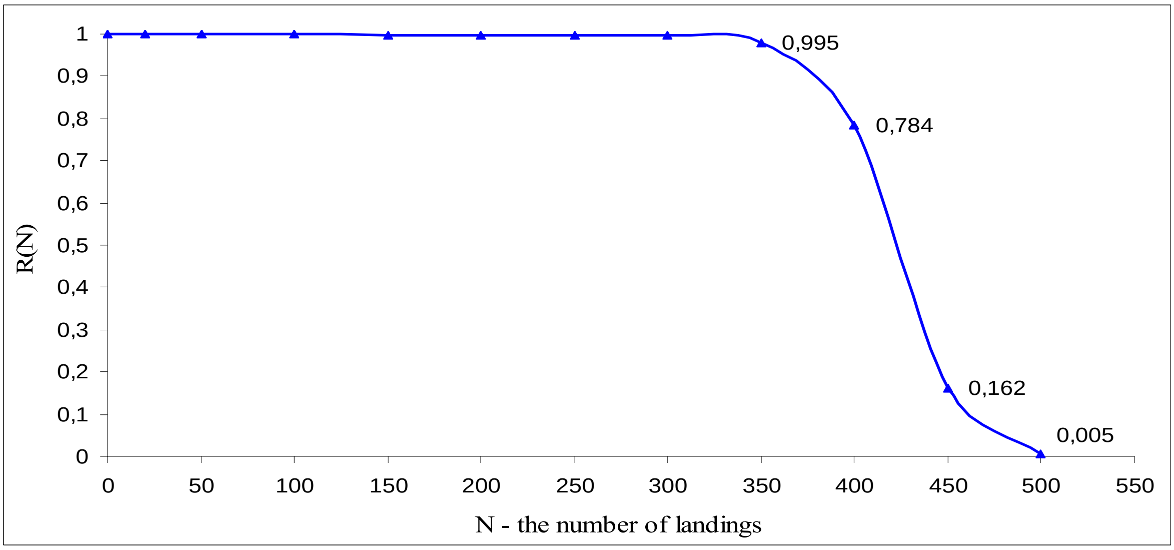


Figure 4. Reliability of an aircraft tyre

Following formulas have been used to estimate reliability of the aircraft tyre:

$$\beta(N) = \frac{z_d - (1-Q)bN}{\sqrt{(1-Q)aN}} \,, \tag{50}$$

$$R_1(N) = e^{-QN}\,, \tag{51}$$

$$R_2(N) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\beta(N)} e^{-\frac{\chi^2}{2}} dx\,. \tag{52}$$

CONCLUSION

Modelling of the wear-out of structural components with their strength affected by loading processes demands that findings of the studies/testing work as well as data from the equipment/systems-operating processes are widely applied. The more accurate and complete results of the testing work on structural components, the more accurate relationships developed to estimate reliability thereof.

What has been done in this paper is relatively simple assumptions made about the component’s operating conditions. The mechanisms of structural components wearing out to finally fail (get damaged/destroyed) are rather complicated; therefore, any attempt to formulate reliability models with account taken of the wear-out processes is a pretty difficult job.

Of great interest are models that take fatigue processes in machine components into consideration. Generation of such models is not an easy job as well, since they require deep and widespread knowledge. The above-presented model of describing the components wear-out, with elements of the random-walk theory applied, may be subject to continuous improvement as soon as new elements of the structure’s wear-out and fatigue are given consideration.

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