PAPER REF: 4652

VIBRATIONS DETECTION AND ANALYSIS IN EQUIPMENTS WITH MCUSUM CHARTS AND FREQUENCIES GRAPHS

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ABSTRACT

The condition monitoring of systems can be the choice to be applied on maritime selected equipment's, chosen by its characteristics and industrial importance. Several statistical methods have been developed and applied in order to improve the performance of equipment´s (Dias *et al*, 2007). In this paper we propose the application of Statistical Process Control (SPC), in order to know the state of an equipment that had work like a prototype, an electric pump (Fig. 1). This prototype is the early work for future maritime applications.

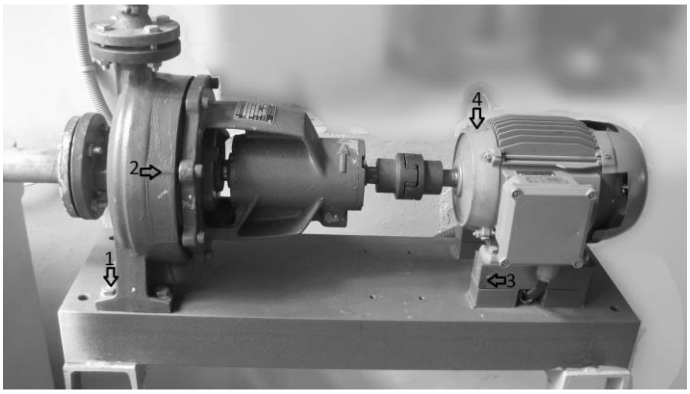

Fig. 1 Eletropump prototype

In previous studies, (Gan, 1991) (Dias *et al*, 2009) (Requeijo *et al*, 2012), has been shown that the univariate control charts are applicable to equipment as well as the traditional multivariate control charts and for *Short Run* charts. We can see an example in Fig.2:

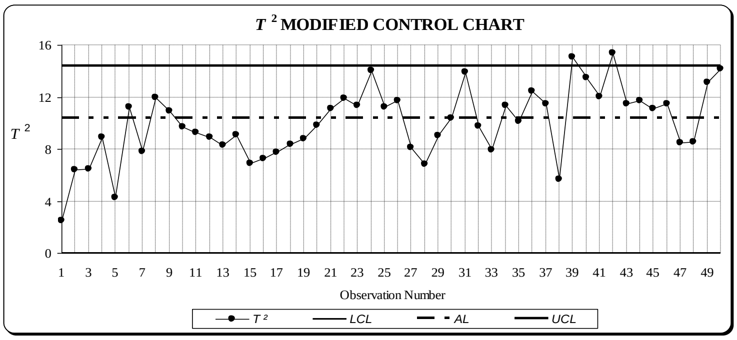

Fig. 2 Traditional Multivariate Control Chart (Lampreia *et Al*, 2012)

The control charts that we propose to be applied are the *MCUSUM* using multivariate data that inherently uses historical data to determine the reliability of equipment. Before its application the Normality and Independence should be checked (Pereira and Requeijo, 2012). With the implementation of these charts, it is expected to be possible the early detection of anomalies. Since the control charts will allow only the fault detection, its diagnosis is now a matter, for it will be used a graphical representation of frequency in order to velocity. When this methods are applied in maritime equipment's, carefully must be taken, because of the external environment influences, machines and meteorological state. Note that, the data used were collected in overall vibration values (RMS-root mean square) in units of velocity (mm/s). With the proposed methodology will be possible to know the *Online* status of equipment when necessary and when requested, resulting in a reduction in maintenance costs and availability increase.

KEY WORDS: *Condition Monitoring, Vibration Detection and Analysis, Multivariate Control Charts CUSUM*

INTRODUCTION

The equipment operation monitoring is critical to an integrated maintenance management, in a way to maximize the availability of resources, with a minimization of costs.

Applying statistical methods to the study of mechanical components or systems can be critical in order to achieve maximum availability of equipment in a certain period time, and to perform diagnosis interventions or repair. (Dias, *et al*, 2007),

Various types of methods and methodologies can be applied to perform data collection in order to monitor the operation of equipment. Some of these methods can be the collecting data by placing fixed sensors on the equipment. The methods of processing the data can be AFTM Accelerated Failure-Time Proportional Hazards Models and the Models, Laplace Test among others. (Lawless, 1982) (Lampreia, 2005)

Note that the data can be treated Offline or Online. In Offline assumes that there is a concept of preventive maintenance, in Online presupposes a kind of predictive maintenance where the data is analyzed and managed proactively. (Requeijo *et Al*, 2012)

Note that the tracking Online can be best to manage the availability of repairable systems.

If the equipment is being analyzed correspond to rotating machines, measurement and vibration analysis is the most used, and the main reasons are because of the results, they cover the detection of various types of faults and are reliable and effective. Note also that the systems often do not require any intervention during the measurement of vibrations. (Alt, 1985) (Lampreia *et Al*, 2012)

In this article will be demonstrated that the vibration analysis is possible without resorting to specific software (Parreira, 2010). To vibration detection will be applied statistical control charts, the type of detection are analyzed in graphical representations (frequency vs speed).

ANOMALIES DIAGNOSIS WITH SPECTRUM ANALYSIS

The application of vibration measurement and analysis on machinery allows the early diagnosis of any anomalies, so technicians can carry out preventive interventions. The advantages of these techniques are (Sampaio, 1999):

- Detection of most faults;
- Detection of faults in its early stages, so we can apply the method of trend analysis;
- Detection of faults without the need to stop systems;
- Allow diagnose the cause of the malfunction.

It's important to refer that the vibrations can be measured by three types of units: displacement, velocity e acceleration. If the acceleration levels are less than 1000 Hz, the best measure unit is the velocity (mm/s), and it's the one that we choose for the present study.

When we analyzed the frequency spectrum the type of anomalies can be decoded, considering the detection frequency and the order. For example: if we had a misalignment, we can find peaks at 1x, 2x and sometimes at 3x.

In this article, in the case study, some types of anomalies are analyzed, and associated to the spectrum frequencies. Because of the field extension, it will not be full developed in the article, but some authors can be consulted (Institute, 2005) (Eshleman, 1999).

STATISTICAL CONTROL CHARTS

As in other types of statistical control with a high number of collected data, the multivariate control charts applied has the phase I and phase II. In phase I will be checked the stability of the equipment and the operating parameters are estimated by the Hotteling chart T^2. In phase II the parameters of vibration will be controlled by *MCUSUM* special control charts.

Phase I

T^2 Traditional Control Charts

T^2 charts for the first phase can be obtained when the number of variables is greater than one.

Given the specific application covered in this article, only the individual observations (*n=1*) charts will be mentioned. (Lampreia *et Al*, 2012)

A. Independent Data

If the observations of p variables in control are independent, we have, $X_{ij} = \mu_j + \varepsilon_{ij}$ where:

x_{ij} is the observation *i* for *variable j*

μ_j is the process mean for the variable *j*

ε_{ij} are *iid* normal random variables with mean zero and standard deviation σ_ε (white noise).

In phase I m individual observations are collected, X_{jk} ($j = 1,2,\ldots,p$ e $k = 1,2,\ldots,m$), where the mean sample, ($\overline{X}_j$), the sample variance (S_{ij}), and the sample covariance S_{jh} are calculated. Based on this statistics, the vector mean, $\overline{X}$, is given by the equation (1) and the covariance matrix S, given by equation (2).

$$\overline{X} = \left(\overline{X}_1, \overline{X}_2, \ldots, \overline{X}_p\right)^T \tag{1}$$

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & \cdots & S_{1p} \\ S_{21} & S_{22} & S_{23} & \cdots & S_{2p} \\ . & . & . & \cdots & . \\ . & . & . & \cdots & . \\ . & . & . & \cdots & . \\ S_{p1} & S_{p2} & S_{p3} & \cdots & S_{pp} \end{bmatrix} \tag{2}$$

The T^2 control charts for each k, are based on the statistic:

$$(T^2)_k = (X_k - \overline{X})^T S^{-1} (X_k - \overline{X}) \tag{3}$$

These vectors, $\overline{X}$ e S, are calculated using the data collected in phase I. The *LCL* and the *UCL* to the phases I, are constant in Table 1.

In table 1, $\beta_{\alpha;p/2;(m-p-1)/2}$ is the right percentile, for a probability α, from the beta distribution with the parameters $p/2$, $(m-p-1)/2$ and $F_{\alpha;p;m-p}$ is the right percentile, for a probability α, from Fisher distribution with p and (m-p) degrees of freedom, respectively numerator and dominator.

It's questionable but the rule applied to calculate the parameters in phase I for T^2 control Charts, with m individual observations is to have m between: $180p \leq m \leq 300p$. (Pereira e Requeijo, 2012)

Table 1 T^2 Control Chart Limits

Chart	*LCL*	*UCL*
Phase I	0	$\frac{(m-1)^2}{m}\beta_{\alpha;p/2;(m-p-1)/2}$

B. Autocorrelated Data

If one of the variables under study have a significant autocorrelation, the T^2 statistic is to be calculated using a model that uses residues for each observation in phase I, replacing the vectors of each of the parameters X_k, $\overline{X}$ e S with the corresponding residues mean vector and covariance residues matrix.

To calculate the residues, it is necessary to build a model for the process with correlated variables. The application of *ARIMA* (*Autoregressive Integrated Moving Average*) models, were suggest and developed by Box and Jenkins. (Pereira e Requeijo, 2012)

The significant autocorrelation in the process are going to be analyzed by studying the Autocorrelation Function (*ACF*) and Partial Auto-Correlation Function (*PACF*). To model the performance of equipment, it is necessary to estimate the *ARIMA* (*p, d, q*) model that fits the data. And the estimated autocorrelation function (*EACF*) are compared to the theoretical autocorrelation function (*ACF*) and the estimated partial autocorrelation function (*EPACF*) with the theoretical partial autocorrelation function (*PACF*). (Pereira e Requeijo, 2012)

Considered stationary processes, the *ACF* and *PACF* for *AR*(*p*), *MA*(*p*) and *ARMA*(*p,q*) the process have distinct characteristics. In the present paper it will be shown that the data obey to a *AR* Model, which is characterized for *ACF* Exponential decrease after a certain lag order, and *PACF* for significant peaks through lags log (p) (Pereira e Requeijo, 2012) (Lampreia et Al, 2012). For more information Pereira e Requeijo, 2012) must be consulted.

A process follows a *ARIMA*(*p,d,q*) model if $\nabla^d X_t$ follows *ARMA*(*p,q*) model.

The model defined by *ARIMA*(*p,d,q*):

$$\Phi_p(\mathrm{B})\nabla^d = \mathrm{X}_t = \Theta_q(\mathrm{B})\varepsilon_t \tag{4}$$

with

$$\Phi_p(B) = (1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p) \tag{5}$$

$$\Theta_q(B) = (1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q) \tag{6}$$

$$B = \frac{X_{t-1}}{X_t} \text{ e } \nabla = \frac{X_t - X_{t-1}}{X_t} \tag{7}$$

Where:

B	- the lags operator;
∇	- the differences operator;
d	- the differentiation order to render a stationary process;
X_t	- the observation at time *t*;
ε_t	- the white noise at time *t* $\left(\varepsilon \sim RB\left(0, \sigma_\varepsilon^2\right)\right)$;
$\Phi(B)$	- the autoregressive polynomial of order *p*;
$\Theta(B)$	- the moving average polynomial of order *q*.

With the model designed, the residues can be estimated by $e_t = X_t - \hat{X}_t$, where $\hat{X}_t$ is the value in the period *t*.

The control chart is built using the residues. The mean and standard deviation are estimated using *ARIMA* models. The process mean is estimated by equations 8 and 9 when the process is modeled by an *AR*(*p*) or by a *MA*(*p*), respectively.

$$E(X) = \mu = \frac{\xi}{1 - \sum_{j=1}^{p} \phi_j} \tag{8}$$

$$E(X) = \mu \tag{9}$$

Phase II

Special Control Charts - *MCUSUM*

The use of a cumulative multivariate chart on data should be considered at the expense of other control charts, like T^2, because it will be more sensitive to little changes in the average. Can be considered two types of multivariate *CUSUM* charts, one controls the mean vector, μ, and the other controls the covariance matrix $\sum$. In the present article, only the one that controls the mean vector will be applied.

MCUSUM Chart to Mean Vector Control

The Multivariate *CUSUM* Modified Chart (*MCUSUMM*) represents the statistical value of the instant, the statistic is defined by Y^*, in the instant *t*, is defined by (Villalobos, 2005):

$$Y_t^* = \left(C_t' \sum^{-1} C_t\right)^{1/2} \tag{10}$$

where

$C_t = 0$ se $C_t^* \le k$

For independent data:

$$C_t = \left(C_{t-1} + X_t - T_L\right)\left(1 - \frac{k}{C_t^*}\right) \text{ se } C_t^* > k \tag{11}$$

$$C_t^* = \left(\left(C_{t-1} + X_t - T_L\right)' \sum^{-1} \left(C_{t-1} + X_t - T_L\right)\right)^{1/2} \tag{12}$$

And for the autocorrelated data:

$$C_t = \left(C_{t-1} + e_t - T_L\right)\left(1 - \frac{k}{C_t^*}\right) \text{ se } C_t^* > k \tag{13}$$

$$C_t^* = \left(\left(C_{t-1} + e_t - T_L\right)' \sum^{-1} \left(C_{t-1} + e_t - T_L\right)\right)^{1/2} \tag{14}$$

with

$C_0 = 0$ e $k > 0$

The multivariate *CUSUM* control charts depend, like in the *CUSUM* control charts, on the $ARL_{InControl}$ value, and on the deviation that we pretend to find.

If the values $Y_t^* > h$, where h is the control limit, we are facing a situation out of control. Considering the design of the best possible chart in which the value of $ARL_{OutofControl}$ is minimized, for the special multivariate *CUSUM* chart, has been proposed by (Crosier, 1988) the values in the next table:

Table 2 *h* from *MCUSUM* chart for $k = 0,5$ (Crosier, 1988)

$ARL_{InControl}$	*P*	*h*
200	2	5,50
	5	9,46
	10	14,9
500	2	6,65
	5	10,9
	10	17,2

METHODOLOGY

In order to define the methodology for processing data, we should take in account the sample size and the type of data to be treated.

The data independence should be study, and the T^2 control chart must be executed in order to define the mean vector and the covariance matrix. If data are autocorrelated, the *ARIMA* model should be applied, and the T^2 chart is executed for the residues, so the mean vector and the covariance matrix are calculated.

Then in the Phase 2 the *MCUSUMM* chart are used to monitor the equipment. In Phase II the equipment are monitored sense the first observation, and only under some circumstances actions are taken. For this, it should be based on international standard values or in the equipment characteristics defined by the manufacturer. (Requeijo *et Al*, 2012)

For the implementation of the equipment monitor by *MCUSUMM* charts the next steps must be taken:

- In Phase I, data must be collected from distinct variables that best defined the equipment state. At least 200 samples of individual observations
- Test variables independence, using the *EACF* and the *EPACF*.
- If data is independent, built a T^2 chart based on that data, check for process stability, data normality and estimate the vibration mean vector and covariance matrix. (Lampreia *et Al*, 2012)
- If autocorrelation exists, set up a model for the auto-correlated variables using the *ARIMA* model. (Lampreia *et Al*, 2012)
- Apply the T^2 control chart for residues, verify the process stability and residues normality, estimate residues mean vector and covariance matrix. (Lampreia *et Al*, 2012)
- To vibration level monitoring, apply the modified *MCUSUM* chart to data collected with forced anomalies and various levels of degradation.
  - Estimate the two limits to control the mean level of vibration, specifically, the Upper Control Limit (*UCL*) and the Alert Value (*AL*).
  - Based on ISO 2372:2003 Standard, establish the vibration level which the system must have an intervention.
  - Establish rules for action on the system. The next are suggested:

- Execute an intervention to detect any anomalous situation when 8 consecutive points above the *AL are* observed.
- Proceed to a maintenance intervention when 5 consecutive points above *UCL* are observed.

- To proceed with the intervention we must know which the anomaly is. For that the frequencies spectrum in order to velocity (mm/s) graph must be studied. When defined which interventions the equipment need, the technicians must proceed with their repaired work.

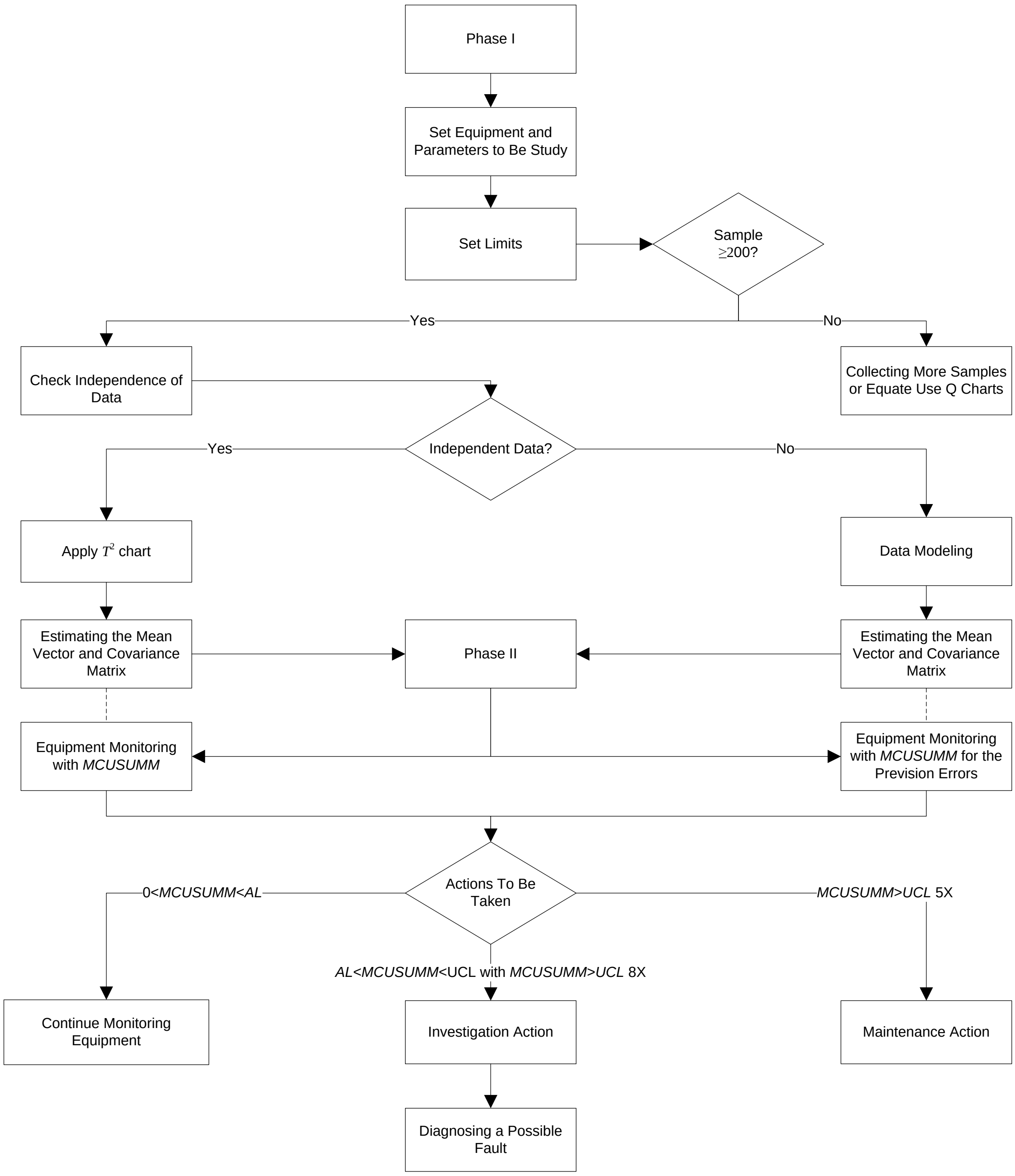

Fig. 3 Methodology Flowchart

CASE STUDY

APPLICATION OF STATISTICAL CONTROL TO EQUIPMENT

As mentioned above, the case study is based on electropump vibration data, which is used for vibration research. Four points for reading data were defined, which is believed to best represent it behavior. An anomaly was further introduced with four state aggravation, in order to test the sensitivity of the charts and diagnosis the anomaly type. The vibration values are treated in units of velocity (mm/s) for vibration global values (RMS) considering a stabilization period of 30 minutes, and collecting data with a 5 minutes interval.

Phase I

The objective of Phase I is the estimation of equipment parameters, so 241 samples with $n=1$ were taken.

At first the electro-pump was in good work condition.

The study begins testing the data independence using software *Statistica*. The sample has autocorrelated data, so that *Shewhart* control charts cannot be directly applied.

To identify the model that fits the data, we compared the *EACF* and *EPACF* with the ACF and PACF, for the four variables.

After modeling, the residues were estimated using the *Statistica* software and the T^2 control chart was applied for Phase I. it was possible to detect point nr 69 above the UCL.

The variable responsible for the out of control point was identified, and the values of X were replaced with the expected values at the same instant.

Then the process was remodeled, and the respective residues were estimated. With the new residues, the T^2 chart was executed.

Fig. 4 and Fig. 5 shows the *EACF* and *EPACF* for the variable 1, where autocorrelation is significant.

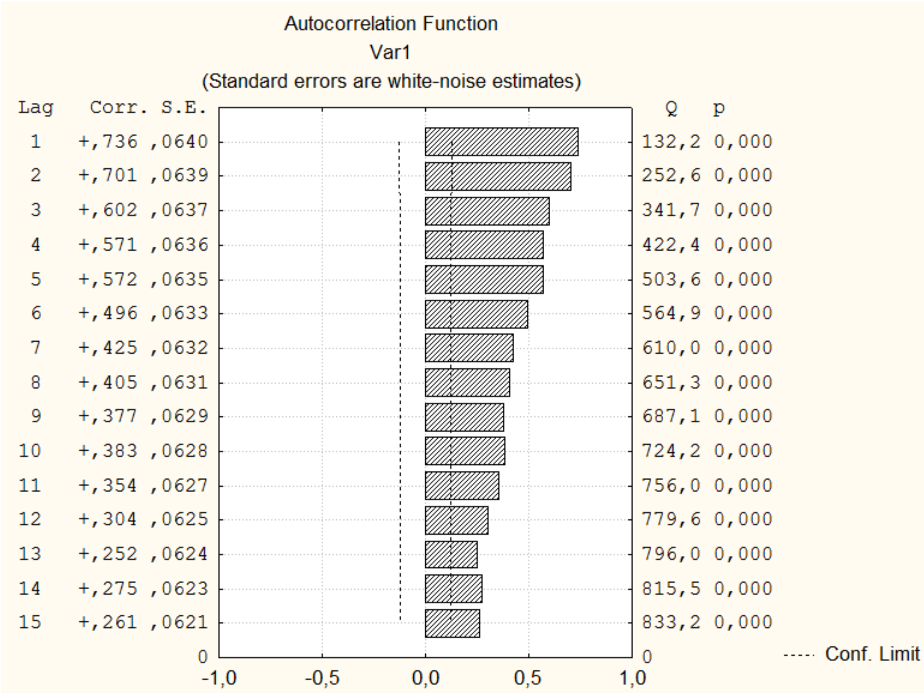

Fig. 4 EACF for variable 1

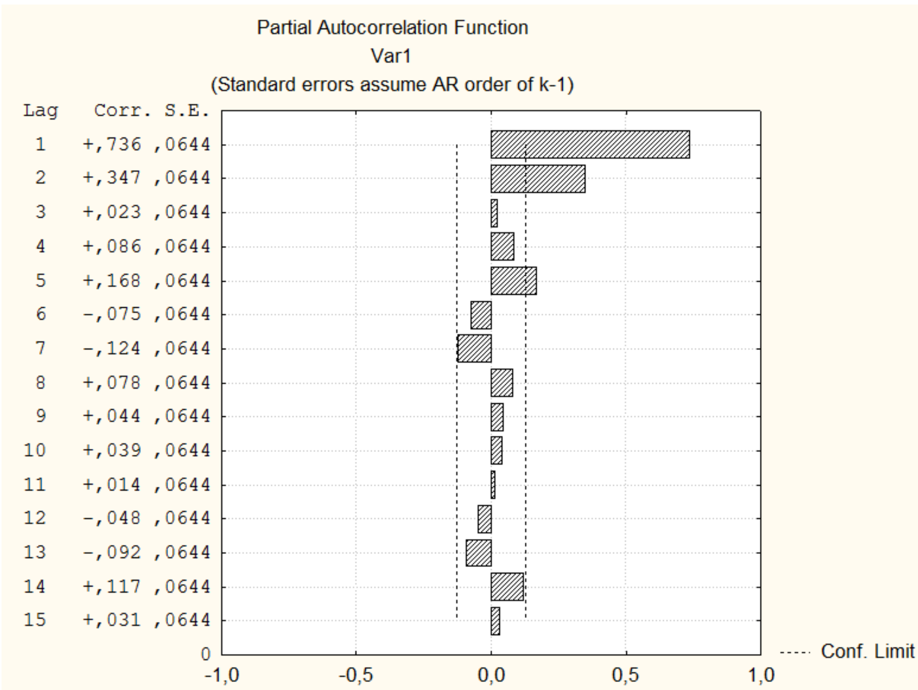

Fig. 5 EPACF for variable 1

In Table 3 we can see the parameters estimations of *ARIMA* model for the four variables.

Table 3 *ARIMA* model parameters adjusted for the four review variables

Variable	1	2	3	4
Model	$AR(2)$	$AR(2)$	$AR(2)$	$AR(2)$
ξ	0.0693	0.1025	0.4759	0.251
Model Parameters	ϕ_1=0.4806 ϕ_2=0.3482	ϕ_1=0.4945 ϕ_2=0.2919	ϕ_1=0.1888 ϕ_2=0.1688	ϕ_1=0.3055 ϕ_2=0.2409

When we analyze the new T^2 Chart, no special causes of variation are shown, so the mean vector (residues and variables) and covariance matrix are estimated. (Requeijo *et Al*, 2012) (Lampreia *et Al*, 2012).

The residues Normality was verified using the Kolmogorov-Smirnov and Shapiro-Wilks Tests. Knowing $D_{Crítico} = \frac{0,886}{\sqrt{N}} = \frac{0,886}{\sqrt{241}} = 0,0581$ for $\alpha = 5\%$, with Kolmogorov-Smirnov Test to Var1 we obtain *d=0,02810*, Fig. 6, so $d < D_{Crítico}$.

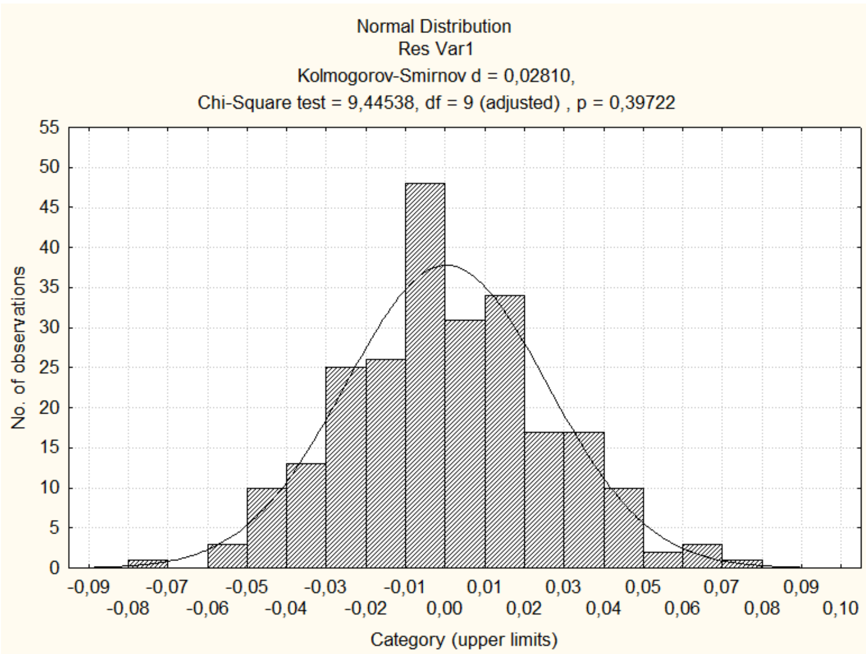

Fig. 6 Normality Study of Var1

All variables followed a Normal distribution.

$$\bar{e} = \begin{bmatrix} 0,00033 \\ 0,00021 \\ 0,00034 \\ 0,00041 \end{bmatrix} \qquad \overline{X} = \begin{bmatrix} 0,406 \\ 0,4810 \\ 0,7400 \\ 0,5220 \end{bmatrix}$$

$$S = \begin{bmatrix} 0,000726 & -0,000057 & 0,000137 & 0,000035 \\ -0,000057 & 0,000686 & -0,000123 & -0,000007 \\ 0,000137 & -0,000123 & 0,007102 & 0,000298 \\ 0,000035 & -0,000007 & 0,000298 & 0,001627 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} 1392,5 & 110,58 & -23,82 & -24,99 \\ 110,58 & 1470,9 & 23,36 & -0,359 \\ -23,82 & 23,36 & 142,74 & -25,57 \\ -24,99 & -0,359 & -25,57 & 619,89 \end{bmatrix}$$

Although 241 observations were used to define the process parameters, in Fig. 7, only 100 are present.

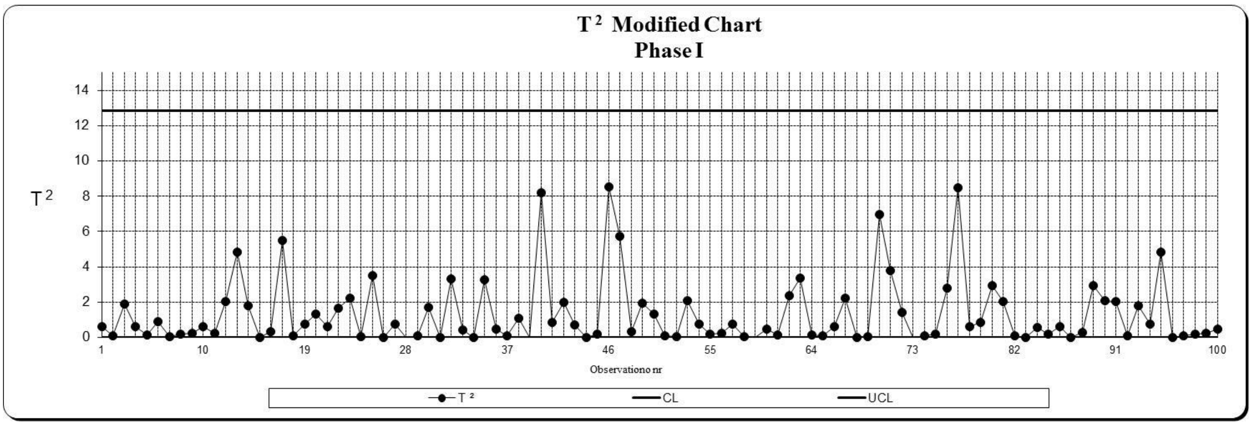

Fig. 7 Carta T2 Modificada na primeira fase e após os valores terem sido revistos.

Fig. 7 Phase I - T^2 Chart

PHASE II

To define the limit of vibrations it was consulted the ISO 2372:2003, which define 1.12 *mm*/s (*RMS*) as the allowable limit for the vibration level of an equipment of class I. The electro-pump under study has a 1,5 *KW* engine, so we specified the value $(T_L)_N = 1{,}12$ as the limit vibration. The vector to consider for the *MCUSUMM* chart is:

$$(T_L)_j = ((T_L)_N - k\sigma))_j = \begin{bmatrix} 0,633 \\ 0,560 \\ 0,127 \\ 0,477 \end{bmatrix}$$

According Croisier (1988) considering *p* equals to 4, and to specify limits for control of *MCUSUMM* charts the table 2 must be consulted, so the h1 is the Alert Level and *h* the Upper Control Limit, table 4.

Table 4 Alert and Control Limits of *MCUSUMM* Chart

AL	*UCL*
h_1	h
9,46	10,9

At this phase, because we are dealing with special multivariate control charts, the limits are set differently. But the observations are analyzed from the first data record. 50 individual observations were read 50 ($n = 1$) with the electric pump in normal operation. To accelerate its degradation was introduced an anomaly, unscrewing the screws of electropump support, and this was aggravated in 4 states gradually.

Applying the *MCUSUMM* to the data with anomalies, till the second aggravation no point is registered in the charts, Fig. 8, so no changes has been detected.

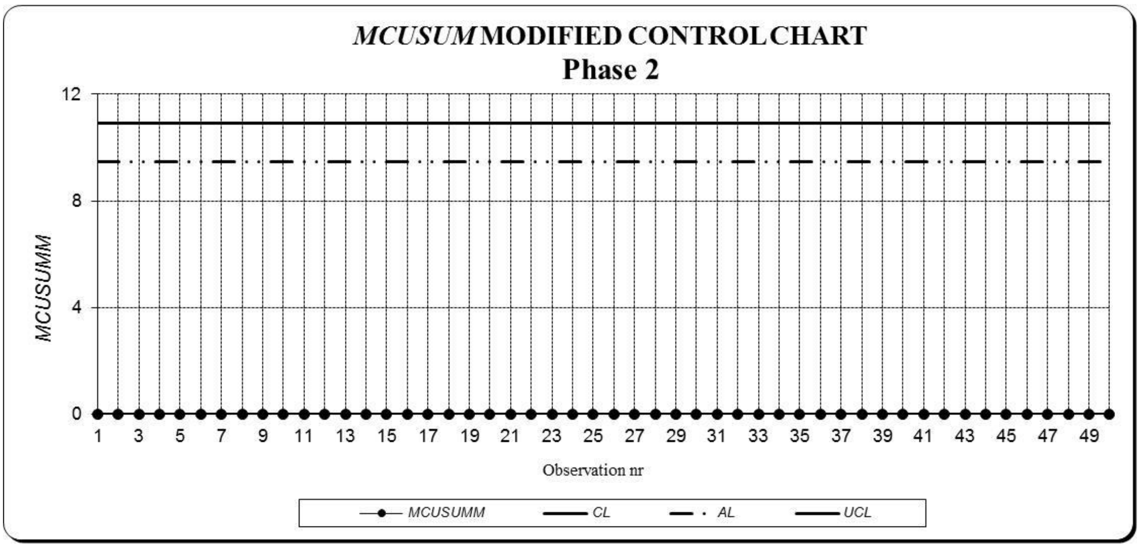

Fig. 8 0, 1ª and 2ª Aggravation *MCUSUMM* Control Chart

For the third aggravation was registered 3 points, but all of them are under control, Fig. 9.

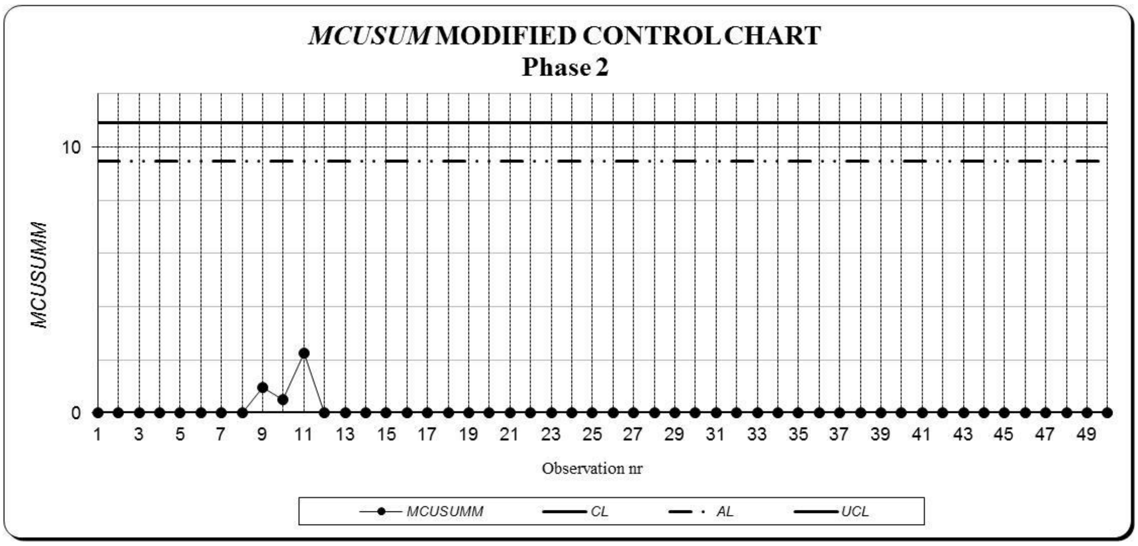

Fig. 9 3ª Aggravation *MCUSUMM* Control Chart

For the fourth aggravation finally this charts shows its high sensitivity, and sense the sixth observation all point are above the UCL, so a maintenance action should be taken in point 10.

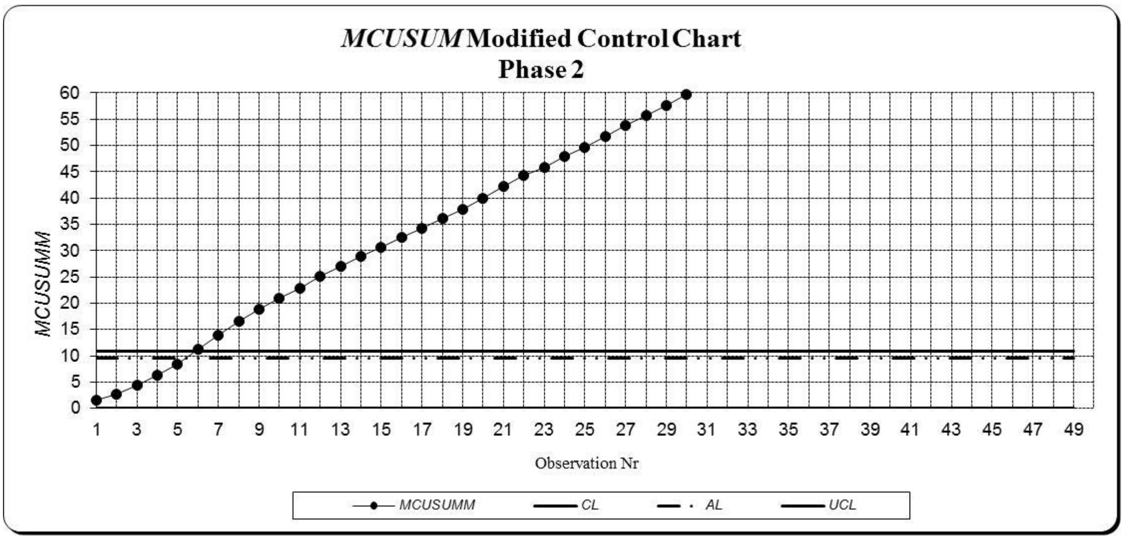

Fig. 10 4ª Aggravation *MCUSUMM* Control Chart

By applying the *MCUSUMM* control charts it is demonstrated the high sensitivity of the chart sense 4th Aggravation.

ANALYSIS OF FREQUENCY SPECTRUM

Is important to refer, that the operation velocity of the electropump is 1500rpm, so the natural frequency is 25Hz.

In this article it was chosen to demonstrate the frequency analysis refers to the Var1 and Var3.

Without any anomalies, for the Var1, we can see the natural frequency at 25Hz at the graphic, Fig. 11.

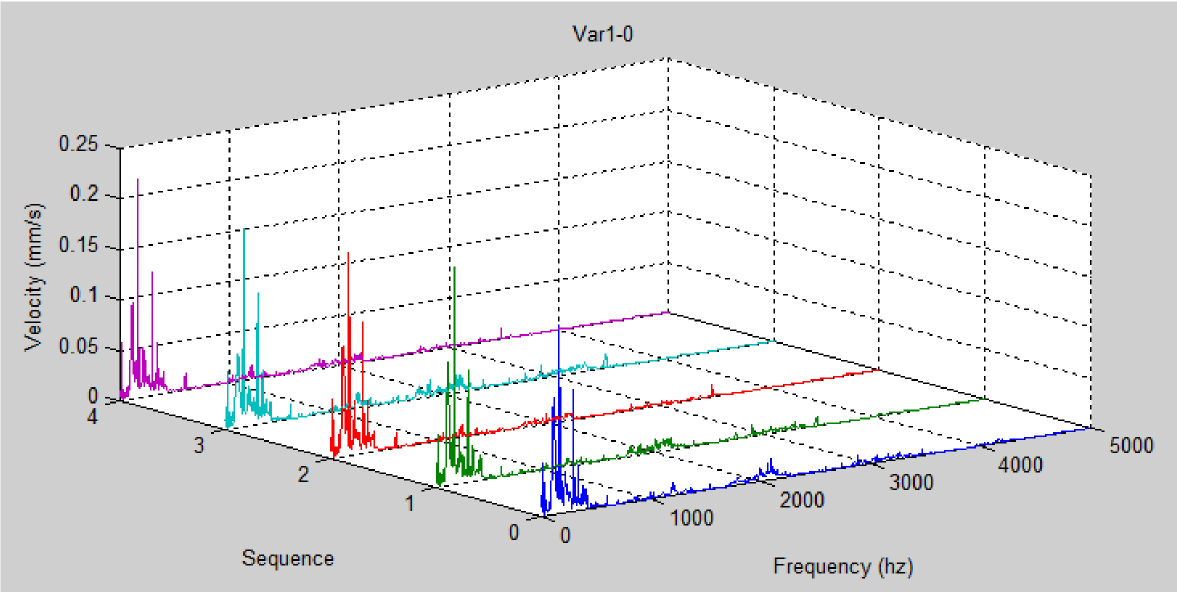

Fig. 11 Var1 Data Analysis – Normal operation samples

In the first aggravation the values get a little higher, especially between the 1750Hz and 2250Hz, where is a resonance suspicion.

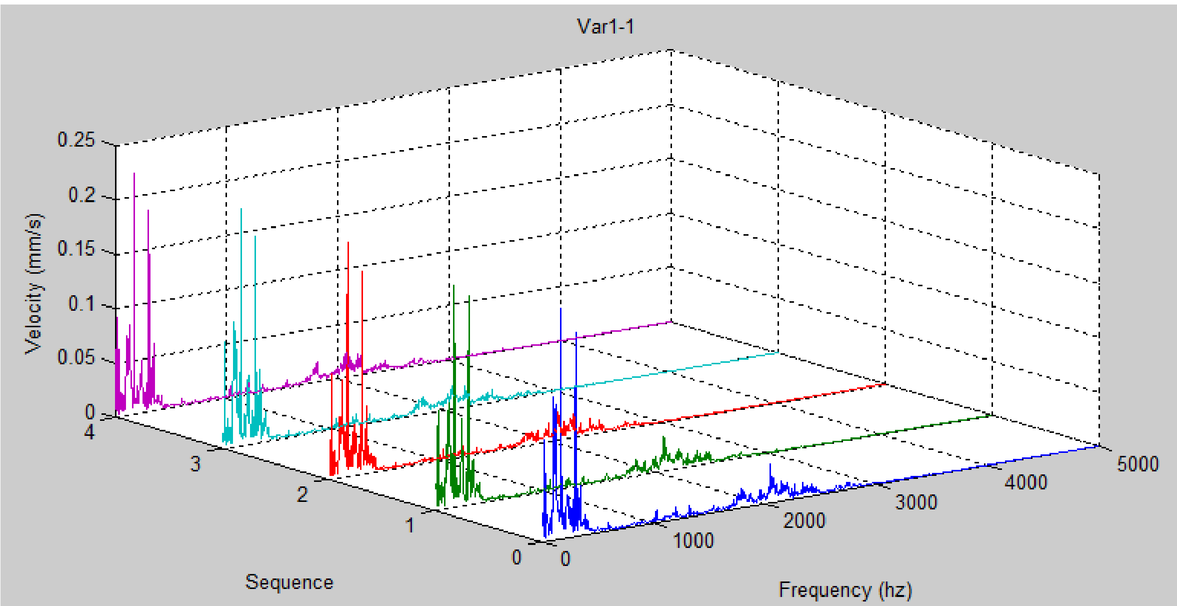

Fig. 12 Var1 Data Analysis – 1th Aggravation

For the second aggravation, by the graph analysis, we verify a peak on the natural frequency 25Hz either, and then another for 125Hz, 171Hz and at 300Hz (12x), and again between 1750Hz and 2250Hz. By the peaks registered, we may probably under a clearance bearing problem, a shaft eccentricity or blades anomaly.

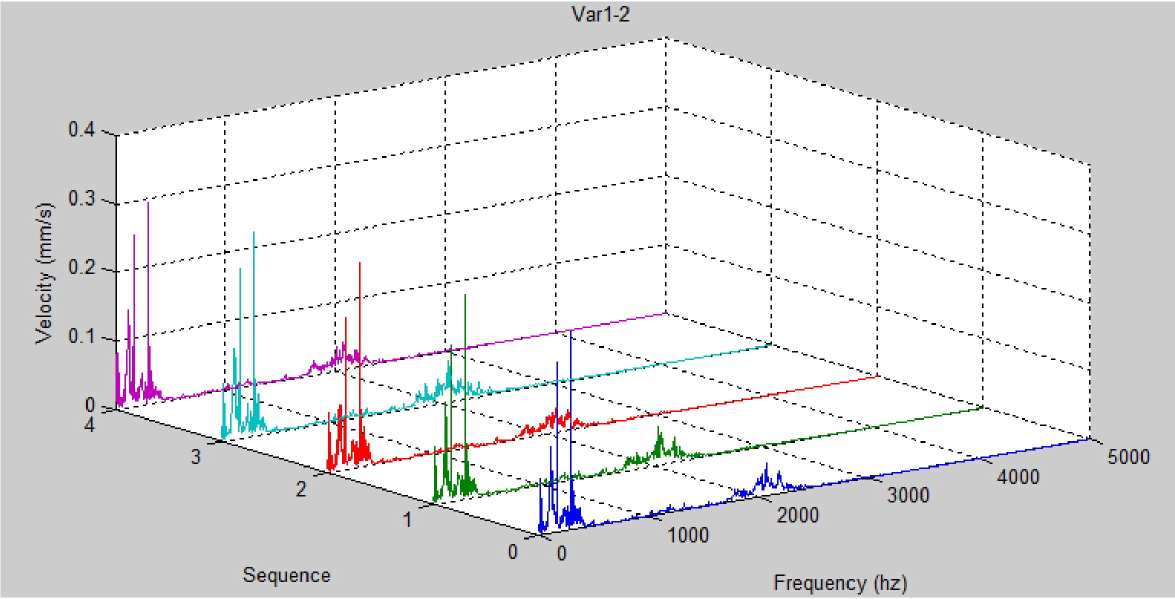

Fig. 13 Var1 Data Analysis – 2th Aggravation

By the 3rd (not represented here) and 4th aggravation the peak at 300hz has a 0,1mm/s increase, the ones with 25hz, 125hz and 171hz had stabilized.

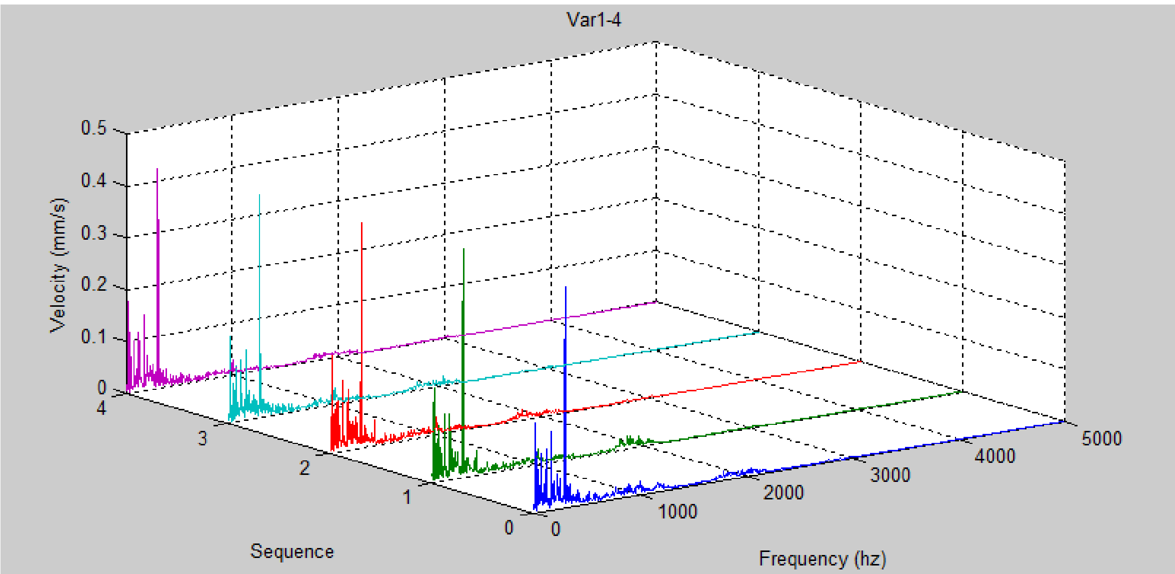

Fig. 14 Var1 Data Analysis – 4th Aggravation

Studying the Var3, with no anomalies we also can see the natural frequency at 25hz, and sidebands.

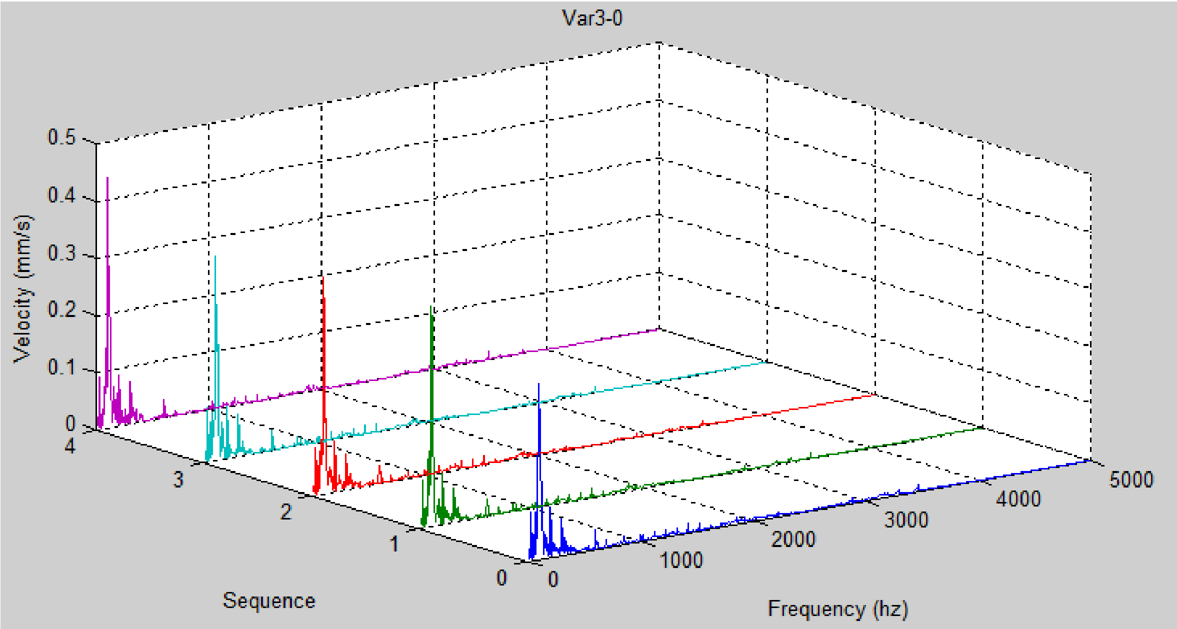

Fig. 15 Var3 Data Analysis – Normal operation samples

In then 1st, 2nd and 3rd aggravation the delineation of the graphs are similar, only the values getting higher with the aggravation. So only the 3rd are represented below.

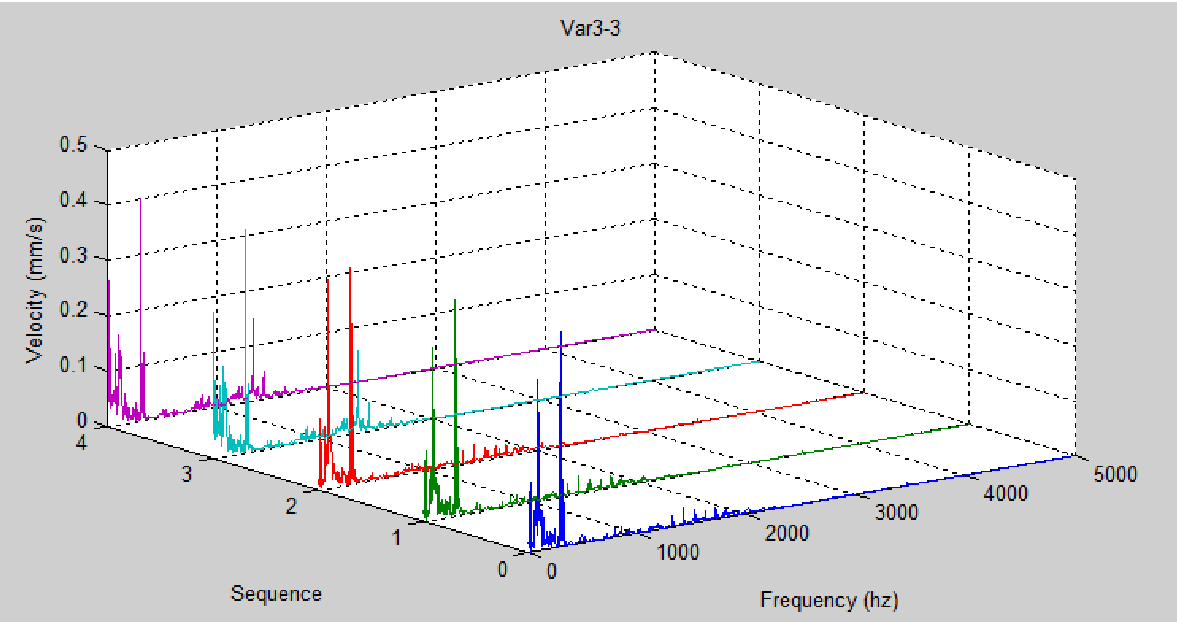

Fig. 16 Var3 Data Analysis – 3rd Aggravation

In the Var3 the 4th Aggravation, the vibration values increase, mostly in the 1x and 3x orders, and in the 12x. And we can see a little peak in 600 hz with sidebands.

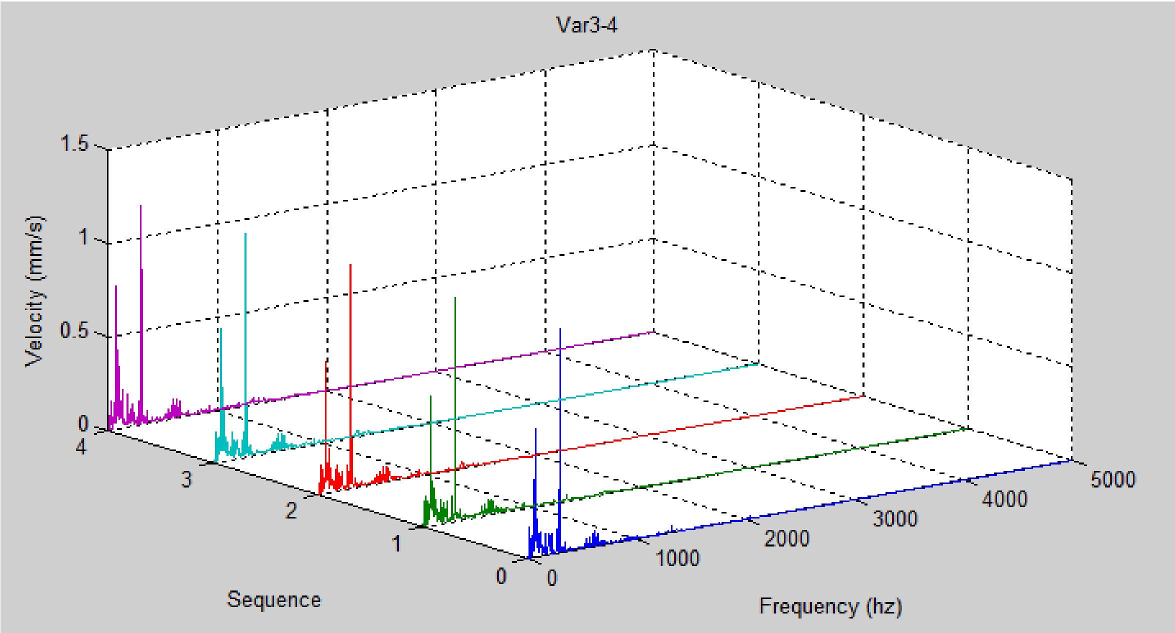

Fig. 17 – Var3 Data Analysis – 4th Aggravation

Mostly the peaks around the natural frequency we thought it are a consequence of the unscrewing. We believe that the unscrewing cause misalignment between the pump and the electrical engine, and imbalance in some other components, such as the rotors and bearings. Although this considerations, it is recommend to disassemble the electropump and do the dimensional control of the spare parts (ex: shaft, bearings) to only substitute the damage ones.

CONCLUSIONS

The Statistical Process Control is applicable to equipment monitoring with the objective of detecting anomalies.

In the Phase I, if the collected data is independent the residues should be applied in the T^2, if they are autocorrelated, the residues are used. And in the Phase II, if the data are independent the *MCUSUMM* Charts are applied directly, if it is not, the prevision errors should be used.

The *MCUCUMM* control charts show high sensitivity, but in this case only when the vibration values became higher. In future these charts should continue to develop and eventual test it to others realities.

If we want to know which anomaly an equipment have and which component will be necessary to substitute, the frequency spectrum should be analyze, and a cascade graph can be used to compare it and take conclusions. More data with other anomalies should be taken to test more the equipment limits and life performance.

The application of SPC and vibration spectrum analysis can be used to condition monitoring. With its application the equipment will be best controlled, more available, have a higher performance, the maintenance and operation costs will decrease.

ACKNOWLEDGMENTS

Portuguese Naval School and CINAV are kindly acknowledged for the use of the machinery workshop and also for the fruitful collaboration of the Faculty of Science and Technology from the Universidade Nova of Lisbon.

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