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A STUDY OF THE IMPLICIT SECULAR EQUATION FOR RAYLEIGH WAVES IN 2D ORTHOTROPY

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ABSTRACT

The paper is concerned with the derivation of an implicit secular equation for Rayleigh waves in a two-dimensional orthotropic linear elastic medium. It has been proven that the Rayleigh waves propagate with no geometric dispersion. It has been found that Rayleigh wave velocity depends significantly (like bulk waves) on the directions of principal material axes. The analytical solution, based on the implicit secular equation, was compared against the finite element and experimental data that had been published by Cerv et al. [4] in 2010. The theory turned out to be in good accordance with experiment.

Keywords: Rayleigh waves, secular equation, orthotropic materials.

INTRODUCTION

Surface acoustic waves propagating on isotropic elastic half-space were first studied by Rayleigh [1]. Hence, today, the surface waves are often referred to as the *Rayleigh waves* (RW). Rayleigh showed that the effect of RW decreased rapidly with depth and their speed of propagation was smaller than that of body waves. Much development has been witnessed since. There were numerous attempts at deriving similar solutions for anisotropic materials, which was recently summarized by Ting [2]. It turned out, however, that one encountered considerable mathematical difficulties, as noted, e.g., by Favretto-Cristini et al. [3], both in forming a solvable secular equation and selecting physically meaningful roots. Although most of the studies on RW concerned crystalline solids there were applications to two-dimensional (2D) composites, as in reference [4] and other fields of material science - see Hess' review [5].

PROBLEM FORMULATION AND SOME RESULTS

The propagation of a Rayleigh wave along the free edge of a semiinfinite 2D orthotropic medium is modeled. It is supposed that corresponding displacement field has the form

$$\begin{aligned} u_1(x_1,x_2,t) &= U_{01}\, e^{q x_2}\, e^{\mathrm{i}k(x_1-ct)}\,, \\ u_2(x_1,x_2,t) &= U_{02}\, e^{q x_2}\, e^{\mathrm{i}k(x_1-ct)}\,, \end{aligned} \tag{1}$$

where k is the wavenumber, c unknown velocity, t time and q a complex parameter dependent on the elastic stiffnesses C_{ij}, c and mass density ρ. It is stipulated that $\mathrm{Re}(q) < 0$. This solution represents a harmonic wave propagating in the positive direction of the x_1-axis. The boundary conditions of the problem are: the edge $x_2 = 0$ is free of tractions, vanishing displacement (or stress) components at infinity, i.e. at $x_2 \to \infty$. The implicit secular equation may be symbolically written as

$$F(C_{ij}, \vartheta, \rho, c) = 0\,, \tag{2}$$

where ϑ represents generally nonzero angle between body and principal material axes. Knowing the material constants of the thin composite SE84LV and angle ϑ, we can evaluate the left-hand side of the equation (2) as a function of velocity c. We obtain just one root $c_R = 1887.776$ m/s. This root corresponds to the Rayleigh-edge wave velocity.

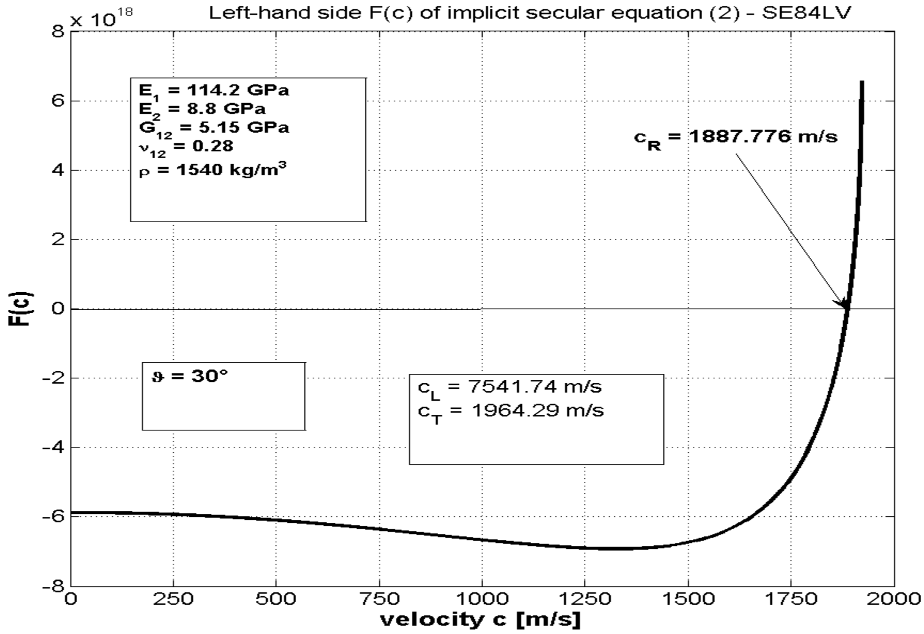

Fig. 1 - Left-hand side $F(c)$ of the implicit secular equation (2) versus velocity c for $\vartheta = 30^\circ$.

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