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DISPLACEMENT AND STRAIN FIELDS ASSESSMENT OF PDMS USING DIGITAL IMAGE CORRELATION

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ABSTRACT

The main goal of this work is the characterization of the hyper-elastic mechanical behaviour of PDMS. The special specimens of PDMS (Sylgard® 184) were tested in a bi-axial tensile machine. The displacement and strain fields were measured using a commercial digital image correlation system (ARAMIS of GOM) during the tensile test. The experimental measurements are compared with numerical simulations, which use the most popular algorithms of constitutive models to characterize the hyper-elastic behaviour.

Keywords: polydimethylsiloxane, hyper-elastic, digital image correlation, finite element method.

INTRODUCTION

The polydimethylsiloxane (PDMS) is an elastomer with very attractive properties for various applications in different fields, like biomedical engineering (Yabuta, 2003) and electronics (Andersson, 2003). In last years, they have been used in the development of micro and nanodevices (Mata, 2005), optical MEMs (Schneider, 2009), among others. These news applications demand a better understanding of PDMS mechanical behavior, which only could be achieved using new experimental and numerical approaches. Until now, most of experimental works are based on uniaxial tensile tests, which are characterized by high level of dispersion of the mechanical properties. In addition, the new applications of PDMS material demand a more detail characterization of their mechanical behavior, being the optical experimental techniques more suitable to supply this information. These materials present a hyper-elastic behavior, presenting high deformations levels, which can only be measured with a few optical techniques. In this work is used the Digital Image Correlation (DIC) optical technique to measure the displacement and strain fields of a specimen during a bi-axial tensile test.

DIGITAL IMAGE CORRELATION (DIC)

The digital image correlation (DIC) is a non-contact measurement technique (Sutton, 1983, 1986, 1991, 1988) and (Bruck, 1989), which method uses a mathematical correlation to estimate the displacement in the surfaces or structures of components subject to thermal or mechanical stresses. The digital image correlation is based on a comparison of speckle pattern between digital images capture in different deformation states. This allows to obtained the surface a speckle pattern must be created on the object surface in order to evaluate the displacement vector field. To increase the DIC efficiency, the image must be divided in

blocks of pixels with randomly distributed gray levels, being each block unique (Sutton, 1988).
The DIC technique only requires the use of white light source (incoherent light), in order to increase the image contrast. The images are digitally recorded using a video camera during the loading tests, to be post-processing the DIC technique. In the reference state, the image is divide is small blocks of pixels or areas, which presents independent speckle pattern. Through the correlation process of these patterns between the reference and the following recorded images, is obtained the correspondent displacement and strains fields produced by the loading. The correlation technique can be explained base on the schematic diagram represented in Fig.1, where f (x, y) the discrete function that defines the grayscale pixel of the original image and f * (x *, y *) is the final pixel of the image (Hu, 1985). The relationship between the two functions is given by:

$$f * (x^{*}, y *) = f\big(x + u(x, y), y + v(x, y)\big) \qquad \text{(Eq. 1)}$$

where u and v represent the displacement field (Fig. 1).

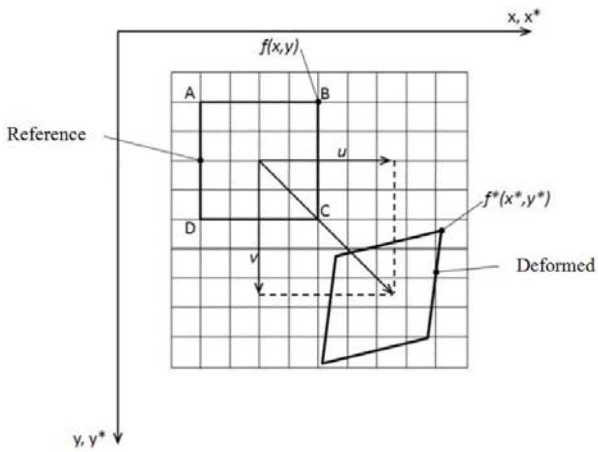


Fig.1 Variation of the initial (reference state) to the final state (deformed state).

The correlation is applied to all patterns in the center of the virtual grating of the initial image, thereby obtaining the displacement field of each network element. Moreover, the strain fields are extracted by analyzing the distortion of the virtual grating.
The displacement field for a random pattern, which is homogeneous and bilinear along the axes x and y is given by:

$$u(x, y) = a_u x + b_u y + c_u xy + d_u \qquad \text{(Eq.2)}$$

$$v(x, y) = a_v x + b_v y + c_v xy + d_v \qquad \text{(Eq.3)}$$

For each loading state, the displacements at points A, B, C and D are computed by iterative processes, thus giving an exact solution. These offsets are calculated during the iteration "i" with the components of the displacement field in the iteration "i-1". This iterative process ends at iteration "n" when the relative position of vertices defined by the virtual grating varies below a critical value, δ_0, defined in the algorithm.
The initial image of pixels moves to a position of sub-pixel on the deformed image. The difficulty lies in defining levels of gray between these points. Therefore, it is used an interpolation function, the most common being the bilinear interpolation of first order (Sutton, 2009). With the mathematical correlation of f(x, y) and f*(x*, y*), it is possible to determine the displacement field u(x, y) and v (x, y). The correlation coefficient can be calculated by the least squares:

$$C_1 = \int_{\Delta M} \left(f(x,y) - f^*(x^*,y^*)\right)^2 dxdy \quad \text{(Eq.4)}$$

where ΔM is the random pattern on the surface. The correlation coefficients are minimized in the determination of the displacement field.
This technique has been used also in the high resolution deformation measurement (Marcellierl, 2001) (P. Hung, 2003).

FINITE ELEMENT METHOD

Currently, the Finite Element Method (FEM) has been used to study the mechanical behavior of hyper-elastic materials. These studies are based on mechanical models obtained from experimental uniaxial tests. To reproduce numerically the nonlinear hyper-elastic behavior of this material is necessary to develop new and more accurate constitutive models. The developments of hyper-elastic models are supported in two different theories: micromechanical model and macro mechanical model. The micromechanical models are developed from the chemical manufacture of the information material, and are based on the concept of the unit cell. The second theory, phenomenological models are based on the material behavior observed during the experimental tests. For development of these models is necessary to know the mechanical behavior of the material, through the experimental tensile test (Holzapfel, 2000).
The Hyper-elastic materials are known to have a non-linear relationship between stress and strain, law not applicable. Thus, the hyper-elastic material behavior is normally defined base on strain energy or stored energy.
The hyper-elastic materials are commonly defined as having nonlinear mechanical properties, presenting large deformations rates. The theory of hyper-elastic material behavior also known, as the Green elastic material, is defined as a function of the Helmholtz free energy, also called the strain energy or stored energy (Ψ). This describes the behavior of this class of materials in terms of energy mechanical, and can be defined according to the following equation:

$$P = \frac{\partial \Psi}{\partial F} \quad \text{(Eq.5)}$$

or, in more general:

$$P = -pF^T + \frac{\partial \Psi}{\partial F} \quad \text{(Eq.6)}$$

being P the first stress tensor of Piola-Kirchhoff, F^T the transposed of deformation gradient, p a multiple of Lagrange obtained according to the state of tension T.
The function of Helmholtz free energy Ψ is a thermodynamic potential, which measures the useful work for a closed thermodynamic system with constant temperature and volume (Pascon, 2008).
A model of hyper-elastic materials depends on the definition of strain energy function which assumes different shapes, according to the material or class of materials considered. This function is obtained from symmetry and thermodynamic energy considerations (Pascon, 2008).
For simplicity, is assumed the material is isotropic and incompressible. As the isotropic material, the strain energy functions (Ψ), depend on the invariants of the deformation.

$$\Psi_{isotrópico} = \Psi(I_1, I_2, I_3) \quad \text{(Eq.7)}$$

where the invariants are define as:

$$I_1 = \sum_{i=1}^{3} \lambda_i^2$$

$$I_2 = \sum_{i,j=1}^{3} \lambda_i^2 \lambda_j^2 \quad i \neq j \quad \text{(Eq.8)}$$

$$I_3 = \prod_{i=1}^{3} \lambda_i^2$$

being, λ_1, λ_2 and λ_3 are the principals deformations.
If the material is too incompressible, the third invariant $I_3 = 1$, equation 7 is defined as:

$$\Psi_1 = \Psi(I_1, I_2) \quad \text{(Eq.9)}$$

From the equation of the Cauchy tensor and the calibration of the main experimental tensile tests (uniaxial and biaxial) are determined the constitutive equations of hyper-elastic models.

RESULTS

The load curves of the bi-axial tensile test are shown in Fig. 2. By observing the load-displacement curve it is possible to verify a typical behaviour of hyper-elastic materials.

The specimen preparation involves the application of a random speckle pattern on the material surface by applying a thin white and black ink coat. A sequence of images of the specimen surface was digital using video CCD camera recorded during the bi-axial tensile test. These were later post-processed by the commercial DIC software in order to extract the displacement and strain fields.

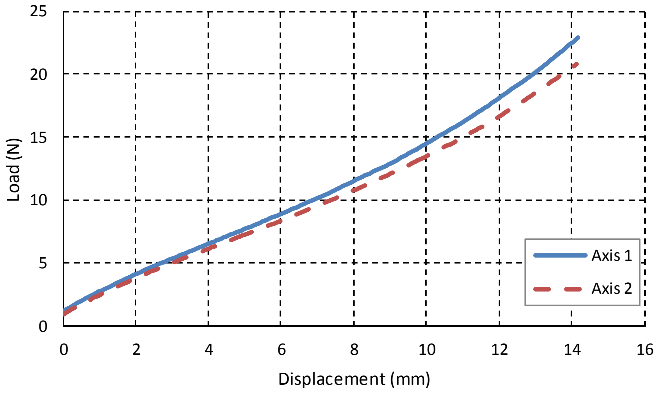
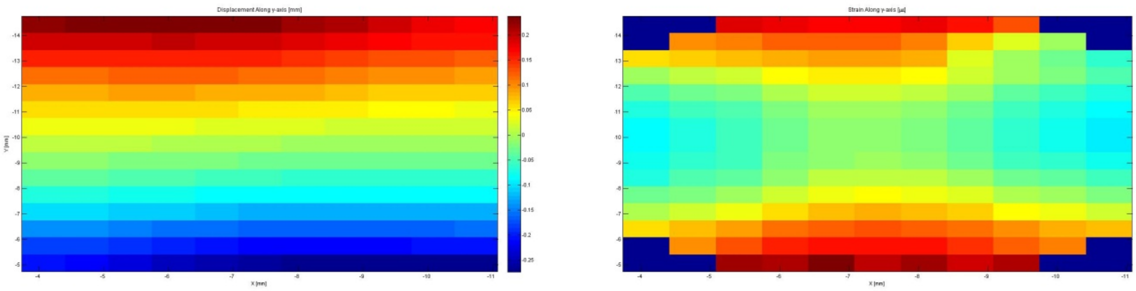


Fig.2 Bi-axial tensile test results

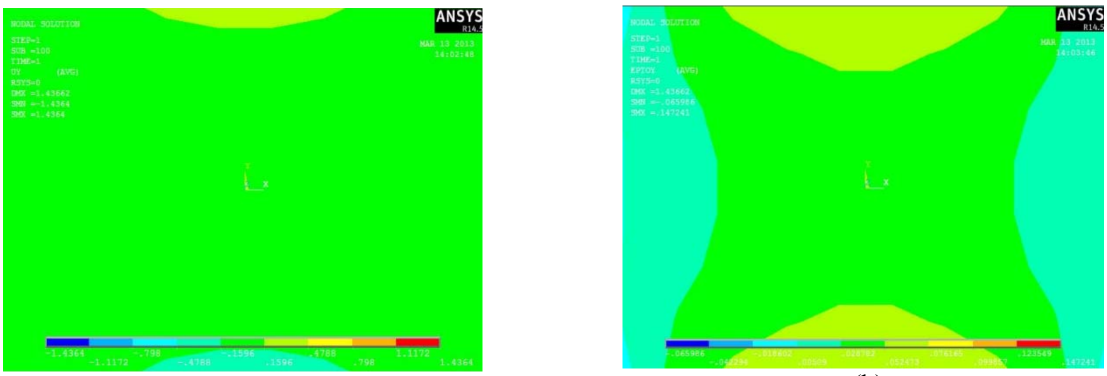
In the Fig. 3 are shown the displacement and strain fields for a load case of 7 N applied in the vertical direction, with a crosshead speed of 5mm/min.



(a) (b)

Fig.3 The experimental results vertical direction, obtained using digital image technique DIC: (a) displacement field; (b) strain field.

A non-symmetry vertical displacement distribution is observed in the Fig. 3, which can be explained by the misalignment of the specimen during the loading test. The correspondent numerical simulations using finite element method are presented in Fig. 4.



(a) (b)

Fig.4 The experimental results vertical direction, obtained using Finite Element Method (FEM): (a) displacement field; (b) strain field.

In the Figure 5 is presented a comparative analysis of the displacement profile for the vertical direction between the experimental measurements and the numerical simulation.
This reveal a similar trend of the material behavior, i.e. there is an approximately linear increase of the displacements.

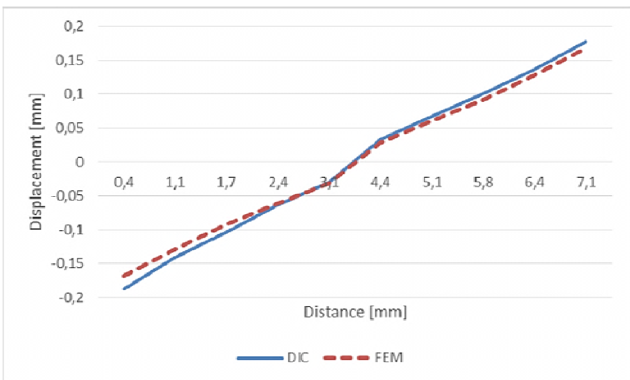


Fig.5 Variation of displacement in the Y direction, measured experimentally with the DIC and determined numerically with FEM.

The results show a small difference in the displacement values between the experimental and numerical simulation, being the average relative error 8.3%, which is considered very acceptable for this type of test.

CONCLUSIONS

The optical technique of Digital Image Correlation proved well adapted for displacement and strain fields measurement of hyper-elastic materials. The results show that this technique correlates well in the displacements when high spatial resolution is used, allowing extracting the information for large deformation amplitudes. This technique is highly dependent on the quality of the random pattern. In this work, it was necessary to test different types of patterns and, it was found that, the use of black ink spray lead to acceptable results. However, further tests are needed in order to improve the quality of the results.

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