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ANN-MCS APPROACH FOR GLOBAL UNCERTAINTY ANALYSIS OF COMPOSITE STRUCTURES

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ABSTRACT

The uncertainty propagation of composite structures is investigated in this work considering descriptive statistical measures of the response variability and sensitivity analysis of system responses inside inverse reliability-based design optimization (RBDO) framework. A study based on sensitivity to uncertainty that allows selecting the important parameters using global sensitivity indices is presented. The uncertainty propagation and the importance measure of input parameters are analysed using an Artificial Neural Network-based Monte Carlo simulation approach (ANN-MCS). The proposed methodology uses the optimal loading conditions obtained solving the inverse RBDO problem.

Keywords: uncertainty, RBDO, composites, global sensitivity, Sobol indices, ANN-MCS

INTRODUCTION

Although several methods have been presented for uncertainty assessment, their efficiency was not proven, in particular when applied to composite structures. The almost totality of sensitivity analyses in applications with composite structures used local importance measures of design parameters. In particular Rais-Rohani and Singh (2004) and Carbillet et al. (2009) studied the sensitivity of reliability index of composite structures with non-linear behaviour and quantified the importance of the random variables using local measures. Although the merit of the proposed approaches Global Sensitivity Analysis (GSA) on the uncertainty response is still unexplored and remains an open issue. So, the uncertainty propagation of composite structures is investigated in this work considering descriptive statistical measures of the system response variability inside GSA framework. In particular, this is implemented using the optimal loading conditions obtained from inverse reliability-based design optimization (RBDO).

UNCERTAINTY PROPAGATION ANALYSIS

The objective of the proposed approach is to study the propagation of uncertainties in input random variables, such as mechanical properties, on the response of composite laminate structures for a specified reliability level. Fig. 1 shows the proposed Artificial Neural Network based Monte Carlo simulation procedure (Conceição António and Hoffbauer, 2013).

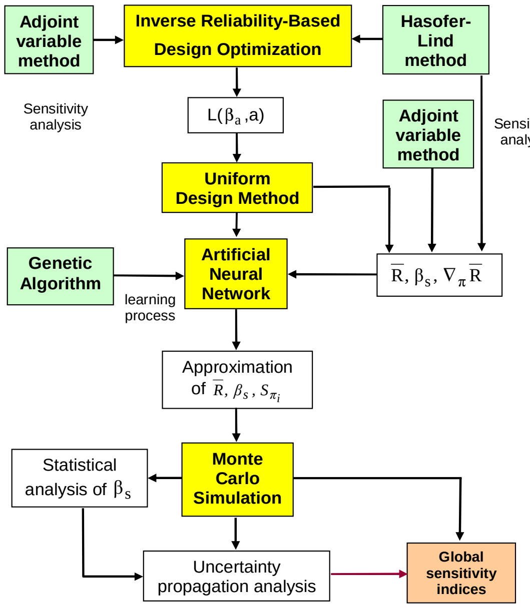

Fig 1: Flowchart of proposed approach for uncertainty propagation analysis

For a target reliability index β_a, the inverse problem can formulated as follows:

$$\underset{\lambda,\, a}{\text{Minimise}} \; \left[\beta_S(\lambda,\, a, \boldsymbol{\mu}_{\boldsymbol{\pi}}) - \beta_a\right]^2 \tag{1}$$

subject to:

$$0 \le a \le \frac{\pi}{2}$$

where β_s, is the structural reliability index, $\boldsymbol{\mu}_{\boldsymbol{\pi}}$ is the realization of random variable $\boldsymbol{\pi}$. The mean values, $\overline{\pi}_i$, of mechanical properties of composite laminates are considered for $\boldsymbol{\mu}_{\boldsymbol{\pi}}$. The design variables are the ply angle, a, and load factor, λ. The vector of applied loads is defined as $\mathbf{L} = \lambda\, \mathbf{L}^{ref}$, where $\mathbf{L}^{ref}$ is the reference load vector and after solution of the problem in equation (1) the corresponding maximum load is computed for each value of ply angle a. This is a conventional RBDO inverse optimization problem. To solve the inverse problem (1), a decomposition of the problem is considered. The Hasofer-Lind method and appropriate iterative scheme based on a gradient method are applied to evaluate the structural reliability index, β_S, in the inner loop (Conceição António, 1995). From the operational point of view, the reliability problem can be formulated as the constrained optimization problem

$$\underset{\mathbf{v}}{\text{Minimize}} \; \beta(\mathbf{v}) = \left(\mathbf{v}^T\, \mathbf{v}\right)^{1/2}$$

$$\text{subject to: } \varphi(\mathbf{v}) = 0 \tag{2}$$

where $\mathbf{v}$ is the vector of the standard normal variables, β is the reliability index and $\varphi(\mathbf{v})$ is the limit state function.

The limit state function that separates the design space into failure ($\varphi(\boldsymbol{\pi}) < 0$) and safe regions ($\varphi(\boldsymbol{\pi}) > 0$) can be written as

$$\varphi(\boldsymbol{\pi}) = \overline{R} - 1 \tag{3}$$

where $\overline{R}$ is the critical *Tsai number*, defined as

$$\overline{R} = Min(R_1,..., R_k,...,R_{N_s}) \tag{4}$$

and N_s the total number of points where the stress vector is evaluated. The *Tsai number*, R_k, which is a strength/stress ratio (Tsai, 1987), is obtained from the Tsai-Wu interactive quadratic failure criterion and calculated at the *k-th* point of the structure solving equation

$$1-\left(F_{ij}\, s_i\, s_j\right)R_k^2 + \left(F_i\, s_i\right)R_k = 0 \tag{5}$$

where s_i are the components of the stress vector, and F_{ij} and F_i are the strength parameters associated with unidirectional reinforced laminate defined from the macro-mechanical point of view (Tsai, 1987). The solution of the reliability problem in equation (2) is referred to, in technical literature, as the design point or most probable failure point (MPP). The bisection method used to estimate the load factor, λ, is iteratively used in the external loop (Conceição António and Hoffbauer, 2009, 2013).

The proposed ANN is organized into three layers of nodes (neurons): input, hidden and output layers. The linkages between input and hidden nodes and between hidden and output nodes are denoted by synapses. These are weighted connections that establish the relationship between input data and output data.

In the proposed ANN-MC approach, each set of input values for the random variable vector $\boldsymbol{\pi}$ is selected using the Uniform Design Method (UDM) (Fang and Wang, 1996). The procedure is based on a UDM table denoted by $U_n(q^s)$, where U is the uniform design, n the number of samples, q the number of levels of each input variable, and s the maximum number of columns of the table. For each UDM table, there is a corresponding accessory table, which includes a recommendation of columns with minimum discrepancy for a given number of input variables. Using the UDM a set of design points belonging to the interval $\left[\overline{\pi}_i - \alpha\,\overline{\pi}_i\,,\,\overline{\pi}_i + \alpha\,\overline{\pi}_i\right]$ is generated, covering a domain centred at mean reference values of the random variables. This method enables a uniform exploration of the domain values necessary in the development of an ANN approximation model guarantying better results after learning procedure (Cheng and Xiao, 2008). The corresponding output data vector contains the critical Tsai number, $\overline{R}$, structural reliability index, β_s, and relative sensitivities S_{π_i} of reliability index with respect to random variables. The concept of relative sensitivity (Cacuci, 2003) of the reliability index is defined as

$$S_{\pi_i} = \left|\frac{\partial \beta_s}{\partial \pi_i}\right| \left|\frac{\overline{\pi}_i}{\beta_s}\right| \tag{5}$$

and its analysis aims to compare the relative importance of input parameters on the response. Fig. 2 shows the topology of the ANN, showing the input and output parameters.

The adopted supervised learning process of the ANN based on a Genetic Algorithm (GA) (Conceição António, 2001) uses the weights of synapses and biases of neural nodes at the hidden and output layers as design variables. A binary code format is used for these variables. The number of digits of each variable can be different depending on the connection between the input-hidden layers or hidden-output layers. A GA is an optimization technique based on the survival of the fittest and natural selection theory proposed by Charles Darwin. The genetic algorithm (Conceição António, 2001) basically performs on three parts: (1) coding and decoding random variables into strings; (2) evaluating the fitness of each solution string; and (3) applying genetic operators to generate the next generation of solution strings in a new population.

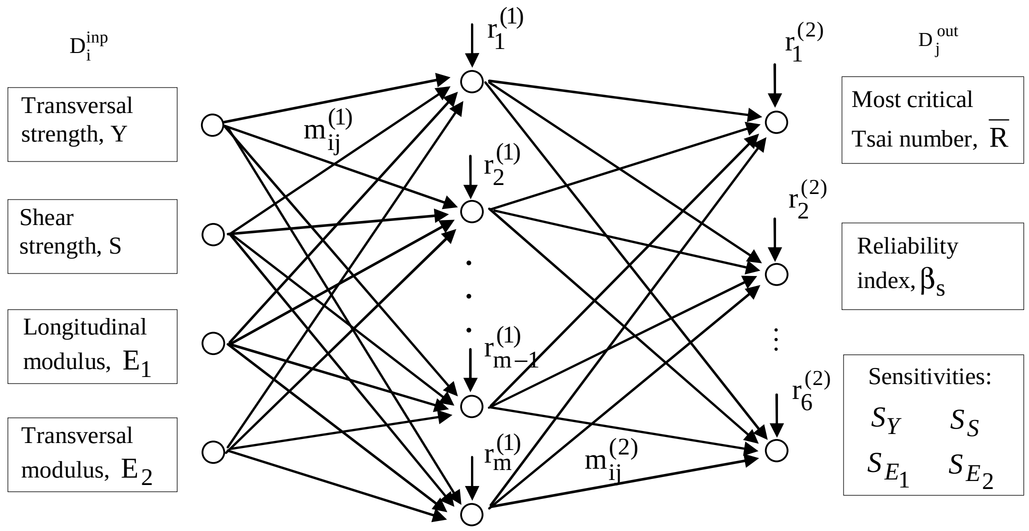

Fig. 2: Artificial Neural Network topology

Three basic genetic operators, namely selection, crossover, and mutation are used in this paper. An elitist strategy based on conservation of the best-fit transfers the best-fitted solution into a new population for the next generation. Once the new population is created, the search process performed by the three genetic operators is repeated and the process continues until the average fitness of the elite group of the current generation no longer shows significant improvement over the previous generation. Further details on creating and using a genetic algorithm for ANN learning can be found in the reference (Conceição António, 2001).

GLOBAL SENSITIVITY ANALYSIS

The local measures of sensitivity are not enough for a full evaluation of the influence of input parameters on structural response uncertainty (Conceição António and Hoffbauer, 2008). The uncertainty analysis on response in the neighbourhood of mean values of input parameters is of limited value. To obtain the influence of individual parameters on the uncertainty at the output structural response Ψ_m Global Sensitivity Analysis (GSA) techniques must be used. Global Sensitivity Analysis denotes the set of methods that consider the whole variation range of inputs and tries to share the output response uncertainty among the input parameters.

Assuming that $\mathbf{X}=(X_1,...,X_n)$ are n independent input parameters and Ψ_m is the performance function of structural response previously defined, an indicator of the importance of an input parameter X_i is the following normalized index

$$S_i = \frac{var(E\langle \Psi_m \,|X_i\rangle)}{var(\Psi_m)} \tag{6}$$

named *first-order sensitivity index* proposed by Sobol (2001). In equation (6) $var(E\langle \Psi_m \,|X_i\rangle)$ is the variance of the conditional expectation and $var(\Psi_m)$ is the variance of Ψ_m. Furthermore, Sobol (2001) proposed a complete variance decomposition of the uncertainty associated with Ψ_m into components depending on individual parameters and interactions between individual parameters. This procedure explains the variance $var(\Psi_m)$ as a contribution of the partial variance associated to each individual parameter. From this decomposition higher order sensitivity indices can be established in particular the second order sensitivity index. The second order index S_{ij} defines the sensitivity of the structural response Ψ_m to the interaction between X_i and X_j, i.e. the portion of the variance of Ψ_m that is not included in the individual effects of X_i and X_j. The sum of all order indices is equal to 1 in case all input parameters are independent.

Since higher order sensitivity indices require tedious calculations only the Sobol first-order sensitivity index is used in the presented work. One of the problems using global sensitivity indices is the computational cost. Due to the large number of input parameters in the uncertainty propagation analysis on composite structures, Finite Element Method evaluations become very expensive. In this work the ANN-based Monte Carlo simulation approach is used for the estimation of GSA indices. To reduce the computational costs the analysis is implemented using groups of input parameters and considering only the Sobol *first-order sensitivity index*. The proposed methodology is based on the algorithm described in (Conceição António and Hoffbauer, 2008, 2013).

RESULTS

Let’s consider an aircraft wing-like composite panel as shown in Fig. 3. The panel thickness is equal to 0.015 m. The structure is clamped along linear side (AB) and free along opposite side. One vertical load with perpendicular direction relatively to OXY plan is applied on point C. The structure is built by one laminate made of a carbon/epoxy composite system as presented in Table 1. A balanced angle-ply laminate with eight layers and stacking sequence $[+a/-a/+45^o/-45^o]_S$ is considered in a symmetric construction. Ply angle *a*, is referenced to the *x*-axis of the reference coordinate, as detailed in Fig. 3. All plies have same thickness.

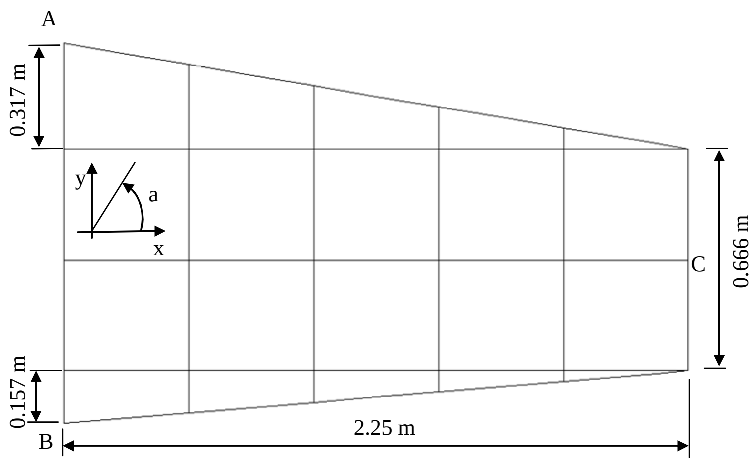

Fig. 3: Geometric definition of aircraft wing-like composite panel

A shell finite element is used for structural analysis. To assess reliability the previously described procedure in equations (2) to (5) is applied considering the vector of random variables $\boldsymbol{\pi} = [E_1, E_2, Y, S]$.

Table 1 Mean reference values of mechanical properties of composite layers

Material	E_1 [GPa]	E_2 [GPa]	G_{12} [GPa]	ν
T300/N5208	181.0	10.3	7.17	0.28
	X ; X' [MPa]	Y ; Y' [MPa]	S [MPa]	ρ [kg/m3]
T300/N5208	1500 ; 1500	40 ; 246	68	1600

The target reliability index is $\beta_a = 3$ and the coefficient of variation of each random variable is set to $CV(\boldsymbol{\pi}) = 6\%$, relatively to the mean values presented in Table 1. The corresponding maximum load is plotted in Fig. 4 and it is used as the reference load for further development of the ANN supported by UDM and GA-based learning procedure.

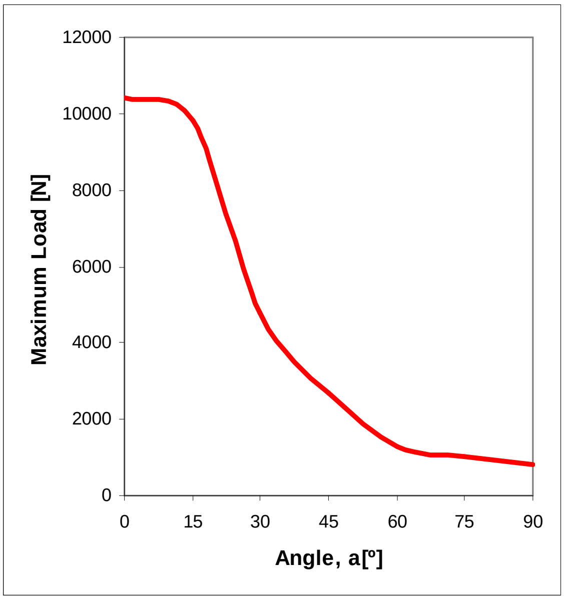

Fig. 4: Maximum load for $\beta_a = 3$, solving the inverse RBDO problem for aircraft wing-like composite panel.

The ANN is developed using UDM points. After obtaining the new optimal ANN for aircraft wing-like composite panel, the uncertainty propagation analysis is performed. A set of random numbers, $N_f = 50$, following a normal distribution $N(0,1)$ and a sample matrix $\mathbf{M}_\alpha$ with dimension $N_r \times (p-1) = 100 \times 3$ are used in GSA algorithm for a total of twenty thousand simulations. The GSA is implemented and the Sobol *first-order sensitivity index* S_i is calculated as a function of ply angle, *a*. Fig. 5 and Fig. 6 show the contribution of each random variable for global variance $var(\Psi_m)$ using two responses functions of the composite structure. The Sobol *first-order sensitivity index* (Sobol, 2001). is used as importance measure and the contribution is represented as a fraction of the total values at each ply angle. Fig. 5 plots the results for structural response analysis based on critical *Tsai number* $\overline{R}$. Similar analysis is performed using the reliability index β_s as response functional of the structure and plotted in Fig. 6. The most important random variable in global variance explanation of $\overline{R}$ is the transversal strength Y for whole domain of ply angle as shown in Fig. 5. Also the shear strength S is important in interval [15º, 45º]. The longitudinal elastic modulus E_1 has relevant importance in interval [45º, 75º] and the elastic transversal modulus E_2 is important for whole domain of ply angle *a*, except for 75º.

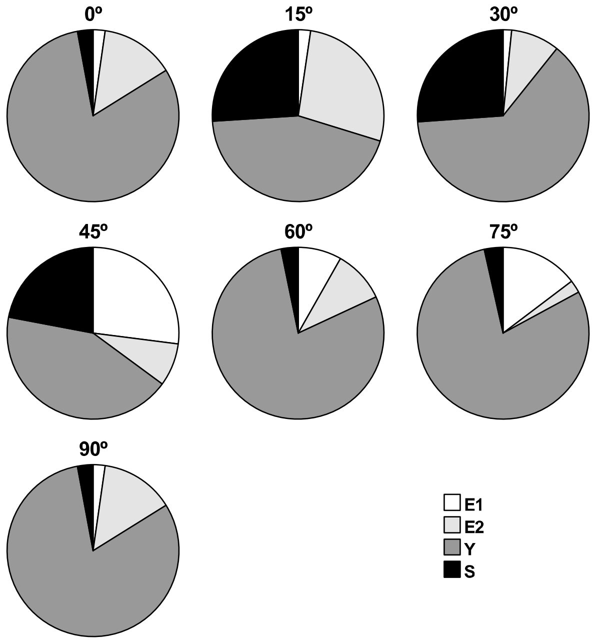

Fig. 5: Global variance $var(\overline{R})$, explained by Sobol first-order sensitivity index S_i for input parameters $\boldsymbol{\pi} = [E_1, E_2, Y, S]$, aircraft wing-like composite panel.

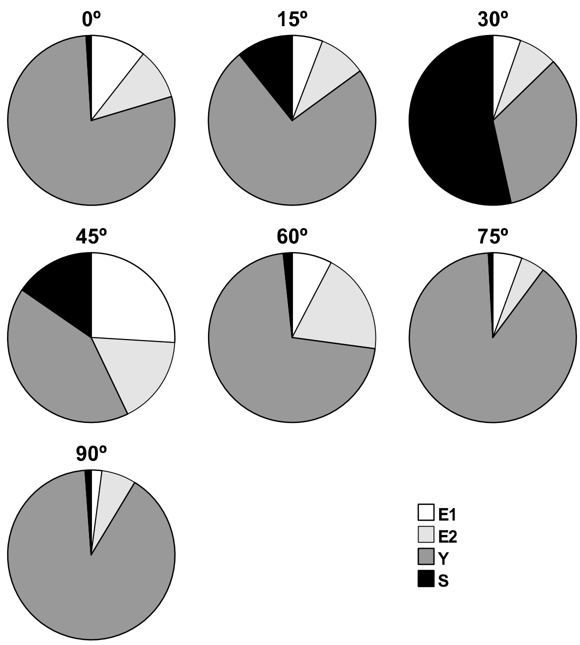

Fig. 6: Global variance $var(\beta_S)$, explained by Sobol first-order sensitivity index S_i for input parameters $\boldsymbol{\pi} = [E_1, E_2, Y, S]$, aircraft wing-like composite panel.

Analysing the results plotted in Fig. 6 it can be concluded that the most important random variable to explain global variance $var(\beta_S)$ is the transversal strength Y except for ply angle a equal to 30º where the shear strength S is the most important. Furthermore, the shear strength has important contribution to explain $var(\beta_S)$ in the interval [15º, 45º]. The balanced contribution of the four random variables $\boldsymbol{\pi} = [E_1, E_2, Y, S]$ for ply angle a equal to 45º is another relevant observation.

The global variance of critical *Tsai number* $\overline{R}$ and of the reliability index β_S can be explained by Sobol indices in different manner when the ply angle $a \in [15^o, 45^o]$. Since $\overline{R}$ is associated to a deterministic analysis and β_S is associated to a probabilistic analysis of failure a different behaviour in uncertainty propagation was expected.

CONCLUSIONS

A study of the anisotropy influence on uncertainties propagation of composites is carried out based on the proposed methodologies. The study proves that the variability of the structural response as a function of uncertainty of the mean values can be very high. This high variability is also corroborated by evaluated relative sensitivity measures. These aspects must be considered for robust design since high structural response variability may induce a drastic reduction in the quality of the optimal design solutions for composite structures. Based on the numerical results, the importance of measuring input parameters on structural response are established and discussed as a function of the anisotropy of composite materials. Some difference was found depending on a deterministic or a probabilistic analysis of structural failure. The uncertainty analysis propagation is very useful in designing laminated composite structures minimizing the unavoidable effects of input parameter uncertainties on structural reliability

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