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PHOTOELASTIC AND NUMERICAL ANALYSIS OF STRESSES DEVELOPED IN GEAR TEETH AND NUMERICAL ANALYSIS OF TWO ORTHOGONAL CYLINDERS IN CONTACT

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ABSTRACT

The double refraction phenomenon called also birefringence is used to analyse the stress field developed in the neighbourhood of the contact zone of two gear teeth that are very important for the transmission of power and motion in various machineries. Experimental photoelastic fringes (isochromatics and isoclinics) obtained on the analyser with plan polarized light allow respectively the determination of the principal stresses difference and their directions. This is of great importance in the design of mechanical components as their shapes and dimensions can be well optimized in order to achieve lower operating stresses. A whole field comparison of the experimental and numerical fringes obtained with the finite elements simulation, and a local analysis of the principal stresses difference, allowed us to validate the numerical approach. Relatively good agreements are obtained. In the second part of the paper, a finite element solution is developed for a 3D contact problem between two orthogonal cylinders. Isochromatic fringes and stresses are obtained for different slices along the cylinder.

Keywords: birefringence, photoelastic fringes, isochromatic, isoclinic, contact, gear tooth

INTRODUCTION

For metallic parts, stress initiation is mainly controlled by shear stress mechanisms. It is therefore very important to determine the type and the amplitude of the imposed mechanical stresses. Photoelastic fringes obtained experimentally with plan polarized light can help designers determine stress fields developed in mechanical parts, particularly in the neighbourhood of the contact zones. This is of great importance in the design of mechanical components as their shapes and dimensions can be well optimized in order to achieve lower operating stresses.

Theoretical studies of these contact stresses can be in some cases very complex. Several methods, experimental as well as numerical, have been used by various authors [1-7] to analyse these types of problems. In this study we use the photoelasticity method to determine the stress field. Experimental photoelastic fringes (isochromatics and isoclinics) obtained on an analyser with plan polarized light allow respectively the determination of the principal stresses difference and their directions. A finite element solution is developed to calculate the photoelastic fringes and the stresses developed in the model particularly in the neighbourhood of the contact zone. Two comparisons are made; the first one is made between experimental and simulated photoelastic fringes, the second one is made between experimental and simulated values of the maximum shear stress along the vertical axis of symmetry to validate the finite elements solution.

EXPERIMENTAL ANALYSIS OF STRESSES

The model is cut in a birefringent material (PLM4) mixed with hardener (PLMH). Poisson's ratio and Young's modulus are respectively equal to $\mu= 0.37$ and $E=2437$ MPa. Before analyzing the model one should determine the fringe constant which depends on the light wavelength and the optical constant of the model material. The light intensity eqn (1), obtained on the analyzer after traveling through the model and the different optical elements, is given by the following relation for a plan polarized light [8]:

$$I = a^2 \sin^2 2\alpha \, \sin^2 \varphi/2 \qquad (1)$$

It is well known that the terms $sin^2 2\alpha$ and $\sin^2\varphi/2$ represent respectively the isoclinic fringes and the isochromatic fringes obtained on the analyzer. The values of the principal stresses difference is given by the following relation:

$$\sigma_1 - \sigma_2 = \frac{Nf}{e} \qquad (2)$$

Where N is the fringe order obtained experimentally from the isochromatic fringe pattern, e is the model thickness and f is the fringe constant of the model ($f = \frac{\lambda}{c}$). A four point bending test which allows to set a constant flexure moment (Fig. 1) is used to record isochromatic fringes that can easily be used to determine the fringe constant necessary to analyze the stress field in the model. After analysis we obtained a value f=10.7 N/mm/fringe.

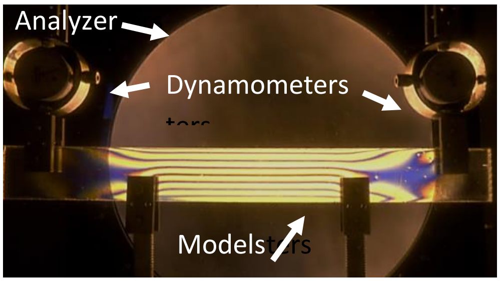


Fig. 1 : The four point loading test

The model mounted on a loading frame (Fig. 2) is then positioned in the polariscope; both plan polarized light and circularly polarized light are used in order to determine the isochromatics and the isoclinics which are the photoelastic fringes used to determine respectively the principal stresses difference and their directions.

Isoclinic fringes are the dark fringes shown by the white arrow (Fig. 3), obtained for different analyzer and polarizer positions. The polarizer and the analyzer set at right angles to each other are rotated simultaneously; isoclinics are registered every 15°. These isoclinics can be easily used to determine the isostatics. The purpose here, however, is to develop a finite element solution which allows obtaining numerically the photoelastic fringes.
We can see that the isoclinics hide partially the isochromatics. A

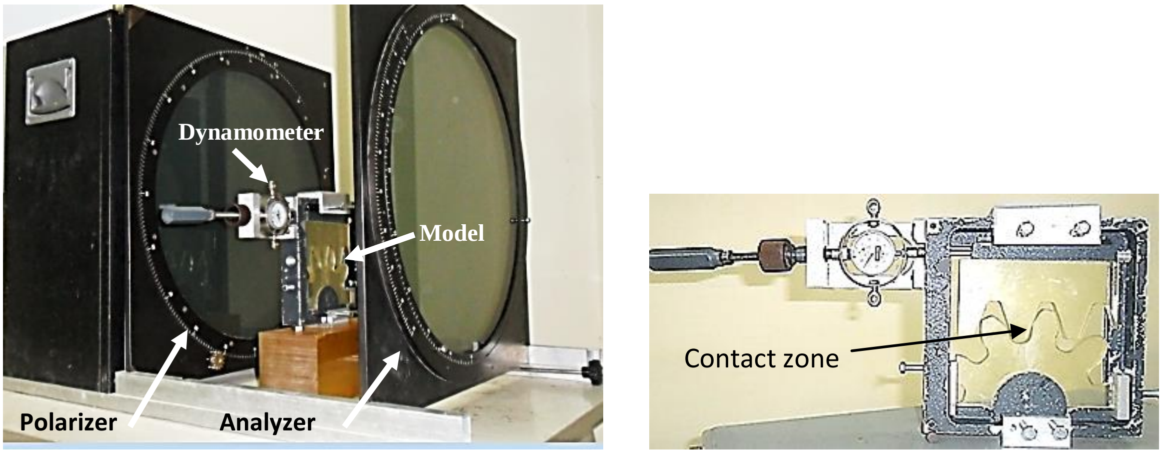


Fig. 2: Experimental set up (left), loading frame (right)

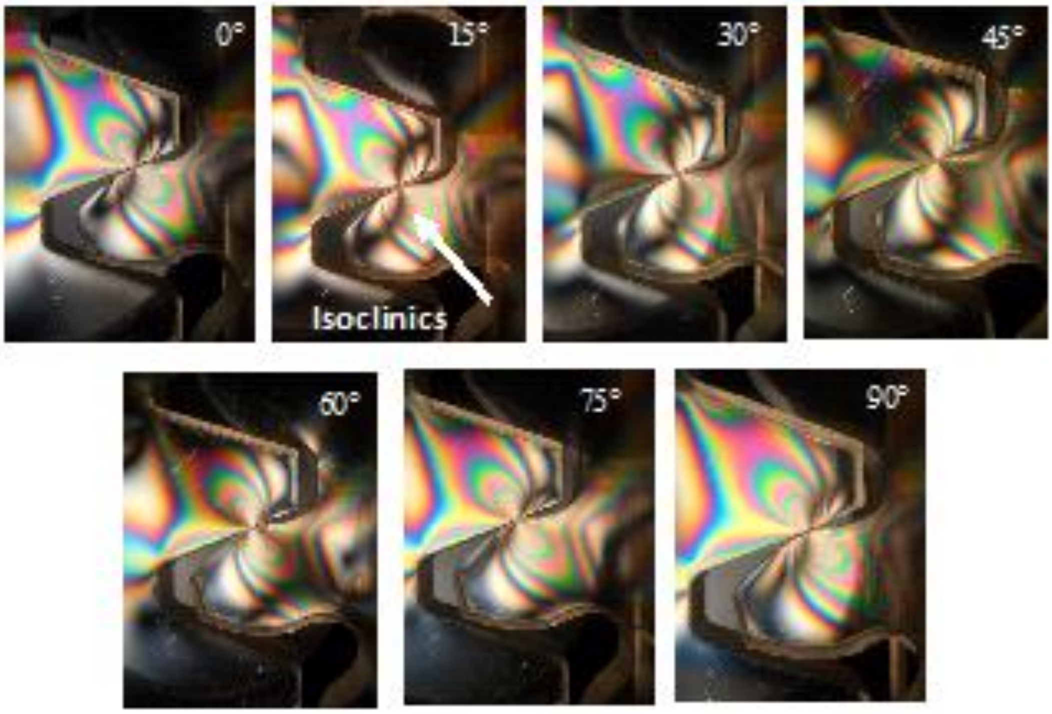


Figure 3: Isoclinics for different polariscope settings

In order to observe clearly the isochromatic fringes, we use a circularly polarized light obtained by adding two quarter wave plates in the polariscope setting. Fringe orders are determined then on the recorded fringe pattern (Fig. 4, left) in order to obtain the graph of the principal stresses difference on a chosen line for comparison purposes with the finite element solution. A close up of the contact area (figure 4, right) allows us to determine correctly the fringe orders in the neighborhood of the contact zone. For the close up image, the model is positioned in the light path in such a way as to record as clearly as possible the isochromatic fringes in the neighborhood of the contact zone in order to obtain accurately the stress values.

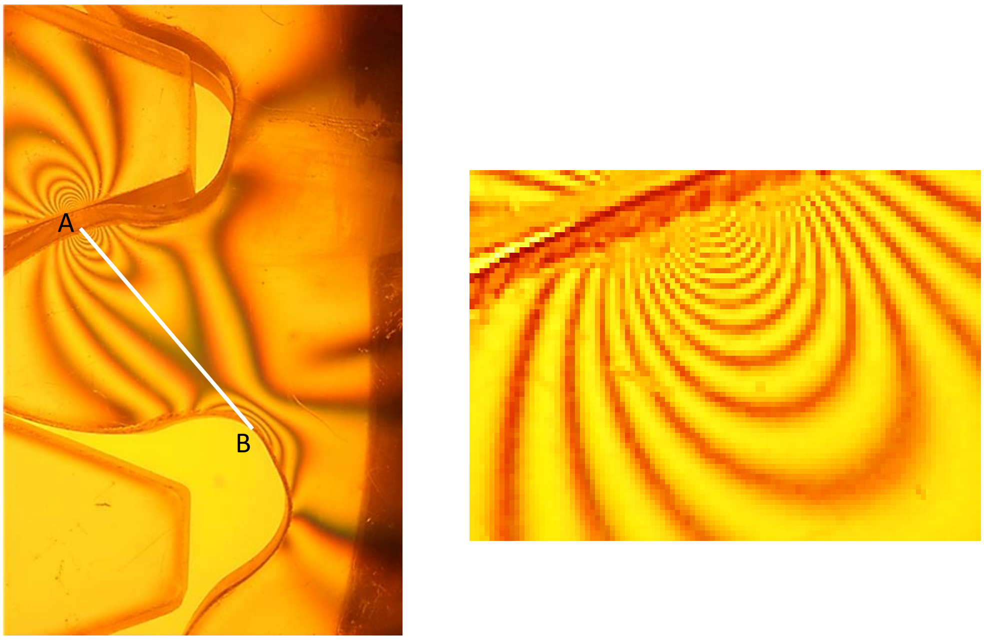


Figure 4: Isochromatic fringes (left), close up of the contact area

NUMERICAL ANALYSIS OF STRESSES

A finite element analysis conducted with "castem" is used to obtain the stress field. In the finite element calculations, we consider that the material behaves everywhere as a purely elastic isotropic material. The following eqn (3) which can be obtained readily from Mohr's circle for stresses is used to evaluate the principal stresses difference at any point of a stressed model.

$$\sigma_1 - \sigma_2 \; = \left(\left(\sigma_x - \sigma_y \right)^2 + 4\tau_{xy}{}^2 \right)^{0.5} \tag{3}$$

Using eqn (1) into eqn (3), the different values of the retardation angle φ can be calculated at any point on the model using the following eqn (4):

$$\varphi \; = \; 2\pi \frac{e}{f} \left(\left(\sigma_x - \sigma_y \right)^2 + 4\tau_{xy}{}^2 \right)^{0.5} \tag{4}$$

- The different values of $\sin^2 \varphi/2$ which represent the isochromatics can therefore be easily calculated over the whole model [6].

- The different values of the isoclinic parameter α can be calculated with eqn (5) which can be obtained readily from Mohr's circle for stresses. The different values of $\sin^2 2\alpha$ give then, readily, the isoclinic fringe pattern.

$$\alpha = \tan^{-1}\left((2\tau_{xy} \, / (\sigma_x - \sigma_y) \right) \tag{5}$$

To achieve a better simulation of the applied load, an imposed displacement is applied The equivalent applied load is calculated then as the sum of the elementary vertical load components at the nodes located at the lower surface of the model which is in contact with the loading frame. The meshing is refined in the neighborhood of the contact zone (Figure 5) to achieve a better simulation.

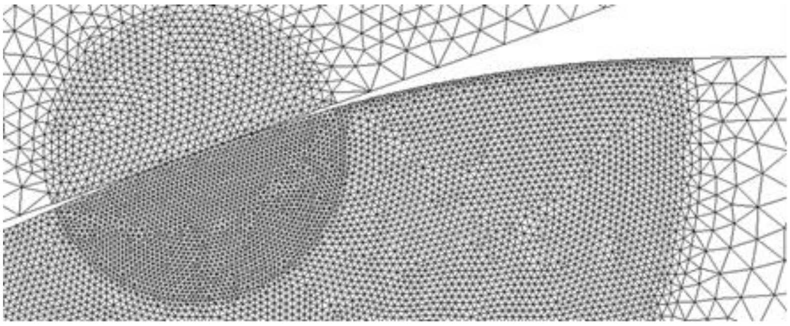
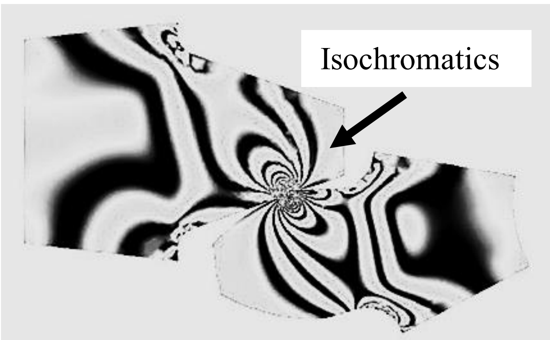


Fig. 5: Meshing in the neighborhood of the contact zone

The program calculates the different values of $\sin^2\varphi/2$ and $\sin^2 2\alpha$. The simulated photoelastic fringe patterns obtained can then be compared to the experimental ones. The simulated isochromatic fringe patterns (Fig. 6) are similar to the isochromatic fringe patterns (Fig. 4) obtained experimentally. Fig. 6 (left) shows the simulated isochromatic fringes calculated with a white background. Fig. 6 (right) shows the simulated isochromatic and isoclinic fringes obtained with a dark background; the isoclinics for a zero degree angle are shown by the arrows.



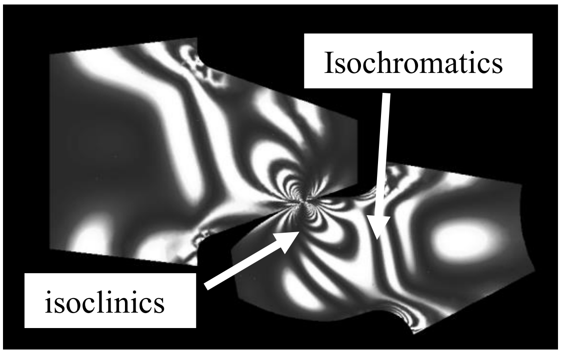


Fig. 6: - Simulated isochromatic fringes (left) - Simulated isochromatic and isoclinic fringes (right)

The simulated isoclinics at zero degree angle (Fig. 6 right) are similar to the experimental isoclinics obtained on Fig.3 for a zero degree angle. We can say that the program gives relatively good results. For comparison purposes the maximum shear stress is obtained along line AB (Fig. 7) numerically as well as experimentally by using the fringe orders recorded along line AB. The maximum shear stress reaches a maximum stress value of approximately 7.8 MPa at a short distance (≈1mm) from the origin which is in good agreement with the theory of contact. As we move away from the contact zone, stresses decrease to lower values then start to increase again as we move close to point B.

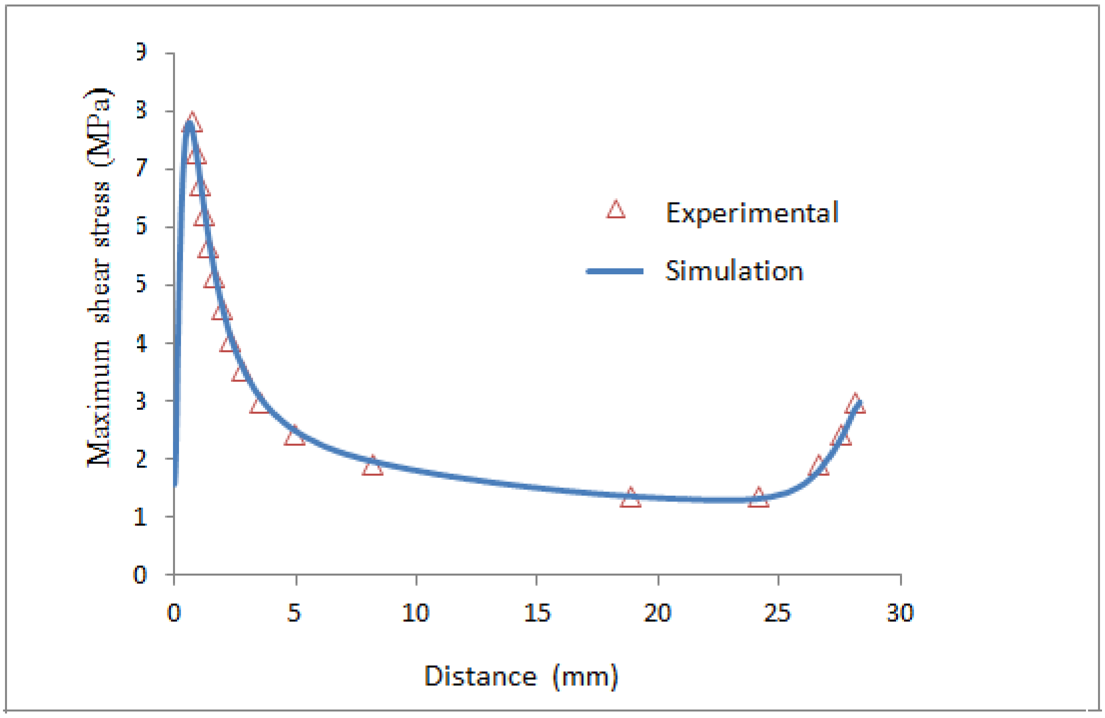


Fig. 7: Maximum shear stress along line AB

The next step of this work is the solution for a 3D contact problem between two orthogonal cylinders, an aluminium rigid cylinder in contact with a deformable epoxy cylinder. Stresses are studied only in the birefringent cylinder.

ANALYSIS OF STRESSES IN TWO ORTHOGONAL CYLINDERS

In the second part of this study, a finite element solution is developed for a 3D contact problem between two orthogonal cylinders (10 mm diameter aluminium cylinder on a 36 mm diameter birefringent material). This situation can be very much encountered in machinery components. The simulated photoelastic fringes are calculated for different slices along the length of the cylinder (Fig. 8). Different approaches of the problem can be adopted [7]. Here, for fringe simulation, we consider the difference and the directions of the principal stresses difference constant along the thickness of the isolated slice. We choose an 8 mm slice thickness which is relatively large in order to observe several fringes.

The stress field can be obtained easily, particularly in the most stressed zones which are of high importance in the design of mechanical components. Stresses can be determined for any desired position in the entire volume of the cylinder. Fig. 9 shows the values of the principal stresses difference for different slices along the cylinder length. Stresses are higher for the zero position; as we move along the y axis stresses decrease to lower values.

We can observe a decrease of the isochromatic fringes in the upper part of the cylinder as we move away from the contact zone, along the cylinder length in the y direction starting from the origin. The first simulated isochromatics for the zero position are sown on the upper left image (Fig.8, isochromatics n°1). The other images correspond to the isochromatics obtained for subsequent slices taken along y direction. They show relatively lower stresses. At the lower part of the cylinder, however, the fringes remain unchanged as the load is uniformly distributed over the cylinder length.

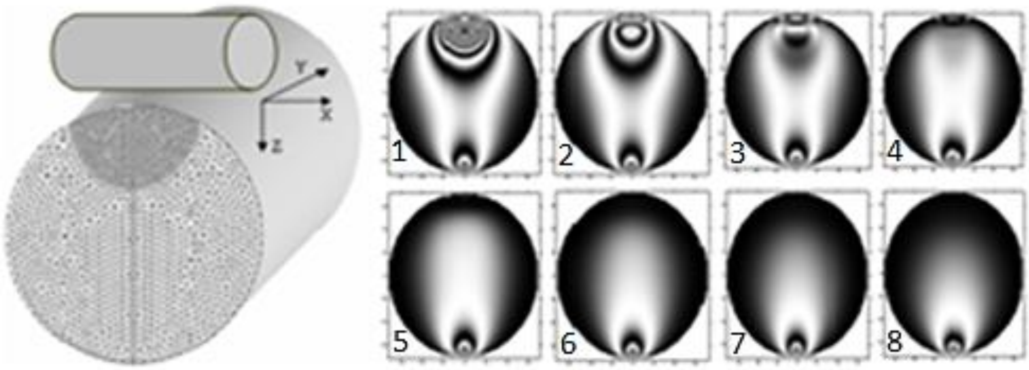


Fig. 8: (From left) Meshing, simulated isochromatics

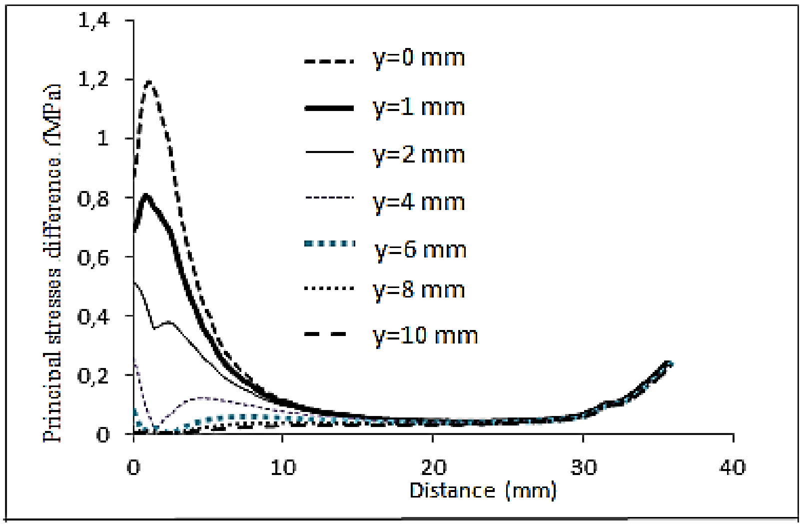


Fig. 9: Principal stresses difference along the vertical axis z for different slices isolated along the y direction

CONCLUSION

We have developed a finite elements solution to obtain stresses in the neighborhood of the contact zone of two gear teeth. Stresses as well as photoelastic fringes are obtained numerically. An experimental solution is conducted in order to validate the simulated results. Good agreements are achieved. As a first approach for tackling 3D stress distribution a finite elements solution is developed for the contact between two orthogonal cylinders. Stresses and photoelastic fringes are obtained for different slices isolated along the cylinder length.

In order to validate this finite element solution stresses and experimental photoelastic fringes can be obtained either by the mechanical slicing method followed by an analysis on a regular polariscope or by the optical slicing method [6,7] which uses two laser plans and the diffusion phenomenon to isolate a slice in any desired position.

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