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A CO-EVOLUTION APPROACH FOR RBDO OF COMPOSITE STRUCTURES UNDER DYNAMIC BEHAVIOR

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ABSTRACT

An approach for optimal design of composite structures aiming minimum weight subject to allowable reliability level under dynamic loading conditions is proposed. The failure probability is obtained using a procedure based on level 2 reliability analyses adapted for dynamic response. Material properties of the ply are considered normal random variables. The optimization process is based on co-evolutionary Genetic Algorithm where a master population evolves together with a net of slave sub-populations. Shredding Genetic Algorithm is used at slave populations aiming to obtain the long-term survival probability of composite structures. The master population evolves searching the minimum weight of structures.

Keywords: dynamic, composites, reliability, optimization, genetic algorithm, co-evolution.

INTRODUCTION

The need to incorporating uncertainties in engineering design has been recognized in previous research (Chiachio et al. 2011). Furthermore, the reliability based optimal design under dynamic response is an emerging research area due to the difficulties associated with coupling optimization and reliability analysis in long-term survival. In the present work it is intended to develop a new model of optimization based on a probabilistic analysis of composite structures under dynamic loading conditions.

The reliability based optimal design under dynamic response is an emerging research area due to the difficulties associated with coupling optimization and reliability analysis (Conceição António, 2000; Kvedaras et al., 2009). In the present work it is intended to develop a simple model of optimization based on probabilistic analysis of composite structures under dynamic loading conditions.

PROBABILISTIC DYNAMIC RESPONSE

Let us consider $\boldsymbol{\pi} = (\pi_1, \pi_2, ..., \pi_n)$ as the non-correlated basic random variables vector of the structural problem for the reliability analysis of composite structures under dynamic response. The random variables are related with uncertainties in the material properties including random laminate fibre orientation, modulus of elasticity, Poisson's ratio and thickness. Mean values and variances characterize the statistical nature of these variables.

Following a discretized form of the probabilistic dynamic equilibrium equation based on the finite element formulation of an isoparametric degenerated shell element, the displacements, velocities and accelerations, $\mathbf{d}$, $\dot{\mathbf{d}}$ and $\ddot{\mathbf{d}}$ respectively, can be defined in terms of nodal variables as

$$\delta\mathbf{d}(\boldsymbol{\pi})^T \left[\mathbf{M}(\boldsymbol{\pi})\, \ddot{\mathbf{d}}(\boldsymbol{\pi}) + \mathbf{C}(\boldsymbol{\pi})\, \dot{\mathbf{d}}(\boldsymbol{\pi}) + \mathbf{p}(\mathbf{d}(\boldsymbol{\pi}),\boldsymbol{\pi}) \right] = \delta\mathbf{d}(\boldsymbol{\pi})^T \mathbf{f} \tag{1}$$

where the mass matrix **M**, the damping matrix **C**, the internal force $\mathbf{p}(\mathbf{d})$ and the external applied load vector $\boldsymbol{f}$ have the following element contributions

$$\mathbf{M}_e(\boldsymbol{\pi}) = \int_{V_e(\boldsymbol{\pi})} \rho \mathbf{N}^T \mathbf{N}\; dV$$

$$\mathbf{C}_e(\boldsymbol{\pi}) = \int_{V_e(\boldsymbol{\pi})} c\, \mathbf{N}^T \mathbf{N}\; dV$$

$$\mathbf{p}_e(\boldsymbol{\pi}) = \int_{V_e(\boldsymbol{\pi})} \mathbf{B}(\boldsymbol{\pi})^T\, \boldsymbol{\sigma}(\boldsymbol{\pi})\; dV$$

$$\mathbf{f}_e = \int_{S_e} \mathbf{N}^T \mathbf{t}\; dS + \int_{V_e} \mathbf{N}^T \mathbf{b}\; dV \tag{2}$$

being S_e and V_e the surface and the volume of the element under consideration, ρ and c are the mass density and the damping parameter respectively, **B** is the standard strain matrix, **N** is the matrix of shape functions and $\boldsymbol{\sigma}$ is the vector of stresses referred to the local coordinates. In Equation (2), **t** is the vector of surface tractions applied on the boundary surface S_e. Since the virtual displacement $\delta\mathbf{d}$ may be arbitrary, Equation (2) is written as

$$\mathbf{M}(\boldsymbol{\pi})\, \ddot{\mathbf{d}}(\boldsymbol{\pi}) + \mathbf{C}(\boldsymbol{\pi})\, \dot{\mathbf{d}}(\boldsymbol{\pi}) + \mathbf{p}(\mathbf{d}(\boldsymbol{\pi}),\boldsymbol{\pi}) = \mathbf{f} \tag{3}$$

For linear elastic problems, the stresses are related to the strains as follows

$$\boldsymbol{\sigma}(\boldsymbol{\pi}) = \mathbf{D}(\boldsymbol{\pi})\, \boldsymbol{\varepsilon}(\boldsymbol{\pi}) = \mathbf{D}(\boldsymbol{\pi})\, \mathbf{B}(\boldsymbol{\pi})\, \mathbf{d}(\boldsymbol{\pi}) \tag{4}$$

and the internal forces can be written as

$$\mathbf{p}(\mathbf{d}(\boldsymbol{\pi}),\boldsymbol{\pi}) = \mathbf{K}(\boldsymbol{\pi})\, \mathbf{d}(\boldsymbol{\pi}) \tag{5}$$

where the structural stiffness matrix **K** results from the contribution of the stiffness of the element e,

$$\mathbf{K}_e(\boldsymbol{\pi}) = \int_{V_e(\boldsymbol{\pi})} \mathbf{B}(\boldsymbol{\pi})^T\, \mathbf{D}(\boldsymbol{\pi})\, \mathbf{B}(\boldsymbol{\pi})\; dV \tag{6}$$

In the present work a consistent mass matrix is considered and Newmark's algorithm together with the predictor-corrector scheme (Hughes and Liu, 1978) is adopted to solve the Equation (3).

RELIABILITY OF COMPOSITES UNDER DYNAMIC LOADING

Limit state functions

The vectors of random displacements and random stresses obtained in the previous section can be incorporated into the first order reliability method (FORM) to derive the safety index and then the probability of structural failure. In this work, reliability is the probability of the structures not failing within a specified time interval. Thus it is necessary to define failure criteria. It will be assumed that the structure fails if the maximum displacement, strain or stress exceeds some specified values. The most critical limit state function of displacement is related to the nodal displacement d_i at the most critical point exceeding an allowable value d_a, within a specified time interval $[0, t_m]$ (Melchers, 1999):

$$\bar{d}(\boldsymbol{\pi}, t_m) = MAX\left[\left| d_i(\boldsymbol{\pi}, t_j)\right| \;\; ; \;\; \mathrm{i} = 1, ..., \mathrm{N_d} \quad , \quad j = 1, ..., t_m \right] \tag{7}$$

$$\varphi_1(\boldsymbol{\pi}, t_m) = d_a - \bar{d}(\boldsymbol{\pi}, t_m) \tag{8}$$

being N_d the number of prescribed nodal displacements. This active strategy is based on the assumption that the displacement field of the shell structures can be represented by the most critical value of the structure.

The second limit state function is established considering the stresses at the ply level. The failure criterion employed for anisotropic materials is a generalised form of the Huber-Mises law (Conceição António, 2001) and can be written as

$$1 - \frac{f_k(\boldsymbol{\sigma}, t_j)}{Y} = 0 \;\; ; \;\; \mathrm{k} = 1, ..., \mathrm{N_s} \tag{9}$$

where Y is the maximum allowable level stress, N_s is the number of points where the stress vector is calculated and $f_k(\boldsymbol{\sigma}, t_j)$ is a failure function evaluated at the *k-th* point and at time t_j. This failure function is defined in terms of the stresses referred to the material axes 1,2,3 as

$$\begin{aligned} f_k{}^2(\boldsymbol{\sigma}, t_j) &= a_1\, \sigma_1{}^2 + 2\, a_{12}\, \sigma_1\, \sigma_2 + a_2\, \sigma_2{}^2 + a_3\, \tau_{12}{}^2 + a_4\, \tau_{13}{}^2 + a_5\, \tau_{23}{}^2 \\ &= \boldsymbol{\sigma}_{1,2,3}^T\, \mathbf{A}\, \boldsymbol{\sigma}_{1,2,3} \end{aligned} \tag{10}$$

being **A** the matrix of the anisotropic strength parameters determined by six independent failure tests. If the material axes 1,2 are rotated by a given angle θ relatively to the reference axes x,y, then the transformation of both stresses and matrix **A** to the global axes is necessary. So, the obtained yield criterion can be rewritten as

$$f_k{}^2(\boldsymbol{\sigma}, t_j) = \boldsymbol{\sigma}^T\, \mathbf{T}^T\, \mathbf{A}\, \mathbf{T}\, \boldsymbol{\sigma} = \boldsymbol{\sigma}^T\, \bar{\mathbf{A}}\, \boldsymbol{\sigma} \tag{11}$$

where **T** is the strain transformation matrix which relates the material system (1,2,3) with the global system (*x, y, z*). The *first ply failure* at time t_j occurs for

$$h_{FPF}(t_j) = MIN\left(1 - \frac{f_k(\boldsymbol{\sigma}, t_j)}{Y} \quad , \quad \mathrm{k} = 1, ..., \mathrm{N_s} \right) = 0 \tag{12}$$

and the second limit state function can be written as

$$\varphi_2(\boldsymbol{\pi}, t_n) = MIN\left(h_{FPF}(t_j) \quad , \quad j = 1, ..., t_m \right) \tag{13}$$

Reliability analysis

Since the boundary of the safety region, called limit state surface, is given by

$$z = \varphi_p(\pi_1, \pi_2, ..., \pi_n, t_m) = 0 \tag{14}$$

the values of $\boldsymbol{\pi}$ belonging to the failure region will satisfy the following inequality:

$$z = \varphi_p(\boldsymbol{\pi}, t_m) < 0 \tag{15}$$

The probability of failure is defined as

$$P_f = P\left[\varphi_p(\boldsymbol{\pi}, t_m) < 0\right] = \int_{\Omega} f(\boldsymbol{\pi}, t_m)\, d\boldsymbol{\pi} \tag{16}$$

where $f(\boldsymbol{\pi})$ is the joint probability density function of $\boldsymbol{\pi}$, $\boldsymbol{\Omega}$ is the failure region, and $\varphi_p(\boldsymbol{\pi})$ is the so-called limit state function which separates the design space into failure ($\varphi_p(\boldsymbol{\pi}) < 0$) and safe ($\varphi_p(\boldsymbol{\pi}) > 0$) regions.

The distribution of the considered basic variables π_i and the considered limit state surface $\varphi_p(\boldsymbol{\pi})$ are known and the probability of failure can be used as a measure of reliability. However, equation (16) cannot be evaluated analytically for realistic structures because the calculation of the integral is difficult. To avoid this feature the moment reliability theory namely the so-called Hasofer-Lind reliability index (Melchers, 1999) is used in this work. The advantage of this method is its invariance with respect to different failure surface formulations for spaces having the same dimension.

The Hasofer-Lind method performs in two steps: the first one consists of projecting the failure surface of equation (14) into the space of standardised variables:

$$u_i = \frac{\pi_i - \overline{\pi}_i}{\sigma_{\pi_i}} \tag{17}$$

where $\overline{\pi}_i$ and σ_{π_i} are, respectively, the mean values and the standard deviations of the basic variables, and the second step measures, in this space, the minimum distance β of the transformed surface

$$\varphi_p(u_1, u_2, ..., u_n, t_m) = 0 \tag{18}$$

to the origin of the axes. A design is considered reliable at β_a level prescribed by an appropriated provision code, if $\beta \geq \beta_a$. The geometric interpretation of this feature can be presented in the following manner: the hypersphere having radius β_a, with its centre at the origin of the axes u_i (corresponding to the mean values of the variables π_i), is required to lie entirely in the transformed safety domain. On the other hand, considering that in the standard normal space the probability density decays exponentially with distance to the origin then the point with maximum probability of failure on the limit-state surface is the point of minimum distance to the origin. From the operational point of view the search for this point can be formulated as a constrained optimisation problem

$$\beta_p = \min_{\mathbf{u} \in A} (\mathbf{u}^T \mathbf{u})^{1/2}$$
$$A = \left\{ \mathbf{u} \mid \varphi_p(\mathbf{u}, t_m) = 0 \right\} \tag{19}$$

where $\mathbf{u}$ is the vector of the standardised variables defined in Equation (17) and the respective solution $\mathbf{u}^*$ is referred in the technical literature as the design point or even as the Most Probable failure Point (MPP).

The assumption of the minimum distance β obtained from the solution of Equation (19) as a measure of reliability is equivalent to considering the discretization at one point only of the safety domain boundary, expressed in the space of the standardised variables. This corresponds to the substitution of the hypersurface by the hyperplane passing through the point defined by $\mathbf{u}^*$. Introducing formally a normal probability distribution function Φ, the first-order approximation of $P_{f,p}$ can be written as

$$P_{f,p} = \Phi(-\beta_p) \tag{20}$$

where β_p is known as the safety index, i.e. the minimum distance from the origin to the limit-state surface $\varphi_p(\mathbf{u}, t_m)$.

Finally, the reliability of the structural system under dynamic load conditions is established as

$$\beta_s = MIN(\beta_{FPF}, \beta_{disp}) \tag{21}$$

being β_{disp} the reliability index associated with the critical displacement limit state established in Equation (8) and β_{FPF} is the reliability index associated with the first ply failure limit state function defined in Equation (13), both of them defined for a specified time interval t_m.

GLOBAL MOST PROBABLE POINT SEARCH

A very important problem in structural reliability analysis of composites is the existence of multiple most probable failure points (MPPs) or failure points of the limit state functions when the optimization problem formulated in Equation (19) is solved. Multiple MPPs are similar to the local minima in structural optimization.

Many problems in structural optimization are stopped once a local minimum is reached. This is an unacceptable procedure in reliability analysis since the local MPP may not represent the worst failure and the actual failure may occur below the predicted level. Only the global MPP represents the actual structural reliability. In some optimisation algorithms the problem of multiple local minima is addressed by checking if all solutions starting from different initial points converge to the same optimum. This method is very costly from the computational point of view and no additional information is done if the problem has or does not have multiples MPPs (Wang and Grandhi, 1995). In this work a methodology based on genetic search aiming the identification of the global MPP (Conceição António, 2000) is proposed. The adopted evolutionary strategy is elitist since a core of genetic material of individuals (solutions) with best fitness is considered while the diversity of the rest of the population is guaranteed. This aspect is important for global most probable point search.

An important aspect of the evolutionary search is the definition of the fitness of individuals, which is related with the objective function and the constraints of the problem. The original minimisation problem is transformed. The genetic algorithms will seek to increase the fitness as it operates and so the constrained minimization problem formulated in Equation (19) is transformed as

$$\begin{aligned} &\text{Maximize } F_p = \overline{C} - \beta_p(\mathbf{u}, t_m) \\ &\text{subject to } \quad \varphi_p(\mathbf{u}, t_m) = 0 \end{aligned} \tag{22}$$

where $\overline{C}$ is a constant large enough to avoid negative fitness. Designs with good fitness and satisfying the constraint have priority in the selection process. Solutions of the problem that violate the constraint are penalized at a graded degree of severity according the difference between the actual and the allowable values of the constraint. Then the fitness function for the optimization problem in Equation (22) is written as

$$Fit_{\,p} = \overline{C} - \beta_p(\mathbf{u}, t_m) - \overline{K}\left|\varphi_p(\mathbf{u}, t_m)\right|^q \tag{23}$$

being the constants q and $\overline{K}$ evaluated considering two constraint violation degrees. A negligible penalty p_o is applied to the constraint violation $\varphi_{p,o}$ that can be tolerated. Very high values $\varphi_{p,1}$ of constraint violation are strongly penalized with a penalty p_1. Using penalties the constants q and $\overline{K}$ are obtained in the following manner:

$$\left(\frac{\varphi_{p,o}}{\varphi_{p,1}}\right)^q = \frac{p_o}{p_1} \tag{24}$$

$$\overline{K} = \frac{p_o}{(\varphi_{p,o})^q} \tag{25}$$

The magnitude of the penalties is related to the magnitude of β_p rather than to the constant $\overline{C}$, a value that is to some extent arbitrary.

RELIABILITY BASED DESIGN OPTIMIZATION

An optimization process based on exploitation of anisotropy of composite materials is proposed aiming the best performance of structural behaviour and minimum weight. The optimization problem is formulated as

$$\text{Minimize } W(\mathbf{x}) \text{ , subject to } \beta_s(\mathbf{x}, t_m) \geq \beta_a$$

$$\text{with } \beta_s(\mathbf{x}, t_m) = MIN\left[\beta_{FPF}(\mathbf{x}, t_m), \beta_{disp}(\mathbf{x}, t_m)\right] \tag{26}$$

where **x** denotes the design variables of the laminated composite materials of the structure, being β_a the allowable reliability index for structural system, β_{disp} the reliability index associated with the critical displacement limit state function and β_{FPF} is the reliability index associated with the first ply failure limit state function, both of them defined for a specified time interval t_m of dynamic loading conditions. In order to obtain structural system reliability index it is necessary to implement the global MPP search. In the proposed approach this search represents the inner optimization problem using the random variables $\boldsymbol{\pi}$ considered in uncertainty propagation analysis. The optimal design aiming the imposed structural reliability level together the weight minimization is defined as the external problem. This last optimization problem is performed using the design variables **x** based on mean values $\overline{\boldsymbol{\pi}}$ of the random variables.

The optimization problem formulated in equation (26) can be solved considering the ply angles θ_i and ply thicknesses $\overline{t}_i$ of shell laminates. In order to improve the efficiency of the search a decomposition of the original optimization problem in (26) is implemented in the proposed approach. The decomposition is based in two levels: in the first one the ply angle θ_i are the design variables considering the objective to improve the performance of structural behaviour, in the second level the weight is minimized based on ply thickness $\bar{t}_i$. Then the optimisation problem is

<u>First level</u>:

$$\text{Maximise } \beta_s(\boldsymbol{\theta}) = MIN\left[\beta_{FPF}(\boldsymbol{\theta}), \beta_{disp}(\boldsymbol{\theta})\right] \qquad \text{over } \boldsymbol{\theta} \tag{27}$$

where $\boldsymbol{\theta}$ denotes the vector of the ply angles of the laminate composite materials of the structure.

<u>Second level</u>:

$$\text{Minimize } W(\bar{\mathbf{t}}, \bar{\boldsymbol{\pi}}) \text{ , subject to } \beta_s(\bar{\mathbf{t}}, t_m) \geq \beta_a \qquad \text{over } \bar{\mathbf{t}} \tag{28}$$

being $\bar{\mathbf{t}}$ denotes the vector of the ply thicknesses of the laminate composite materials.

In order to obtain every reliability index it is necessary to implement the global MPP search at each optimization level solving the problem formulated in Equation (22). In the proposed approach this represents the inner optimisation problem. The optimal design aiming the weight minimization is defined as the external problem.

Thus a co-evolutionary Genetic Algorithm (Conceição António, 2001, 2006) is proposed where two kinds of populations are identified: a master population denoted by $\mathbf{P}^t$ and a net of n small size sub-populations denoted by $\mathbf{V_S}_p$ as shown in Figure 1. The master population performs the evolutionary process associated with the external optimization problem of minimum weight while the sub-populations evolve linked to MPP search inner problems. A Shredding Genetic Algorithm (Conceição António, 2001; Deng et al., 2005) is used to solve these inner problems. In the proposed approach the uniform design method is used to approximate the sampling space aiming to accelerate the MPP search.

Each chromosome has two segments activated in alternative way when the evolutionary process passes from the master population to the slave sub-populations $\mathbf{V_S}_p$. In both cases a binary code format is used to manipulate the exchange data.

The Shredding Genetic Algorithm and the Genetic Algorithm applied to the master population are based on four principal operators: selection, crossover, implicit mutation and replacement of similar individuals. These operators are supported by an elitist strategy that always preserves a core of best individuals of the population that is transferred into the next generations. In the crossover process, the two progenitors are selected randomly: one belongs to the population group with best fitness (elite) and the second one is selected from the remaining group with lower fitness. Then the structured stochastic exchange data based on a multi-point combination technique called *Parametrized Uniform Crossover* (Spears and DeJong 1991) is applied to the binary string of the selected chromosomes. This crossover performs with a probability of choice for genes from the chromosome with best fitness. The offspring group will take part of the population into the next generation that will be formed by the crossover operator.

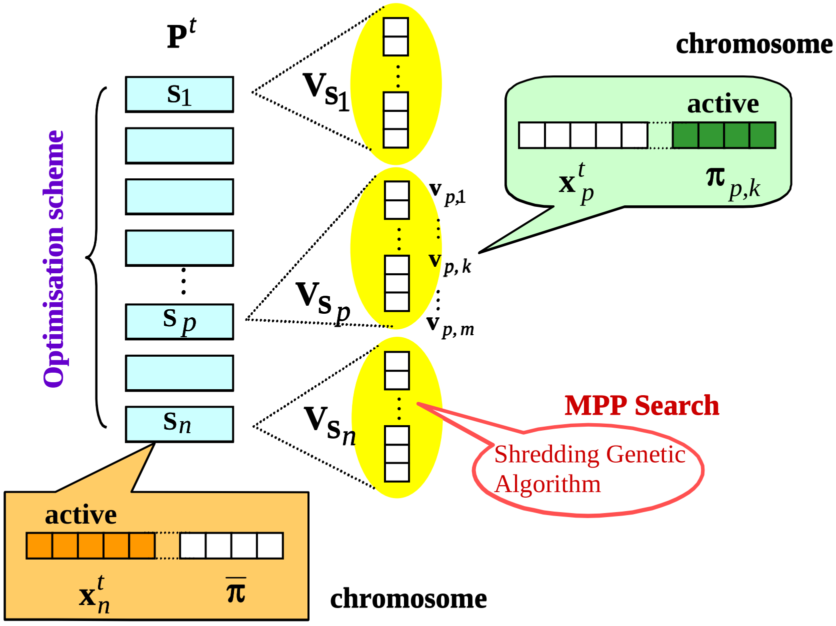


Fig.1 Morphology and linkage of the populations for co-evolutionary Genetic Algorithm

To avoid the rising of local minima a chromosome set which genes are generated randomly is introduced into the population. This operation is called mutation and is quite different from classic techniques where a reduced number of genes is changed. The mutation operator guarantees the diversity of the population in each generation (Conceição António, 2001, 2006).

To control the genetic diversity, a scheme that detects individuals belonging to the same neighbourhood has been implemented. The analysis is made from the genetic point of view. The best individual is kept in the population and others generated randomly replace the similar ones.

The co-evolutionary Genetic Algorithm performs as follows:

1. **For** t=0, random generation of the initial master population $\mathbf{P}^0$, considering the mean values $\bar{\boldsymbol{\pi}}$ of the random variables;
2. For slave sub-populations,

Do p=1 to n,

2.1 Random generation of the initial sub-population $\mathbf{V}_{\mathbf{S}_p}$ with small size and considering the $\mathbf{x}^t_p$ values of the design variables;

2.2 Evolution of the $\mathbf{V}_{\mathbf{S}_p}$ sub-population over $\boldsymbol{\pi}_{p,k}$ using the Shredding Genetic Algorithm $\Rightarrow$ MPP search for each limit state function;

2.3 Definition of the structural reliability index for the design solution $\mathbf{x}^t_p$.

end do

3. Check the convergence criterion on the evolutionary process of the master population. If the evolutionary process converges stop, otherwise continue;
4. Evolution of the master population $\mathbf{P}^t$ over the design vector $\mathbf{x}^t_p$ using the mean values $\bar{\boldsymbol{\pi}}$ of the random variables. The evolutionary process is based on:

Selection using an elitist strategy;
Crossover based on the *Parametrized Uniform Crossover*;
Implicit Mutation for diversity increasing;
Replacement of individuals: to control genetic diversity.

5. **Do** $t \rightarrow t+1$,

Transfer the elite group of $\mathbf{P}^{t}$, the offspring and the mutation group into the next generation $\Rightarrow$ creation of the master population $\mathbf{P}^{t+1}$;

6. **Go to** Step 2.

In the proposed approach sampling space reduction variance techniques together with weak convergence conditions are used to accelerate the MPP search.

NUMERICAL RESULTS

A deep thin spherical shell is adopted as an example for reliability based design under dynamic response. The cap with a central angle of *120°*, radius of *0.508* m and thickness of $7.6\times10^{-3}\,\mathrm{m}$ is clamped on its boundary.

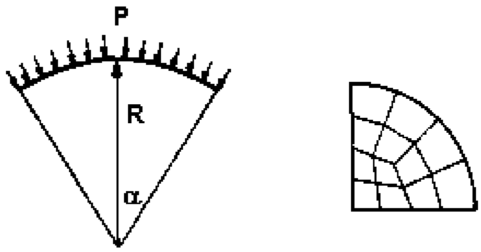
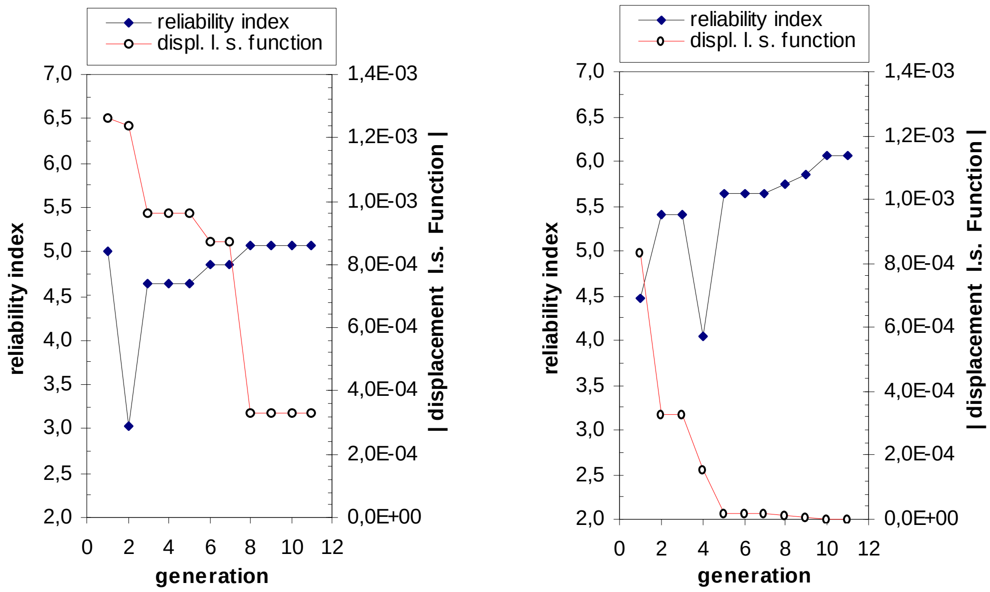


Fig. 2 Shallow spherical shell subjected to a suddenly applied uniform pressure

The shell is made of a symmetric laminate with 6 layers having the same thickness. The random variables π_i considered in the numerical example are the longitudinal modulus E_1 of the ply material. The three random variables are not correlated and have normal distribution functions. The mean values are $\overline{\pi}_i = E_x = 206.9$ GPa , $i = 1,2,3$ and the standard deviations are $\sigma_{\pi_i} = 0.06\,\overline{\pi}_i$, $\mathrm{i} = 1,2,3$. The mass density of the cap is $3.26\times10^{3}\,kg\ \mathrm{m}^{-3}$ and the composite material has the following properties:

$$\begin{aligned} &E_y = 206.9\ \text{GPa},\quad \nu = 0.3\ ,\quad G_{xy} = G_{xz} = G_{yz} = 106.2\ \text{GPa} \\ &\overline{\sigma}_o = \sigma_{ox} = \sigma_{oy} = \sigma_{o45} = 689.7\,\text{MPa} \\ &\tau_{o12} = \tau_{o13} = \tau_{o23} = 397.9\,\text{MPa} \end{aligned} \tag{29}$$

A suddenly applied pressure load of 4.138 MPa is applied on the shell. The imposed service condition is based on the absolute value of the maximum allowable vertical displacement $d_a = 8.9\times10^{-1}$ mm within a specified time interval $t_m = 6.0\times10^{-4}\,\mathrm{s}$. A binary code format with five digits is used for the chromosome codification of each random and design variable. The master population has twelve individuals and the slave sub-populations evolve with ten individuals. The elite, offspring and mutation groups of all populations have equal sizes. Figure 3 shows the evolution history of the MPP search associated with the best-fitted individual for two generations of the reliability maximisation problem. The efficiency and the robustness of the MPP search are evident in both cases. In particular the absolute value of the displacement limit state (l.s.) function is close to zero at MPP.



a) b)
Fig. 3 Inner reliability index calculation for the best solution at:
a) 1st generation and b) 19th generation of the reliability maximisation scheme

For the final solution of the structural reliability maximisation the transient response of the structure is shown in Fig. 4. This final optimal solution corresponds to the best one at 19th generation which MPP search is presented in Fig. 3 b). The dynamic behaviour corresponding to the mean values is compared with the structural response for the MPP. The final solution has a maximum absolute displacement close to $d_a = 8.9 \times 10^{-1}$ mm as expected.

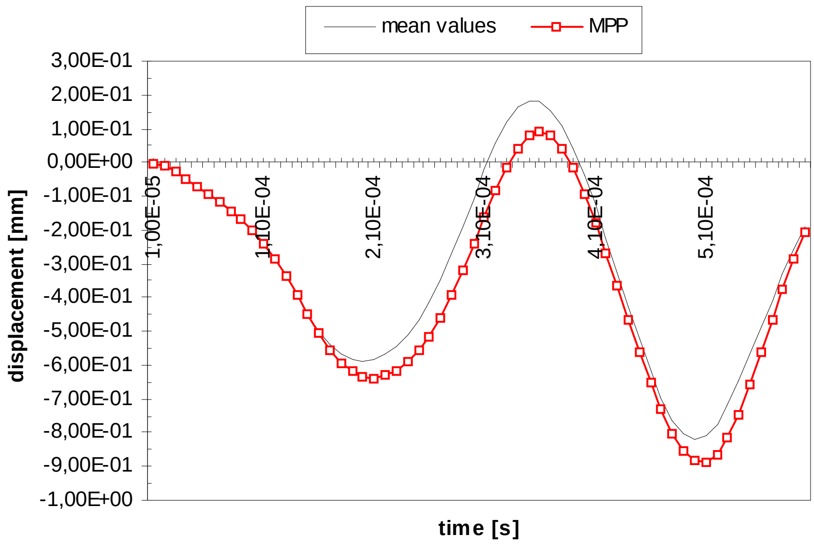


Fig. 4 Central deflection time history for the mean values of random variables and the MPP values for the best solution of the reliability maximisation problem

Using the proposed numerical model formulated in Equation (22) the structural reliability is maximised. The optimal results for the ply angles and the corresponding MPP are presented in Table 1. The algorithm is checked considering a fixed number of generations (k=12) without mean fitness evolution for the elite group.

Table 1 Optimal values and associated most probable failure point (MPP)

Design Variables	$\theta_1 = -2{,}9°$	$\theta_2 = 66{,}8°$	$\theta_3 = 26{,}1°$
MPP Values [GPa]	$\pi_1 = 183{,}6$	$\pi_2 = 230{,}1$	$\pi_3 = 182{,}0$

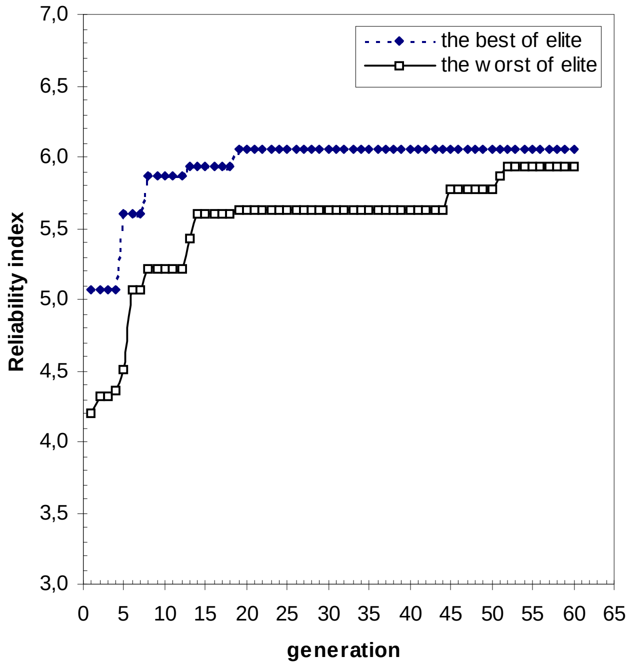


Fig. 5 Evolutionary history of reliability index maximisation

The genetic search history for the solution of the optimisation problem given in Equation (27) is shown in Fig. 5. The solution is matured after 19 generations what can be considered a good trial considering the number of design and random variables intervening in the global search. It can be observed that the worst fitted individual of the elite group continues to be improved and at the end of the optimisation procedure is close to the best one.

Genetic search works with a population of solutions instead of classical gradient based methods where only one solution is considered for a particular iteration. This way the fitness of the individuals of a population always improves. The efficiency of the presented evolutionary process is measured by the success rate of individuals coming from crossover or mutation, actually enter the elite group and the worst individuals of this group are eliminated.

CONCLUSIONS

A new methodology aiming the structural reliability maximisation and weight minimization of laminated composite structures under dynamic loading was presented. Reliability in dynamic response is defined as the probability of the structure does not fail within a specified time interval. To define the failure criteria and the associated limit state functions it is assumed that the structure fails if the maximum displacement, maximum strain or maximum stress exceeds some specified values. A version of the Hasofer-Lind method adapted for dynamic response and a Shredding Genetic Algorithm were developed and applied with success. The optimisation process is based on a co-evolutionary Genetic Algorithm where a master population evolves together with a net of slave sub-populations. The proposed topology aims to link the Most Probable failure Point search with the maximisation of structural reliability and weight minimization. In particular, an example using a displacement limit state function was presented and the results for the reliability analysis show the robustness of the proposed approach.

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REFERENCES

Chiachio M, Chiachio J, Rus G. Reliability in composites – A selective review and survey of current development. Composites: Part B, 2012, 43, p. 902-913.

Conceição António CA. An approach to reliability based design of composite structures under dynamic response. ECCOMAS 2000 - European Congress on Computational Methods in Applied Sciences and Engineering, incorporating the VI International Conference on Computational Plasticity (COMPLAS VI), 11-14 Setembro, Barcelona, Spain, 2000.

Conceição António CA. A hierarchical genetic algorithm for reliability based design of geometrically non-linear composite structures. Composite Structures, 2001, 54, p. 37-47.

Conceição António CA. A hierarchical genetic algorithm with age structure for multimodal optimal design of hybrid composites. Structural and Multidisciplinary Optimization, 2006, 31, P. 280-294.

Deng L, Ghosn M, Shao S. Development of a shredding genetic algorithm for structural reliability. Structural Safety, 2005, 27, p. 113-131.

Hughes TJR, Liu WK. Implicit-explicit finite elements in transient analysis: stability theory. J. Applied Mechanics, 1978, 45, p. 371-374.

Kvedaras AK, Kudzys A, Valiunas B. Reliability verification for composite structures of annular cross section. Mechanics of Composite Materials, 2009, 45, p. 407-414.

Melchers RE. Structural reliability analysis and prediction. 2nd Edition, John Wiley & Sons, Chichester, UK, 1999.

Spears WM, DeJong SK. On the Virtues of Parametrized Uniform Crossover. Proceedings of the Fourth International Conference on Genetic Algorithms, 1991, p. 230-236.

Wang L, Grandhi RV. Intervening variables and constraint approximations in safety index and failure probability calculations. Structural Optimization, 1995, 10, p. 2-8.