PAPER REF: 4082

THE INFLUENCE OF TEMPERATURE FIELD ON VIBRATION OF NANO, MICRO AND NANO BEAMS

Jurij Avsec[1]

[1]Faculty of Energy Technology, University of Maribor, Krsko, Slovenia
(*)*Email:* jurij.avsec@um.si

ABSTRACT

Mechanics and thermodynamics are two very important scientific disciplines to solve many interdisciplinary problems. Temperature variations can significantly change the vibration characteristics of macro-, micro- and nano-structures. In the presented article we have studied the microbeams and nanotubes vibration under thermal effects. Microbeams and nanotubes will be very important in future industry like MEMS and NEMS industry. For the physical explanation of vibration of nanotubes classical mechanics is valid with some limitations. We have taken into account the influence of thermal force, axial force in rotating shaft and also gyroscopic effect at the same time. The effect of temperature-dependent material properties was considered primary with respect to temperature variations. On the basis of our analytical model is possible to determine the vibrational characteristics in very wide region of temperatures. In the presented paper is shown for the first time in scientific literature the combination of temperature, gyroscopic effects and rotor speeds on shaft and beam vibrations. For nanobeams vibration we have taken into account the nonlocal and local beam models.

Keywords: vibration of continuous systems, vibration of microbeams, vibration of nanobeams

INTRODUCTION-IMPORTANT THERMOPHYSICAL PROPERTIES FOR CALCULATION OF VIBRATION CHARACTERISTICS

The solid structures are consisted from atoms and molecules. The basis of the relative motion of molecules and atoms in the solid structures as well as the effect of attractive and repulsion forces are intermolecular and intramolecular interactions between the electrons and nuclei [1]. In the solid structures are dominant effects important to crate solid structure intermolecular forces and vibration of atoms in molecules. To model vibration of engineering structures we need to take into account the value of thermophysical properties like Young modulus, the coefficient of linear expansion…. From the thermomechanics discipline we know that all thermodynamic properties are dependent on temperature field. But in the micro and nano world we need to take into account also other effects. In the presented article we will also try to show that in some cases also the dimension of vibrating element has the influence on thermophysical properties.

One of the great scientific and technical advancement at the end of 20^{th} century and at the start at the 21^{st} century is the creation of nanomaterials and nanotechnology. The area which cover all important problems from that field is called in the broadest sense the mechanics. Regarding the cross sectional diameter we can divide mechanics into some subdisciplines:[1]

macromechanics 10^{-4}-10^{-5} m,
mesomechanics 10^{-5}-10^{-7} m,
micromechanics 10^{-7}-10^{-8} m,
nanomechanics: 10^{-8}-10^{-9} m.

Since the atomic level (interatomic distance in a crystal lattice) has an order of one to several Å (10^{-10} m) the nanolevel is restricted to 10^{-9} m.

The vibrations of beams and microbeams is of vital importance in mechanical engineering. Mechanical machines very often operate under diverse temperature conditions. In internal combustion engines, rocket systems, movement of the satellites, MEMS and NEMS the conditions are particularly temperature-sensitive. Thermodynamic effects are frequently ignored in research, which may yield totally incorrect results. Literature [3] shows that even the slightest temperature change leads to huge alteration of the clamped beam vibration properties. Contrary as in papers [1-3], the impact in the present paper is not neglected of a change in thermodynamic properties, which have to be taken into consideration at major temperature changes. Carbon nanotubes in dependence of the chiral angle can be classified into three types: armchair, zigzag and chiral. Numerous studies are available on the physical properties of armchair and zigzag carbon nanotube in the literature. However, only a limited portion of the literature studied nanotubes in dependence of temperature field. This article develops a model that analyzes the frequency of the chiral single-walled carbon nanotubes (SWCNTs) subjected to a thermal vibrations by using Timoshenko beam model, including the effect of rotary inertia and shear deformation. The Timoshenko model we have compared with with Euler model.
Carbon nanotubes could be classified into single wall nanotubes (SWNT) and multi wall nanotubes (MWNT). On the basis of molecular simulation many researchers found the solution that modulus of elasticity is no more constant, is dependent on diameter of nanotube and thickness of nanotube [4,5,13]. On the basis of molecular dynamics calculation we could express equations for surface Young modulus and Poisson number for armchair SWNT:

$$Y_s = \frac{4\mu K_p}{\sqrt{3}(\lambda + 3\mu)} \tag{1}$$

where:

$$\nu = \frac{\lambda - \zeta\mu}{\lambda + \zeta\mu},\ \lambda = \frac{7 - \cos(\pi / n)}{34 + 2\cos(\pi / n)},\ \mu = \frac{K_\theta^2}{K_\rho r_0^2} \tag{2}$$

The above equation are obtained on the basis of continuous mechanics and molecular simulation [3,4], where Y_s means surface Young modulus [13] and ν Poisson number. From the Figs. 1-3 we could see that material properties are temperature and also size dependent.

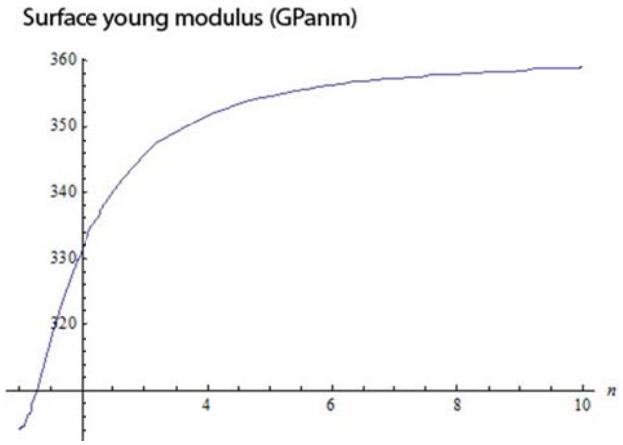

Figure 1: Young modulus of armchair nanotubes

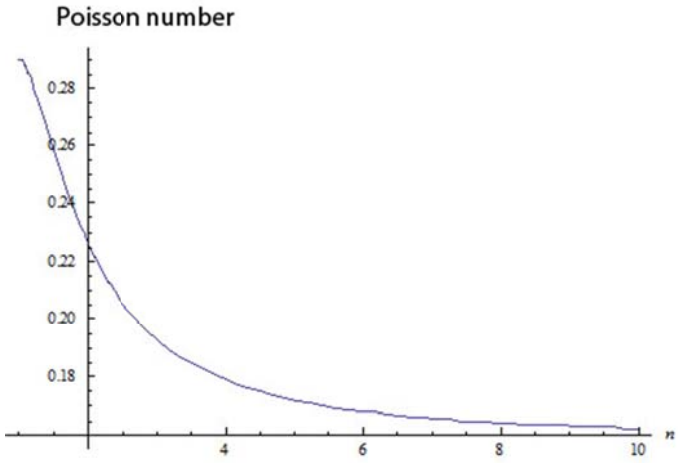

Figure 2: Poisson number of armchair nanotubes

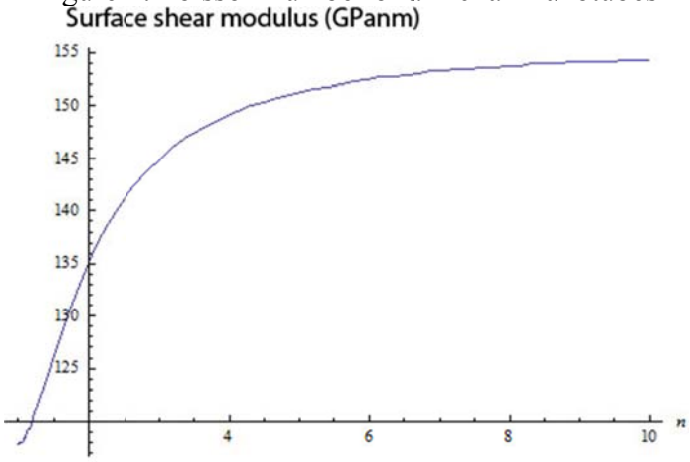

Figure 1: Shear modulus of armchair nanotubes

The Young modulus of carbon nanotube and linear expansion coefficient are dependent also on temperature field. On the basis of Prakash [6] molecular dynamics simulation is obtained the next relation for modulus of elasticity and linear expansion coefficient α for SWNT:

$$Y_s = Y_s(1 - 0.000075T) \tag{3}$$

$$\alpha = \frac{1}{l}\left(\frac{dl}{dT}\right) = \frac{10^{-18}T^2 - 2*10^{-15}*T + 10^{-13}}{10^{-18}T^3 - 10^{-15}T^2 + 10^{-13}T + 3*10^{-8}} \quad (4)$$

LOCAL EULER-BERNOULI AND TIMOSHENKO BEAM MODELS UNDER THERMAL STRESSES

Let us assume that the support is homogenous, having the same temperature over its entire length. As a result of thermal expansion, an additional axial force F_T occurs:

$$F_T = \alpha\theta EA \quad (5)$$

In equation (7) α is the linear thermal extension coefficient, θ is the temperature difference between the actual and initial or reference temperature. The equation by means of which we can resolve the problem using the axial force is as follows according to Wear, Timoshenko and Young [4]:

$$\mathrm{EI}\frac{\partial^4 \mathrm{w}(\mathrm{x},\mathrm{t})}{\partial \mathrm{x}^4} + \mathrm{F_T}\frac{\partial^2 \mathrm{w}(\mathrm{x},\mathrm{t})}{\partial \mathrm{x}^2} + \rho\mathrm{A}\frac{\partial^2 \mathrm{w}(\mathrm{x},\mathrm{t})}{\partial \mathrm{t}^2} = 0, \quad (6)$$

where E means Young modulus, I area moment of inertia, A area, ρ density of material, t time and w the displacement. Using the method of separation of variables $\mathrm{w}(\mathrm{x},\mathrm{t}) = \mathrm{X}(\mathrm{x})\Omega(\mathrm{t})$ and introducing the new functions, Equation (8) can be written down in a slightly less complicated way:

$$c^2\frac{X''''(x)}{X(x)} + 2\gamma\frac{X''(x)}{X(x)} - \frac{\ddot{\Omega}(t)}{\Omega} = \omega^2, \quad (7)$$

where the partial derivatives have been replaced with total derivatives.

$$\ddot{\Omega}(t) + \omega^2\Omega(t) = 0 \quad (8)$$

$$X''''(x) + 2\gamma X''(x) - \beta^4 X(x) = 0 \quad (9)$$

In Equation (11), the new symbols represent the following functional relations:

$$\beta^2 = \frac{\omega}{c}, c^2 = \frac{EI}{\rho A}, \gamma = \frac{F_T}{2EI} \quad (10)$$

Thus, a general solution to Equations (4) and (5) are ($\lambda = \sqrt{\beta^4 + \gamma^2}$) [1-4]:

$$X(x) = C_1\cos\left(\sqrt{\lambda+\gamma}x\right) + C_2\cosh\left(\sqrt{\lambda-\gamma}x\right) + C_3\sin\left(\sqrt{\lambda+\gamma}x\right) + C_4\sinh\left(\sqrt{\lambda-\gamma}x\right) \quad (11)$$

$$\Omega(t) = A\sin(\omega t) + B\cos(\omega t) \quad (12)$$

In the equation (11) the value of λ (where is hidden the influence of angular frequency ω) and three of four constants of integration C_1, C2, C_3 and C4 are determined from the boundary conditions. The fourth constant is possible to find in the combination with the constants A and B in Equation (12). For a given beam at defined temperature the values by λ depend upon the boundary conditions [5-9]. Using boundary conditions, the following solutions can be analytically computed ($\Gamma = L^2\gamma, \Lambda = L^2\lambda$):

For supported-simply supported beam regarding boundary conditions we obtain the next equation:

$$sin(\Lambda + \Gamma) = 0 \tag{13}$$

With the known angular frequencies ω_n of individual modes of vibration is possibly to calculate X_n and Ω_n of individual modes of vibration. To determine the solution for the displacement we have to solve the equation [5-9]:

$$w(x,t) = \sum_{i=1}^{\infty} (A_n\, sin(\omega_n t) + B_n\, cos(\omega_n t)) X_n(x), \tag{14}$$

where the modal shapes can be shown to be orthogonal:

$$\int_0^l X_n(x) X_m(x) dx = 0 \quad for\ n \neq m \tag{15}$$

The model presented in our paper is fully analytical, but if compared with the measured results it points to a large deviation from reality [1, 2]. The biggest problem of this model is that in the mathematical model in question the clamped wall can fully withstand the beam for the beam to have a constant length all the time. The above assumption is not realistic. As a result, a new model was designed to reduce to at least to some extent the huge differences between the analytical results and the measured values.

LOCAL TIMOSHENKO BEAM MODEL

The Timoshenko beam model [4] includes the effect of rotary inertia and shear deformation. The Timoshenko vibrational beam model gives the next expression:

$$\frac{EI}{\rho A}\frac{\partial^4 Y}{\partial x^4} + \frac{F_T}{\rho A}\frac{\partial^4 Y}{\partial x^4} + \frac{\partial^2}{\partial t^2} - \frac{l}{A}\left(1 + \frac{E}{KG}\right)\frac{\partial^4 Y}{\partial x^2 \partial t^2} + \frac{I}{A}\frac{\rho}{KG}\frac{\partial^4 Y}{\partial t^4} \tag{16}$$

$$K = \frac{2(1+\mu)}{4+3\mu} \tag{17}$$

In the Eq. (16) is I dynamic moment of inertia of the beam, nanotube, K is the shear coefficient of nanotube, μ is the Poisonn′s ratio. F_t presents additional thermal force:

$$F_t = \alpha TEA \tag{18}$$

The solution of eq. (16) could be expressed as:

$$Y(x,t) = y(x)e^{-i\omega t} \tag{19}$$

In the Eq (29) we call ω angular frequency On the above approximations the following dimensionless forms can be expressed as:

$$\frac{d^4\eta}{d\zeta^4}+\left[(\alpha+\beta)\Gamma^2+\delta\right]\frac{d^2\eta}{d\zeta^4}-\left(1-\Gamma^{2\alpha}\alpha\beta\right)\Gamma^2\eta \tag{20}$$

$$\eta=\frac{y}{L};\zeta=\frac{x}{L},\alpha=\frac{l}{AL^2},\beta=\frac{El}{KGAL^2} \tag{21}$$

$$\Gamma=\frac{\rho A\omega^2L^4}{El},\delta=\frac{F_TL^2}{El} \tag{22}$$

Where δ represents the effect of thermal vibration of the frequency of SWCNT. The general solution of Eq. (30) could be expressed as

$$\eta(\xi)=C_1\cos\left(\sqrt{\lambda-\gamma}\varepsilon\right)+C_2\cosh\left(\sqrt{\lambda+\gamma}\varepsilon\right)+C_3\sin\left(\sqrt{\lambda-\gamma}\varepsilon\right)+C_4\sinh\left(\sqrt{\lambda+\gamma}\varepsilon\right) \tag{23}$$

When the solution integrate with boundary condition for support-simply support nanotube model we obtain the solution:

$$\sin(\lambda-y)=0 \tag{24}$$

$$\Gamma^4-\frac{1+(n\pi)^{2(\alpha+\beta)}}{\alpha\beta}\Gamma^2+\frac{n^2\pi^2}{\alpha\beta}\left(-n^2\pi^2+\delta\right)=0 \tag{25}$$

For the case of Euler-Bernouli beam ($\alpha=\beta=0$) we obtain the next equation:

$$\Gamma=n\pi\sqrt{-\delta+(n\pi)^2} \tag{26}$$

RESULTS AND DISCUSSION

The presented mathematical model was used to calculate thermodynamic properties of state of pure aluminum microbeam. Table 1 contains the main important data of the beam. The aluminum beam is very interesting, particularly due to relatively high expansion coefficients. In the presentd section we have calculated vibrational characteristics for supported-simply supported systems. For carbon nanotubes we have used data for Young modulus and linear expansion coefficient shown in Prakash Thesis [7]. Indicated in Figures 2 and 3 show angular frequency for nanotubes for 1st, second and third order. From both Figures we see that when we have relatively long nanotube (L/D>10) the results for Timoshenko and Euler-Bernouli model give similar results, contrary when we have short nanotubes the Timoshenko model gives much better results for 1st and higher orders..

Table 1: Fundamental constants for aluminum beam

	Beam
Length (m)	$6.35\cdot10^{-2}$
Width (m)	$2.04\cdot10^{-2}$
Thickness (m)	$1.62\cdot10^{-3}$
Young modulus (N/m^2)	$6.9\cdot10^{10}$
Volume expansion coefficient (1/K)	$24\cdot10^{-6}$ K^{-1}
Spring constant (N/m)	$1.553\cdot10^{5}$
Density (kg/m^3)	2780

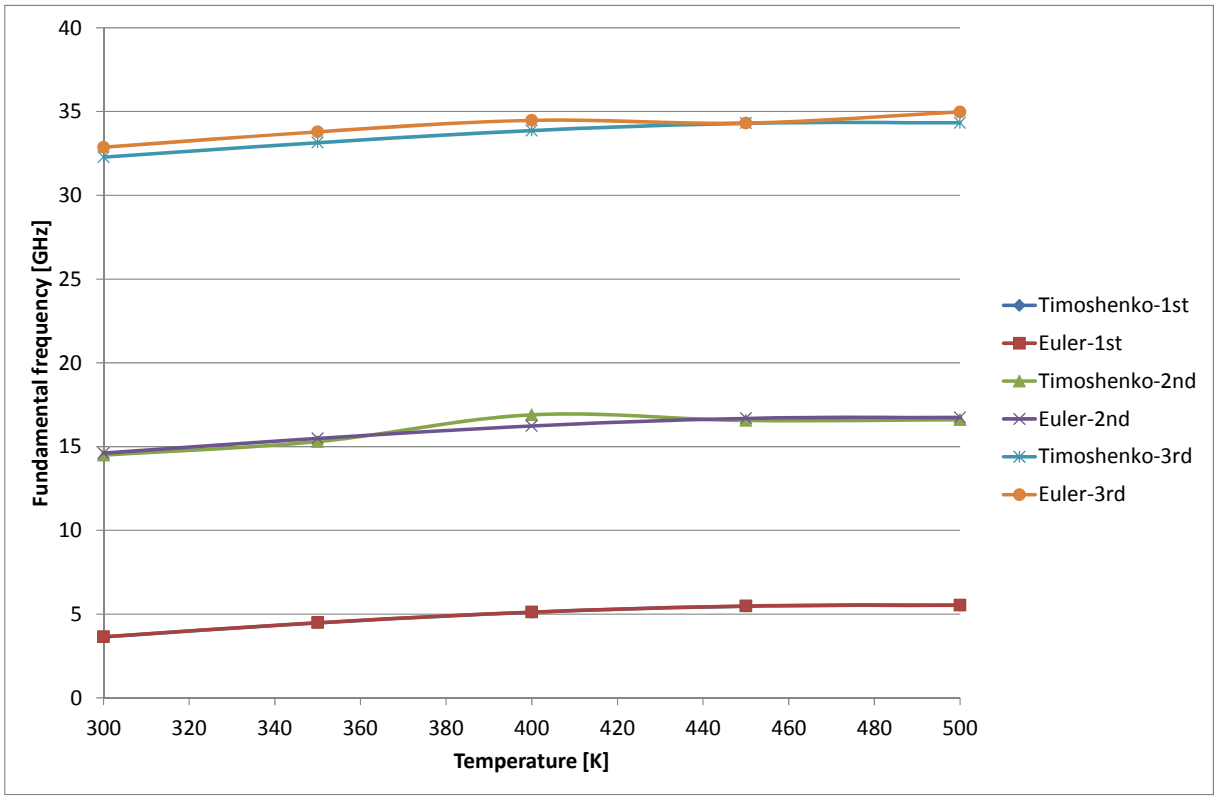

Figure 2: Fundamental frequency for nanotube with L/D=40 with Timoshenko and Euler-Bernouli model

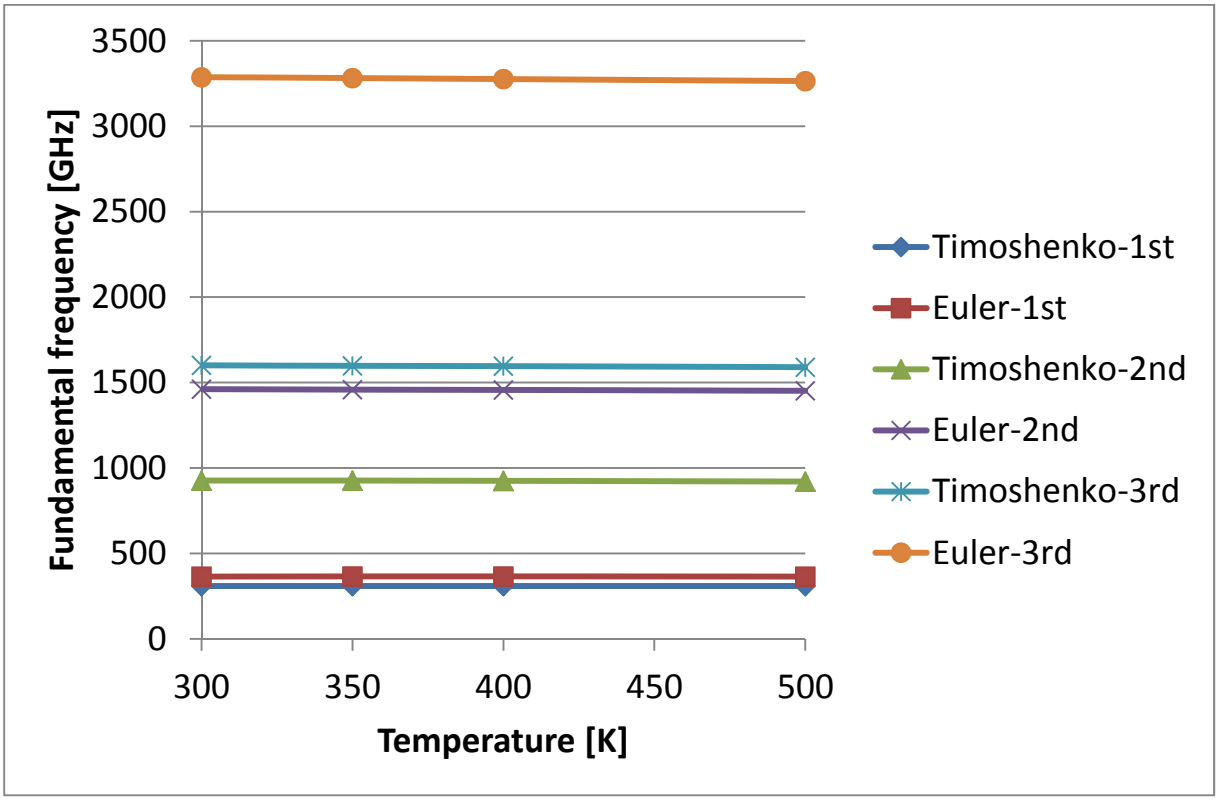

Figure 3: Fundamental frequency for nanotube with L/D=4 with Timoshenko and Euler-Bernouli model

CONCLUSION

The presented article shows the vibration characteristics for beams, minibeams and macrobeams in dependence of temperature field. The presented aanalysis also shows where the results of Euler-bernouli and Timoshenko deviate essentialy. In the presented article is also for the first time in scientific literature also presented analytical solution also for higher vibration orders for nanobeams.

REFERENCES

[1] **Guz, A.; Rushitskii, Ya**., ″Nanomaterials on the Mechanics of Nanomaterials″, International Applied Mechanics, Vol. 39, No. 11, 2003, pp.:1271-1293

[2] Marques, R.F., Inman, D.J. Rade, D.J., , "Assesment of adaptive techniques for the control of structures subject to temperature variations", 12th ICSV Conference, 2005, Lisbon, Portugal.

[3] Avsec, J., , "The Influence of Temperature Field on Vibration Characteristics of Beams and Rotating Shafts, ‚Journal of Vibroengineering. vol. 11, iss. 4, pp.665-672, 2009.

[4] Hsu, J.-C., Chang, R.-P., Chang W.-J., Resonance frequency of chiral single-walled carbon anotubes using Timoshenko beam theory, 2008, Phys. Let. A, Vol. 372, pp.: 2757.2759.

[5] Chang, T., Geng, J., Guo, X., Prediction of chirality-, and size dependent elastic properties of single-walled carbon nanotubes via molecular mechanics model, proc. Roy. Soc.A, 2006, Vol. 462, pp. 2524-2540.

[6] Weaver, W., S. Timoshenko, Young, D.H., "Vibration problems in engineering", John Wiley & Sons, 1974, New York.

[7] Prakash N., Determination of coefficient of thermal expansion of single-walled carbon nanotubes using molecular dynamics simulation, PhD Thesis, Florida state University, 2005.

[10] Bahzad, M., Bastami, A.R., Effect of centrifugal force on natural frequency of lateral vibration of rotating shafts, Journal of Sound and Vibration, 2004, Vol. 274, No. 3, pp. 985-995

[11] Avsec, J., Oblak, M., "Vibrational analysis of temperature dependent materials for clamped isotropic beam under thermal effects", Journal of Sound and Vibration, vol. 308, iss. 3/5, str. 514-525, 2007.

[12] Weaver, W., S. Timoshenko, Young, D.H., "Vibration problems in engineering", John Wiley & Sons, 1974, New York.

[13] Chang, T., Gao, H., Size-dependent elastic properties of a single walled carbon nanotube via a molecular mechanics model, Journal of mechanics and physics of Solids, 2003, Vol. 51, pp.: 1059-1074.