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GENERALIZED FVDAM THEORY FOR PERIODIC MATERIALS UNDERGOING FINITE DEFORMATIONS

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ABSTRACT

The recently constructed generalized finite-volume theory for two-dimensional linear elasticity problems on rectangular domains is further extended to make possible simulation of periodic materials with complex microstructures undergoing finite deformations. This is accomplished by embedding the generalized finite-volume theory with newly incorporated finite-deformation features into the 0th order homogenization framework, and introducing parametric mapping to enable efficient mimicking of complex microstructural details. The higher-order displacement field representation within subvolumes of the discretized unit cell microstructure, expressed in terms of elasticity-based surface-averaged kinematic variables, substantially improves interfacial conformability and pointwise traction and non-traction stress continuity between adjacent subvolumes. These features enable application of much larger deformations in comparison with the standard finite-volume direct averaging micromechanics (FVDAM) theory developed for finite-deformation applications.

Keywords: finite-volume micromechanics, 0th-order homogenization, higher-order displacement field, parametric mapping, finite deformation.

INTRODUCTION

Given the documented success of the generalized finite-volume theory for linear plane elasticity problems (Cavalcante and Pindera, 2012a,b), in this contribution we further extend its theoretical framework in order to enable analysis of periodic materials with complex microstructures undergoing large deformations. This is accomplished by formulating the governing equations in the finite-deformation domain based on the Lagrangian description of material deformation within the 0th order homogenization theory, and incorporating parametric mapping to enable efficient modeling of microstructural details of a periodic material such as inclusions and porosities with curved boundaries, amongst others (Cavalcante et al., 2011).

The generalized finite-volume direct averaging micromechanics (FVDAM) theory is constructed in a manner that enables systematic specialization to lower order versions through separation of surface-averaged displacement, rotation and curvature contributions towards interfacial conformability between adjacent subvolumes. Each reduction in order indicates which kinematic and static features of the local subvolume response are abandoned, establishing clear connection between mathematics and physics of the subvolume's deformation characteristics relative to its neighbors.

RESULTS AND CONCLUSIONS

In this section we examine the effect of the higher-order displacement field variables which represent surface-averaged rotational and curvature effects for a hexagonal array of cylindrical porosities with 25% volume content, Figure 1. As no analytical solution is available for this case, a Q9-based finite element formulation is employed for verification and comparison. An incremental-iterative scheme based on Newton-Raphson approach was employed for both the generalized FVDAM theory and finite element method.

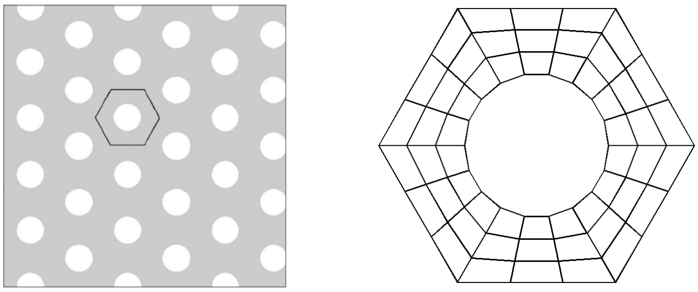

Fig. 1 Hexagonal array of circular porosities (left) and the coarsest meshes used in the analysis with 18x3 subvolumes/elements (right).

The matrix is modeled by the compressible Mooney-Rivlin material with the strain energy density function given by

$$W = c_1\left(I_1/I_3^{1/3} - 3\right) + c_2\left(I_2/I_3^{2/3} - 3\right) + c_3(J-1)^2$$

where I_1, I_2 and I_3 are the right Cauchy-Green deformation tensor invariants and $J = det\boldsymbol{F}$. For consistency with linear elasticity, $c_1 + c_2 = \mu/2$ and $c_3 = \kappa/2$, where μ and κ are shear and bulk moduli, respectively. The matrix phase parameters are listed in Table 1.

Table 1 – Elastic parameters of the compressible Mooney-Rivlin material.

Material	***E (MPa)***	***ν***	***κ (MPa)***	***μ (MPa)***	***c_1 (MPa)***	***c_2 (MPa)***
Rubber matrix	4.00	0.45	13.3333	1.3793	0.6897	0.00

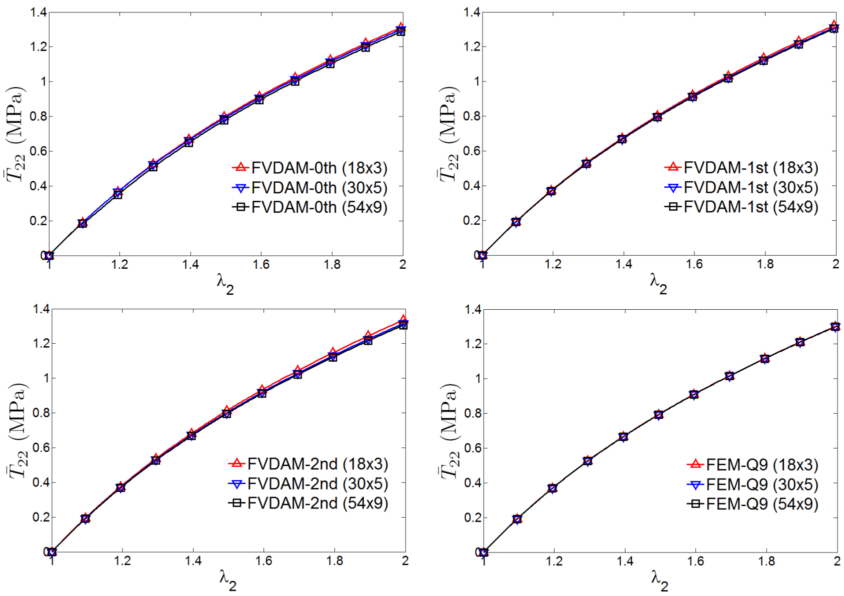

Fig. 2 Homogenized response obtained from the numerical solutions using different mesh discretizations for a uniaxial macroscopic transverse loading $\bar{T}_{22} \neq 0$.

The applied pure transverse loading was carried out until a maximum transverse stretch of $\lambda_2 = 2.0$ was attained. The resulting homogenized $\bar{T}_{22} - \lambda_2$ stress-stretch response predicted by the FVDAM theories and the finite element analysis is shown in Figure 2 for three mesh discretizations ranging from 18x3 to 54x9 which were sufficient to produce converged homogenized response. Despite quick convergence of the homogenized response with mesh refinement for the 0th order theory, the interfacial interpenetrations due to rotational differential of adjacent subvolume faces become substantial at large stretches, and do not completely vanish with mesh refinement, thereby affecting the local stress fields. This is seen in Figure 3 which compares the local transverse Cauchy stress $\sigma_{22}(Y_2, Y_3)$ distributions predicted by the FVDAM and finite element approaches and plotted in the deformed configurations at the final stretch of $\lambda_2 = 2.0$ for the two mesh discretizations of 18x3 and 54x9 subvolumes/elements. In contrast, smooth distributions are observed for the 1st and 2nd order FVDAM theories which converge to the finite element results with sufficient mesh refinement, which in this case is the 54x9 subvolume mesh.

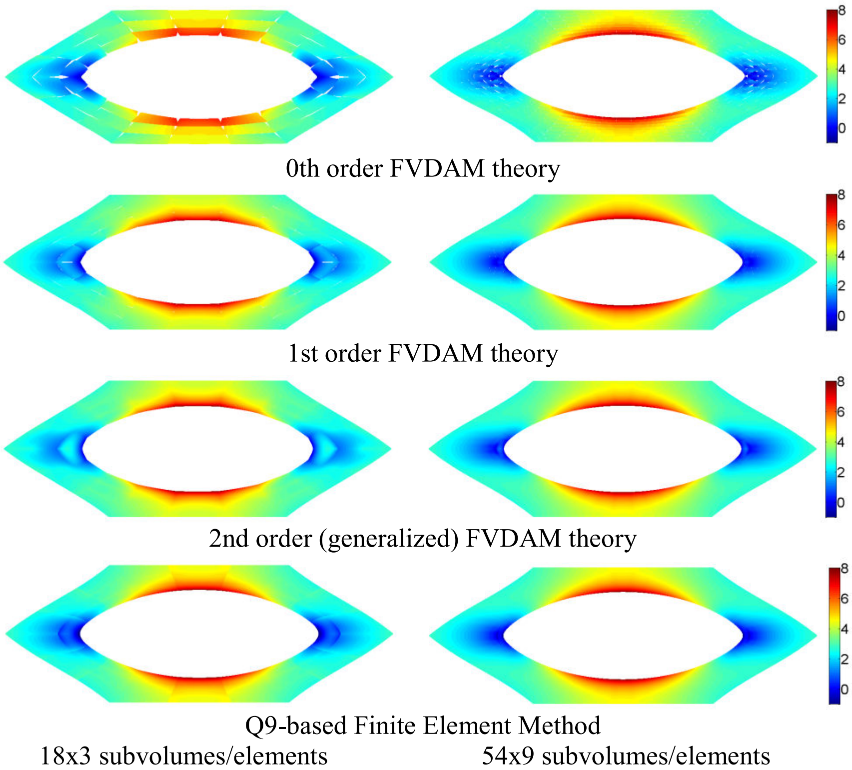

Fig. 3 Comparison of the Cauchy stress fields $\sigma_{22}(Y_2, Y_3)$ obtained from the numerical solutions using different mesh discretizations for a uniaxial macroscopic transverse loading $\bar{T}_{22} \neq 0$ at the macroscopic stretch $\lambda_2 = 2.0$.

The results indicate that the addition of higher-order terms in the local fluctuating displacement field representation, which provides the basis for the higher-order theory's framework, substantially improves the conformability of adjacent subvolumes by reducing and eliminating interfacial interpenetrations. This is a direct result of satisfying continuity of the newly introduced surface-averaged kinematic variables which represent interfacial rotations and curvatures, and it is particularly important in situations involving large deformations.

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