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SOME APPROACHES FOR DETERMINATION OF EFFECTIVE PROPERTIES FOR ACTIVE MULTIPHASE COMPOSITES

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ABSTRACT

The work discusses the uniform approaches to the effective modules determination of multiphase composite materials with the coupling of physic-mechanical fields based on the effective moduli method from the composite mechanics, modeling of the representative volumes with account for microstructure and the finite element technologies for solving the problems for the representative volumes. In order to provide an example, the models of porous piezoelectric, magnetoelectric, porous elastic and thermoelastic media are considered.

Keywords: active composite, coupled fields, effective moduli, finite element modeling, magnetoelectric composite, porous elastic composite, porous piezoceramics

INTRODUCTION

In the recent time there has been observed an increased interest to the investigations of composite materials of complex structure that exhibit very effective properties for many practical applications. For example, porous piezoelectric ceramics have extraordinary high volumetric piezosencitivity, low acoustic impedance and extended frequencies bandwidth. In connection to this, the use of porous piezocomposite materials enables to improve considerably the main parameters of ultrasonic piezo emitters. Two-phase magnetoelectric composites consisting of piezo- and magnetoactive phases demonstrate the ability to mutual transformation of magnetic and electric fields, at that each single phase does not have such property. Modern magnetoelectric composites have high effectiventess of the magnetoelectric transformation, relatively high temperatures of phase transitions and considerable technological resource. The problems of determination of effective properties still remain actual for more traditional composites with field coupling, for example, thermo- and poroelastic porous bodies, especially with account for their microstructure. Mathematical and computer investigations of composite materials of complex structure enable to explain and simulate some of their important characteristics and predict the effectiveness of different relations and structure couplings of constitutive phases.

In present paper we have developed the effective moduli method and finite element technique for porous elastic materials, thermoelastic, piezoelectric and magnetoelectric (electromagnetoelastic) composites.

THE MODELS OF MAGNETOELASTIC AND ELECTROELASTIC (PIEZOELECTRIC) MATERIALS

Let Ω be a representative volume of multiphase composite heterogeneous body, $\Gamma=\partial\Omega$ is its boundary, $\mathbf{n}$ is the outward unit normal vector, $\mathbf{x}=\{x_1,x_2,x_3\}$ is the vector of the special coordinates. Further we will use these notations for all composite materials under consideration.

In the framework of the static theory of magnetoelectroelasticity the displacement vector $\mathbf{u}$, the electric potential φ and the magnetic potential ϕ for heterogeneous magnetoelectric material in the volume Ω should satisfy the following system of differential equations

$$\nabla\cdot\boldsymbol{\sigma}=0\,,\qquad \nabla\cdot\mathbf{D}=0\,,\qquad \nabla\cdot\mathbf{B}=0\,, \tag{1}$$

$$\boldsymbol{\sigma}=\mathbf{c}^{E,H}\cdot\cdot\boldsymbol{\varepsilon}-\mathbf{e}^{H*}\cdot\mathbf{E}-\mathbf{q}^{E*}\cdot\mathbf{H}\,, \tag{2}$$

$$\mathbf{D}=\mathbf{e}^{H}\cdot\cdot\boldsymbol{\varepsilon}+\boldsymbol{\varepsilon}^{S,H}\cdot\mathbf{E}+\boldsymbol{\alpha}^{S}\cdot\mathbf{H}\,, \tag{3}$$

$$\mathbf{B}=\mathbf{q}^{E}\cdot\cdot\boldsymbol{\varepsilon}+\boldsymbol{\alpha}^{S*}\cdot\mathbf{E}+\boldsymbol{\mu}^{S,E}\cdot\mathbf{H}\,, \tag{4}$$

$$\boldsymbol{\varepsilon}=\frac{1}{2}(\nabla\mathbf{u}+\nabla\mathbf{u}^{*})\,,\qquad \mathbf{E}=-\nabla\varphi\,,\qquad \mathbf{H}=-\nabla\phi \tag{5}$$

Here (1) are the field equations, (2) – (4) are the constitutive equations, $\boldsymbol{\sigma}$ is the stress second rank tensor, $\mathbf{D}$ is the vector of electric displacement (electric induction), $\mathbf{B}$ is the vector of magnetic induction, $\mathbf{c}^{E,H}$ is the forth rank tensor of elastic stiffness, calculated at constant electric and magnetic fields, $\mathbf{e}^{H}$ is the third rank tensor of piezomodules, calculated at constant magnetic field, $\mathbf{q}^{E}$ is the third rank tensor of magnetostriction modules, calculated at constant electric field, $\boldsymbol{\varepsilon}^{S,H}$ is the second rank tensor of dielectric permittivities, calculated at constant strains and magnetic field, $\boldsymbol{\alpha}^{S}$ is the second rank tensor of magnetoelectric coupling coefficients, calculated at constant strains, $\boldsymbol{\mu}^{S,E}$ is the second rank tensor of dielectric permittivities, calculated at constant strains and electric field, $\boldsymbol{\varepsilon}$ is the strain tensor, $\mathbf{E}$ is the electric field intensity vector, $\mathbf{H}$ is the magnetic field intensity vector.

For heterogeneous magnetoelectric medium its modules are the functions of coordinates, i.e. $\mathbf{c}^{E,H}=\mathbf{c}^{E,H}(\mathbf{x})$, $\mathbf{e}^{H}=\mathbf{e}^{H}(\mathbf{x})$, $\mathbf{q}^{E}=\mathbf{q}^{E}(\mathbf{x})$, $\boldsymbol{\varepsilon}^{S,H}=\boldsymbol{\varepsilon}^{S,H}(\mathbf{x})$, $\boldsymbol{\alpha}^{S}=\boldsymbol{\alpha}^{S}(\mathbf{x})$, $\boldsymbol{\mu}^{S,E}=\boldsymbol{\mu}^{S,E}(\mathbf{x})$, and they can vary significantly in the volume Ω for composite material.

Magnetoelectric composites are of interest because they consist of two phases $\Omega=\Omega_m\cup\Omega_e$: piezomagnetic phase Ω_m and piezoelectric phase Ω_e, and these phases can be mixed together. We note that $\mathbf{e}^{H}(\mathbf{x})=0$ and $\boldsymbol{\alpha}^{S}(\mathbf{x})=0$ for $\mathbf{x}\in\Omega_m$, $\mathbf{q}^{E}(\mathbf{x})=0$ and $\boldsymbol{\alpha}^{S}(\mathbf{x})=0$ for $\mathbf{x}\in\Omega_m$, that is both phases separately do not have magnetoelectric coupling. However for composite magnetoelectric medium due to the coupling of the magnetic and mechanical fields at the piezomagnetic phase Ω_m and the coupling of the electric and mechanical fields at the piezoelectric phase Ω_e as the result we get the coupling of magnetic and electric fields that does not exist at each separate phase. Recently magnetoelectric composites became of interest to many researchers so the number of works devoted to the modeling of the effective properties of these composites has increased considerably (Challagulla, 2011; Dinzart, 2009; Lee, 2005; Lu, 2011; etc.).

Piezoelectric composites are studied much better and have numerous practical applications (Rybyanets, 2010). The system of differential equations for piezoelectric materials is obtained as the special case of (1) – (3), (5), if we set $\mathbf{q}^{E}(\mathbf{x})=0$, $\boldsymbol{\alpha}^{S}(\mathbf{x})=0$ and do not take into account the magnetic components. Then the remaining modules for piezoelectric composites will also be the functions that vary strongly in Ω, i.e. $\mathbf{c}^{E,H} = \mathbf{c}^{E} = \mathbf{c}^{E}(\mathbf{x})$, $\mathbf{e}^{H} = \mathbf{e} = \mathbf{e}(\mathbf{x})$, $\boldsymbol{\varepsilon}^{S,H} = \boldsymbol{\varepsilon}^{S} = \boldsymbol{\varepsilon}^{S}(\mathbf{x})$. The best known piezoelectric composites are related with piezoceramic materials. They are represented by the classes of various connectivity types, such as ceramics-ceramics, ceramics-polymer and ceramics-pores (pore piezoceramics). It is also possible to obtain more complex structures, such as ceramics-polymer-pores.

It is well known, that the connectivity of the composites play an important part in the determining their effective properties and main characteristics. The classification (Newnham, 1986) became widely used for two-phase piezoelectric materials. According to this classification, the composite connectivity is written as $m-n$, where $m,n = 0,1,2,3$, at that the first number indicates the dimension of the main active (piezoceramic) phase whereas the second number indicates the dimension of the second phase (piezoceramics, polymer or pore). Here the dimension of the phase refers to the fact that this phase in the composite can be connected in one, two or three special dimensions or have no common points of contact with each other in the material volume (0-connectivity). In this work we restrict our attention to the consideration of mixture models of the composites of 3-0 and 3-3 connectivity. For the porous piezoceramics of these connectivity types in (Nasedkin, 2005, 2011; Domashenkina, 2011) the investigations on the applicability of the effective moduli method and finite element method for the correct determination of their effective constants were carried out. Further on it will be shown that the technologies developed according to (Nasedkina, 2012) can be applied for other multiphase composites.

EFFECTIVE MODULI METHOD FOR MAGNETOELECTRIC COMPOSITES

On the boundary Γ of magnetoelectric composite body we will consider the mechanical stress vector $\mathbf{p}$, the surface density of electric charges q_e and the surface density of magnetic charges q_m

$$\mathbf{p} = \mathbf{n}\cdot\boldsymbol{\sigma}, \qquad q_e = -\mathbf{n}\cdot\mathbf{D}, \quad q_m = -\mathbf{n}\cdot\mathbf{B}. \tag{6}$$

As usual, we will denote the volume-averaged quantities in the broken brackets

$$\langle ... \rangle = \frac{1}{\Omega}\int_{\Omega}(...)\, d\Omega \quad . \tag{7}$$

Following a substantiation of a method of effective moduli for elastic media, for piezoelectric body we will formulate the auxiliary statements. These statements are proved under similar techniques, as for an elastic body (Pobedria, 1984). The following states are found on the basis of effective moduli theory.

Lemma 1. These representations take place for volume averaging field characteristic by means of appropriate values on boundary Γ

(a) $$\boldsymbol{\varepsilon} = \frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^*) \quad \Rightarrow \quad \langle\boldsymbol{\varepsilon}\rangle = \frac{1}{2\Omega}\int_{\Gamma}(\mathbf{un}^* + \mathbf{nu}^*)\, d\Gamma, \tag{8}$$

(b) $$\mathbf{E} = -\nabla\varphi, \qquad \mathbf{H} = -\nabla\phi \quad \Rightarrow \quad \langle\mathbf{E}\rangle = -\frac{1}{\Omega}\int_{\Gamma}\mathbf{n}\varphi\, d\Gamma, \ \ \langle\mathbf{H}\rangle = -\frac{1}{\Omega}\int_{\Gamma}\mathbf{n}\phi\, d\Gamma, \tag{9}$$

(c) $$\mathbf{p} = \mathbf{n}\cdot\boldsymbol{\sigma}; \qquad \nabla\cdot\boldsymbol{\sigma} = 0 \quad \Rightarrow \quad \langle\boldsymbol{\sigma}\rangle = \frac{1}{2\Omega}\int_{\Gamma}(\mathbf{px}^* + \mathbf{xp}^*)\, d\Gamma, \tag{10}$$

(d) $\quad q_e = -\mathbf{n}\cdot\mathbf{D}; \quad \nabla\cdot\mathbf{D} = 0 \quad \Rightarrow \quad \langle\mathbf{D}\rangle = -\frac{1}{\Omega}\int_\Gamma \mathbf{x} q_e \, d\Gamma,$ (11)

(e) $\quad q_m = -\mathbf{n}\cdot\mathbf{B}; \quad \nabla\cdot\mathbf{B} = 0 \quad \Rightarrow \quad \langle\mathbf{B}\rangle = -\frac{1}{\Omega}\int_\Gamma \mathbf{x} q_m \, d\Gamma.$ (12)

◄ For proof statings (8)–(12) we can use component-wise notations, formulas (5) and Gauss divergence theorem:

$$\langle\varepsilon_{ij}\rangle = \frac{1}{2\Omega}\int_\Omega (u_{i,j} + u_{j,i})\, d\Omega = \frac{1}{2\Omega}\int_\Gamma (n_j u_i + n_i u_j)\, d\Gamma,$$

$\langle E_i\rangle = -\frac{1}{\Omega}\int_\Omega \varphi_{,i}\, d\Omega = \frac{1}{2\Omega}\int_\Gamma n_i\varphi\, d\Gamma$, and similarly for $\langle H_i\rangle$.

In addition for proof (10) – (12) we use the formulas

$(\sigma_{ik}x_j)_{,k} = \sigma_{ik,k}x_j + \sigma_{ij}, \qquad (\sigma_{jk}x_i)_{,k} = \sigma_{jk,k}x_i + \sigma_{ji},\ \sigma_{ik,k} = 0;\ \sigma_{ij} = \sigma_{ji} \Rightarrow$

$\sigma_{ij} = [(\sigma_{ik}x_j)_{,k} + (\sigma_{jk}x_i)_{,k}]/2,$

$(D_i x_j)_{,i} = D_{i,i}x_j + D_i;\ D_{i,i} = 0$, and similarly for $(B_i x_j)_{,i}$.

Therefore,

$$\langle\sigma_{ij}\rangle = \frac{1}{2\Omega}\int_\Omega (\sigma_{ik}x_j + \sigma_{jk}x_i)_{,k}\, d\Omega = \frac{1}{2\Omega}\int_\Gamma (n_k\sigma_{ik}x_j + n_k\sigma_{jk}x_i)\, d\Gamma = \frac{1}{2\Omega}\int_\Gamma (p_i x_j + p_j x_i)\, d\Gamma,$$

$\langle D_i\rangle = \frac{1}{\Omega}\int_\Omega (D_i x_j)_{,i}\, d\Omega = \frac{1}{\Omega}\int_\Gamma n_i D_i x_j\, d\Gamma = -\frac{1}{\Omega}\int_\Gamma q_e x_j\, d\Gamma$, and similarly for $\langle B_i\rangle$.►

Lemma 2.

(a) Let $\mathbf{u} = \mathbf{x}\cdot\boldsymbol{\varepsilon}_0|_\Gamma$ for $\forall\,\mathbf{x}\in\Gamma$, where $\boldsymbol{\varepsilon}_0 = \boldsymbol{\varepsilon}_0{}^* = \text{const}$. Then we have $\langle\boldsymbol{\varepsilon}\rangle = \boldsymbol{\varepsilon}_0$.

(b) Let $\varphi = -\mathbf{x}\cdot\mathbf{E}_0|_\Gamma$ for $\forall\,\mathbf{x}\in\Gamma$, where $\mathbf{E}_0 = \text{const}$. Then we have $\langle\mathbf{E}\rangle = \mathbf{E}_0$.

(c) Let $\phi = -\mathbf{x}\cdot\mathbf{H}_0|_\Gamma$ for $\forall\,\mathbf{x}\in\Gamma$, where $\mathbf{H}_0 = \text{const}$. Then we have $\langle\mathbf{H}\rangle = \mathbf{H}_0$.

(d) Let $\mathbf{p} = \mathbf{n}\cdot\boldsymbol{\sigma}_0|_\Gamma$ for $\forall\,\mathbf{x}\in\Gamma$, where $\mathbf{p}$ is the stress vector from (6) and $\boldsymbol{\sigma}_0 = \text{const}$. Then we have $\langle\boldsymbol{\sigma}\rangle = \boldsymbol{\sigma}_0$.

(e) Let $q_e = -\mathbf{n}\cdot\mathbf{D}_0|_\Gamma$ for $\forall\,\mathbf{x}\in\Gamma$, where q_e is the surface density of electric charges from (6) and $\mathbf{D}_0 = \text{const}$. Then we have $\langle\mathbf{D}\rangle = \mathbf{D}_0$.

(f) Let $q_m = -\mathbf{n}\cdot\mathbf{B}_0|_\Gamma$ for $\forall\,\mathbf{x}\in\Gamma$, where q_m is the surface density of magnetic charges from (6) and $\mathbf{B}_0 = \text{const}$. Then we have $\langle\mathbf{B}\rangle = \mathbf{B}_0$.

◄ For proof (a)–(f) we apply the corresponding statement (a)–(e) from Lemma 1 and transfer from integration on Γ to integration on Ω by Gauss divergence theorem.►

Lemma 3.

(a) Let $\mathbf{u} = \mathbf{x}\cdot\boldsymbol{\varepsilon}_0|_\Gamma$ for $\forall\,\mathbf{x}\in\Gamma$, where $\boldsymbol{\varepsilon}_0 = \boldsymbol{\varepsilon}_0{}^* = \text{const}$, and the equilibrium equation $\nabla\cdot\boldsymbol{\sigma} = 0$ takes place. Then we have $\langle\boldsymbol{\varepsilon}\cdot\cdot\boldsymbol{\sigma}\rangle/2 = \langle\boldsymbol{\varepsilon}\rangle\cdot\cdot\langle\boldsymbol{\sigma}\rangle/2$.

(b) Let $\varphi = -\mathbf{x}\cdot\mathbf{E}_0|_\Gamma$ for $\forall\,\mathbf{x}\in\Gamma$, where $\mathbf{E}_0 = \text{const}$, and the equation of quasielectrostatics $\nabla\cdot\mathbf{D} = 0$ takes place. Then we have $\langle\mathbf{D}\cdot\mathbf{E}\rangle/2 = \langle\mathbf{D}\rangle\cdot\langle\mathbf{E}\rangle/2$.

(c) Let $\phi = -\mathbf{x}\cdot\mathbf{H}_0|_\Gamma$ for $\forall\,\mathbf{x}\in\Gamma$, where $\mathbf{H}_0 = \text{const}$, and the equation of quasimagnetostatics $\nabla\cdot\mathbf{B} = 0$ takes place. Then we have $\langle\mathbf{B}\cdot\mathbf{H}\rangle/2 = \langle\mathbf{B}\rangle\cdot\langle\mathbf{H}\rangle/2$.

(d) Let $\mathbf{p} = \mathbf{n}\cdot\boldsymbol{\sigma}_0|_\Gamma$ for $\forall\,\mathbf{x}\in\Gamma$, where $\boldsymbol{\sigma}_0 = \text{const}$, $\mathbf{p}$ is the stress vector from (6), and the equilibrium equation $\nabla\cdot\boldsymbol{\sigma} = 0$ takes place. Then we have $\langle\boldsymbol{\varepsilon}\cdot\cdot\boldsymbol{\sigma}\rangle/2 = \langle\boldsymbol{\varepsilon}\rangle\cdot\cdot\langle\boldsymbol{\sigma}\rangle/2$.

(e) Let $q_e = -\mathbf{n}\cdot\mathbf{D}_0|_\Gamma$ for $\forall\,\mathbf{x}\in\Gamma$, where $\mathbf{D}_0 = \text{const}$, q_e is the surface density of electric charges from (6), and the equation of quasielectrostatics $\nabla\cdot\mathbf{D} = 0$ takes place. Then we have $\langle\mathbf{D}\cdot\mathbf{E}\rangle/2 = \langle\mathbf{D}\rangle\cdot\langle\mathbf{E}\rangle/2$.

(f) Let $q_m = -\mathbf{n}\cdot\mathbf{B}_0|_\Gamma$ for $\forall\,\mathbf{x}\in\Gamma$, where $\mathbf{B}_0 = \text{const}$, q_m is the surface density of magnetic charges from (6), and the equation of quasimagnetostatics $\nabla\cdot\mathbf{B} = 0$ takes place. Then we have $\langle\mathbf{B}\cdot\mathbf{H}\rangle/2 = \langle\mathbf{B}\rangle\cdot\langle\mathbf{H}\rangle/2$.

◄ (a) If $\mathbf{u} = \mathbf{x}\cdot\boldsymbol{\varepsilon}_0|_\Gamma$ for $\mathbf{x}\in\Gamma$, $\nabla\cdot\boldsymbol{\sigma} = 0$, that it is correct the following sequence of identities

$$\frac{1}{2}\langle\boldsymbol{\varepsilon}\cdot\cdot\boldsymbol{\sigma}\rangle = \frac{1}{2\Omega}\int_\Omega \nabla\mathbf{u}\cdot\cdot\boldsymbol{\sigma}\,d\Omega = \frac{1}{2\Omega}\int_\Gamma(\mathbf{n}\cdot\boldsymbol{\sigma})\cdot\mathbf{u}\,d\Gamma = \frac{1}{2\Omega}\int_\Gamma(\mathbf{n}\cdot\boldsymbol{\sigma})\cdot\boldsymbol{\varepsilon}_0\cdot\mathbf{x}\,d\Gamma =$$

$$= \frac{1}{2\Omega}\int_\Omega \nabla\cdot(\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}_0\cdot\mathbf{x})\,d\Omega = \frac{1}{2\Omega}\int_\Omega \boldsymbol{\sigma}\cdot\cdot\boldsymbol{\varepsilon}_0\,d\Omega = \frac{1}{2\Omega}\int_\Omega \boldsymbol{\sigma}\,d\Omega\cdot\cdot\boldsymbol{\varepsilon}_0 = \frac{1}{2}\langle\boldsymbol{\sigma}\rangle\cdot\cdot\langle\boldsymbol{\varepsilon}\rangle.$$

(b) If $\varphi = -\mathbf{x}\cdot\mathbf{E}_0|_\Gamma$ for $\mathbf{x}\in\Gamma$, $\nabla\cdot\mathbf{D} = 0$, then $\frac{1}{2}\langle\mathbf{D}\cdot\mathbf{E}\rangle = -\frac{1}{2\Omega}\int_\Omega \mathbf{D}\cdot\nabla\varphi\,d\Omega =$

$$= -\frac{1}{2\Omega}\int_\Omega \nabla\cdot(\mathbf{D}\varphi)\,d\Omega = -\frac{1}{2\Omega}\int_\Gamma \mathbf{n}\cdot\mathbf{D}\varphi\,d\Gamma = \frac{1}{2\Omega}\int_\Gamma \mathbf{n}\cdot\mathbf{D}(\mathbf{x}\cdot\mathbf{E}_0)\,d\Gamma =$$

$= \frac{1}{2\Omega}\int_\Omega \nabla\cdot(\mathbf{D}(\mathbf{x}\cdot\mathbf{E}_0))d\Omega = \frac{1}{2\Omega}\int_\Omega \mathbf{D}\cdot\mathbf{E}_0 d\Omega = \frac{1}{2}\langle\mathbf{D}\rangle\cdot\langle\mathbf{E}_0\rangle = \frac{1}{2}\langle\mathbf{D}\rangle\cdot\langle\mathbf{E}\rangle$, and similarly for (c).

(d) If $\mathbf{p} = \mathbf{n}\cdot\boldsymbol{\sigma}_0|_\Gamma$, $\nabla\cdot\boldsymbol{\sigma} = 0$, then $\frac{1}{2}\langle\boldsymbol{\varepsilon}\cdot\cdot\boldsymbol{\sigma}\rangle = \frac{1}{2\Omega}\int_\Omega \nabla\mathbf{u}\cdot\cdot\boldsymbol{\sigma}\,d\Omega = \frac{1}{2\Omega}\int_\Gamma(\mathbf{n}\cdot\boldsymbol{\sigma})\cdot\mathbf{u}\,d\Gamma =$

$$= \frac{1}{2\Omega}\int_\Gamma(\mathbf{n}\cdot\boldsymbol{\sigma}_0)\cdot\mathbf{u}\,d\Gamma = \frac{1}{2\Omega}\int_\Omega \nabla\mathbf{u}\cdot\cdot\boldsymbol{\sigma}_0\,d\Omega = \frac{1}{2\Omega}\int_\Omega \boldsymbol{\varepsilon}\cdot\cdot\boldsymbol{\sigma}_0\,d\Omega = \frac{1}{2}\langle\boldsymbol{\varepsilon}\rangle\cdot\cdot\langle\boldsymbol{\sigma}_0\rangle = \frac{1}{2}\langle\boldsymbol{\varepsilon}\rangle\cdot\cdot\langle\boldsymbol{\sigma}_0\rangle$$

(e) If $q_e = -\mathbf{n}\cdot\mathbf{D}_0|_\Gamma$, $\nabla\cdot\mathbf{D} = 0$, then $\frac{1}{2}\langle\mathbf{D}\cdot\mathbf{E}\rangle = -\frac{1}{2\Omega}\int_\Omega \mathbf{D}\cdot\nabla\varphi\,d\Omega =$

$$= -\frac{1}{2\Omega}\int_\Omega \nabla\cdot(\mathbf{D}\varphi)\,d\Omega = -\frac{1}{2\Omega}\int_\Gamma \mathbf{n}\cdot\mathbf{D}\varphi\,d\Gamma = -\frac{1}{2\Omega}\int_\Gamma \mathbf{n}\cdot\mathbf{D}_0\varphi\,d\Gamma = -\frac{1}{2\Omega}\int_\Omega \nabla\cdot(\mathbf{D}_0\varphi)\,d\Gamma =$$

$$= -\frac{1}{2\Omega}\int_{\Omega} \mathbf{D}_0 \cdot \nabla \varphi \, d\Omega = \frac{1}{2\Omega}\int_{\Omega} \mathbf{D}_0 \cdot \mathbf{E} \, d\Omega = \frac{1}{2}\langle \mathbf{D}_0 \rangle \cdot \langle \mathbf{E} \rangle \text{, , and similarly for (f).} \blacktriangleright$$

In accordance with four equivalent fundamental forms of constitutive equations we will introduce the moduli of magnetoelectric medium:

– $\varepsilon E H$ -form (2) – (4) in independent variables $\boldsymbol{\varepsilon}$, $\mathbf{E}$, $\mathbf{H}$,

– $\sigma E H$ -form in independent variables $\boldsymbol{\varepsilon}$, $\mathbf{E}$, $\mathbf{H}$

$$\boldsymbol{\varepsilon} = \mathbf{s}^{T,H} \cdot\cdot \boldsymbol{\sigma} + \mathbf{d}^{H*} \cdot \mathbf{E} + \mathbf{r}^{E*} \cdot \mathbf{H}, \tag{13}$$

$$\mathbf{D} = \mathbf{d}^{H} \cdot\cdot \boldsymbol{\sigma} + \boldsymbol{\varepsilon}^{T,H} \cdot \mathbf{E} + \boldsymbol{\gamma}^{T} \cdot \mathbf{H}, \tag{14}$$

$$\mathbf{B} = \mathbf{q}^{E} \cdot\cdot \boldsymbol{\sigma} + \boldsymbol{\gamma}^{T*} \cdot \mathbf{E} + \boldsymbol{\mu}^{T,E} \cdot \mathbf{H}, \tag{15}$$

and analogously for $\varepsilon D H$, $\sigma D H$, $\varepsilon E B$, $\sigma E B$, $\varepsilon D B$, $\sigma D B$ -forms.

Let Ω be the representative volume of heterogeneous piezoelectric materials. We will determine the effective moduli $\tilde{\mathbf{c}}^{E,H}$, $\tilde{\mathbf{e}}^{H}$, $\tilde{\mathbf{q}}^{E}$, $\tilde{\boldsymbol{\varepsilon}}^{S,H}$, $\tilde{\boldsymbol{\alpha}}^{S}$, $\tilde{\boldsymbol{\mu}}^{S,E}$ by the following technique similarly for elastic and piezoelectric composites (Nasedkin, 2011).

We consider the static magnetoelectroelastic problem for representative volume Ω with Eqs. (1) – (5) and the following boundary conditions

$$\mathbf{u} = \mathbf{x} \cdot \boldsymbol{\varepsilon}_0, \qquad \varphi = -\mathbf{x} \cdot \mathbf{E}_0, \quad \phi = -\mathbf{x} \cdot \mathbf{H}_0, \quad \mathbf{x} \in \Gamma = \partial\Omega. \tag{16}$$

We call the problem (1) – (5), (16) the problem I and denote the solution of this problem by $\mathbf{u}^{\mathrm{I}}$, φ^{I}, φ^{I}. From the obtained solution and (2) – (5) we can find $\boldsymbol{\varepsilon}^{\mathrm{I}}$, $\mathbf{E}^{\mathrm{I}}$, $\mathbf{H}^{\mathrm{I}}$, $\boldsymbol{\sigma}^{\mathrm{I}}$, $\mathbf{D}^{\mathrm{I}}$ and $\mathbf{B}^{\mathrm{I}}$, where $\boldsymbol{\varepsilon}^{\mathrm{I}} = \boldsymbol{\varepsilon}(\mathbf{u}^{\mathrm{I}})$ and so on. Note, that from lemma 2 for the problem I the following relations take place $\langle \boldsymbol{\varepsilon}^{\mathrm{I}} \rangle = \boldsymbol{\varepsilon}_0$, $\langle \mathbf{H}^{\mathrm{I}} \rangle = \mathbf{H}_0$ and $\langle \mathbf{E}^{\mathrm{I}} \rangle = \mathbf{E}_0$.

We supply in conformity to the initial heterogeneous medium some "equivalent" homogeneous medium with effective moduli $\tilde{\mathbf{c}}^{E,H}$, $\tilde{\mathbf{e}}^{H}$, $\tilde{\mathbf{q}}^{E}$, $\tilde{\boldsymbol{\varepsilon}}^{S,H}$, $\tilde{\boldsymbol{\alpha}}^{S}$ and $\tilde{\boldsymbol{\mu}}^{S,E}$. The constitutive equations for "equivalent" medium, analogous (1) – (4), are in the forms

$$\boldsymbol{\sigma}_0 = \tilde{\mathbf{c}}^{E,H} \cdot\cdot \boldsymbol{\varepsilon}_0 - \tilde{\mathbf{e}}^{H*} \cdot \mathbf{E}_0 - \tilde{\mathbf{q}}^{E*} \cdot \mathbf{H}_0, \tag{17}$$

$$\mathbf{D}_0 = \tilde{\mathbf{e}}^{H} \cdot\cdot \boldsymbol{\varepsilon}_0 + \tilde{\boldsymbol{\varepsilon}}^{S,H} \cdot \mathbf{E}_0 + \tilde{\boldsymbol{\alpha}}^{S} \cdot \mathbf{H}_0, \tag{18}$$

$$\mathbf{B}_0 = \tilde{\mathbf{q}}^{E} \cdot\cdot \boldsymbol{\varepsilon}_0 + \tilde{\boldsymbol{\alpha}}^{S*} \cdot \mathbf{E}_0 + \tilde{\boldsymbol{\mu}}^{S,E} \cdot \mathbf{H}_0. \tag{19}$$

For problem I we accept the following equations such as relations for definition of effective moduli from (17) – (19)

$$\langle \boldsymbol{\sigma}^{\mathrm{I}} \rangle = \boldsymbol{\sigma}_0; \qquad \langle \mathbf{D}^{\mathrm{I}} \rangle = \mathbf{D}_0, \qquad \langle \mathbf{B}^{\mathrm{I}} \rangle = \mathbf{B}_0. \tag{20}$$

The moduli, found from these conditions, are marked with superscripts "I". Note, that with lemma 3 the average energies are equal for heterogeneous and for "equivalent" homogeneous magnetoelectric media

$$\langle \boldsymbol{\sigma}^{\mathrm{I}} \cdot\cdot \boldsymbol{\varepsilon}^{\mathrm{I}} + \mathbf{D}^{\mathrm{I}} \cdot \mathbf{E}^{\mathrm{I}} + \mathbf{B}^{\mathrm{I}} \cdot \mathbf{H}^{\mathrm{I}} \rangle / 2 = (\boldsymbol{\sigma}_0 \cdot\cdot \boldsymbol{\varepsilon}_0 + \mathbf{D}_0 \cdot \mathbf{E}_0 + \mathbf{B}_0 \cdot \mathbf{H}_0)/2. \tag{21}$$

Now, by using (17) – (20), we can select such boundary conditions, at which the obvious expressions for effective moduli are obtained. For example, we consider problem I (1) – (5), (16) with

$$\boldsymbol{\varepsilon}_0 = \varepsilon_0(\mathbf{e}_k\mathbf{e}_m + \mathbf{e}_m\mathbf{e}_k)\,; \qquad \mathbf{E}_0 = 0\,, \qquad \mathbf{H}_0 = 0\,, \tag{22}$$

where k, m are some fixed numbers ($k, m = 1, 2, 3$); $\mathbf{e}_k$ are the unit vectors of Cartesian basis. Then, from (17) – (22) we obtain

$$\tilde{c}^{E,H\,\mathrm{I}}_{ijkm} = \left\langle \sigma^{\mathrm{I}}_{ij} \right\rangle / (2\varepsilon_0)\,; \quad \tilde{e}^{H\,\mathrm{I}}_{jkm} = \left\langle D^{\mathrm{I}}_j \right\rangle / (2\varepsilon_0)\,, \quad \tilde{q}^{E\,\mathrm{I}}_{jkm} = \left\langle B^{\mathrm{I}}_j \right\rangle / (2\varepsilon_0) \tag{23}$$

If in the problem I (1) – (5), (16) we accept

$$\boldsymbol{\varepsilon}_0 = 0\,; \qquad \mathbf{E}_0 = E_0\mathbf{e}_k\,, \qquad \mathbf{H}_0 = 0\,, \tag{24}$$

then from (17) – (20), (24) we find

$$\tilde{e}^{H\,\mathrm{I}}_{kij} = -\left\langle \sigma^{\mathrm{I}}_{ij} \right\rangle / E_0\,; \quad \tilde{\varepsilon}^{S,H\,\mathrm{I}}_{jk} = \left\langle D^{\mathrm{I}}_j \right\rangle / E_0\,, \quad \tilde{\alpha}^{S\,\mathrm{I}}_{kj} = \left\langle B^{\mathrm{I}}_j \right\rangle / E_0\,. \tag{25}$$

Finally, if in the problem I (1) – (5), (16) we accept

$$\boldsymbol{\varepsilon}_0 = 0\,; \qquad \mathbf{E}_0 = 0\,, \qquad \mathbf{H}_0 = H_0\mathbf{e}_k\,, \tag{26}$$

then from (17) – (20), (26) we obtain

$$\tilde{q}^{E\,\mathrm{I}}_{kij} = -\left\langle \sigma^{\mathrm{I}}_{ij} \right\rangle / H_0\,; \quad \tilde{\alpha}^{S\,\mathrm{I}}_{jk} = \left\langle D^{\mathrm{I}}_j \right\rangle / H_0\,, \quad \tilde{\mu}^{S,E\,\mathrm{I}}_{kj} = \left\langle B^{\mathrm{I}}_j \right\rangle / H_0\,. \tag{27}$$

Note, that the quantities $\left\langle \sigma^{\mathrm{I}}_{ij} \right\rangle$, $\left\langle D^{\mathrm{I}}_j \right\rangle$ and $\left\langle B^{\mathrm{I}}_j \right\rangle$ in (23), (25) and (27) are different, since they are calculated from the solutions of the problems I with different boundary conditions (16): (22), (24) and (26).

However, for magnetoelectric media it is possible to suggest the other ways of introduction of effective moduli, considering the problems with other mechanical, electric and magnetic boundary conditions from lemma 2. Just, we can consider the following problems:

—problem II with boundary conditions for stress vector $\mathbf{p}$, electric potential φ and magnetic potential ϕ

$$\mathbf{p} = \mathbf{n}\cdot\boldsymbol{\sigma}_0\,, \qquad \varphi = -\mathbf{x}\cdot\mathbf{E}_0\,, \quad \phi = -\mathbf{x}\cdot\mathbf{H}_0\,, \quad \mathbf{x}\in\Gamma\,, \tag{28}$$

—problem III with boundary conditions for displacement $\mathbf{u}$, surface density of electric charges q_e and magnetic potential ϕ

$$\mathbf{u} = \mathbf{x}\cdot\boldsymbol{\varepsilon}_0\,, \qquad q_e = -\mathbf{n}\cdot\mathbf{D}_0\,, \quad \phi = -\mathbf{x}\cdot\mathbf{H}_0\,, \quad \mathbf{x}\in\Gamma\,, \tag{29}$$

— problem IV with boundary conditions for stress vector $\mathbf{p}$, surface density of electric charges q_e and magnetic potential ϕ

$$\mathbf{p} = \mathbf{n}\cdot\boldsymbol{\sigma}_0\,, \qquad q_e = -\mathbf{n}\cdot\mathbf{D}_0\,, \quad \phi = -\mathbf{x}\cdot\mathbf{H}_0\,, \quad \mathbf{x}\in\Gamma\,, \tag{30}$$

— problems V – VIII with similar boundary conditions for problems I – IV, but with the change the boundary condition for magnetic potential ϕ ($\phi = -\mathbf{x}\cdot\mathbf{H}_0$, $\mathbf{x}\in\Gamma$) to the boundary condition for surface density of electric charges q_m ($q_m = -\mathbf{n}\cdot\mathbf{B}_0$, $\mathbf{x}\in\Gamma$).

In the all these problems the field equations (1) of equilibrium, electrostatic and magnetostatic are considered. For problem I the constitutive equations (2) – (4) in the $\varepsilon E H$ -form are used and originally the effective moduli $\tilde{\mathbf{c}}^{E,H\alpha}$, $\tilde{\mathbf{e}}^{H\alpha}$, $\tilde{\mathbf{q}}^{E\alpha}$, $\tilde{\boldsymbol{\varepsilon}}^{S,H\alpha}$, $\tilde{\boldsymbol{\alpha}}^{S\alpha}$ and $\tilde{\boldsymbol{\mu}}^{S,E\alpha}$ are defined, where α is the number of problem ($\alpha = \mathrm{I}, \mathrm{II}, ..., \mathrm{VIII}$). Respectively, for problem II the constitutive equations (13) – (15) in the $\sigma E H$ -form are used and moduli $\tilde{\mathbf{s}}^{T,H\,\mathrm{II}}$, $\tilde{\mathbf{d}}^{H\,\mathrm{II}}$, $\tilde{\mathbf{r}}^{E\,\mathrm{II}}$, $\tilde{\boldsymbol{\varepsilon}}^{T,H\,\mathrm{II}}$, $\tilde{\boldsymbol{\gamma}}^{T\,\mathrm{II}}$ and $\tilde{\boldsymbol{\mu}}^{T,E\,\mathrm{II}}$ are obtained; and analogously for problems III – VIII .

Note, that from lemma 3 for the all these problems, similar for the problem I, the average energy is conserved, i.e. the relation (21) is satisfies with replace superscript "I" by "II", "III" etc.

In any of these problems from obtained effective moduli we can find and another moduli from another constitutive equations for "equivalent" homogeneous medium. Eqs. (22) – (27) and similar Eqs. for another problems allow us to obtain a full set of effective moduli for magnetoelectric media with arbitrary anisotropy class. Use of different constitutive equations from eight types of problems can be useful for determination of effective moduli of the inhomogeneous structures making at work mainly one-dimensional or simple movements, for example, for rods, plates and disks, etc.

RESULTS AND CONCLUSIONS

In particular the theory of the effective moduli method for magnetoelectric composites with piezoelectric and piezomagnetic phases were developed. The basic propositions for the average field characteristics that generalize the approaches developed for the elastic and piezoelectric media were formulated. Eight static magnetoelectroelastic problems for a representative volume that allow finding the effective moduli of an inhomogeneous magnetoelectric composite were specified. These problems are different by the boundary conditions on the surfaces of the representative volume, which provide the constant gradients of field characteristics for the case of homogeneous media distribution: 1) mechanical displacements, electric and magnetic potentials linearly dependent on the coordinates; 2) mechanical displacements and magnetic potential linearly dependent on the coordinates and constant normal component of the vector of electric displacement; 3) constant stress vector and electric and magnetic potentials linearly dependent on the coordinates; 4) constant stress vector and constant normal component of the vector of electric displacement and magnetic potentials linearly dependent on the coordinates. In the four another problems the boundary conditions for magnetic potential are replaced by the conditions for constant normal component of the vector of magnetic flux density. Respective equations for calculation of the full set of the effective moduli for magnetoelectric media with arbitrary anisotropy were derived.

In (Domashenkina, 2011; Nasedkin, 2011) various models of representative volumes with different connectivities were considered, including the models for the structures of highly porous materials with account for inhomogeneities of the properties for individual finite elements. Based on these approaches and using finite element method the effective modules for various magnetoelectric composites having wide range of the electric or magnetic volume fractions were calculated.

The similar technics were applied for modeling of effective properties for porous elastic, thermoelastic and porous piezoelectric materials (Nasedkina, 2012). The finite element computations were made with the help of the finite element package ANSYS and special computer programs written in marcolanguage APDL ANSYS. Using the computation results the influence of different structures of the representative volumes and account for local inhomogeneities on the effective modules was analyzed (Nasedkin, 2005, 2011; Domashenkina, 2011). Also the comparison of the calculated main characteristics of the composites of complex structures with the known experimental data was made.

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