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FLUID, NANOFLUID AND MAGNETIC SLIP FLOW IN RECTANGULAR AND CIRCULAR MICROCHANNELS AND MINICHANNELS

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ABSTRACT

The presented paper shows the development of mathematical model for the calculation of transport properties on the basis of statistical nano-mechanics and further analytical calculation of fluid flow in mini and micro-channels.

Keywords: fluid flow, magnetic flow, nanofluid flow, thermophysical properties

INTRODUCTION

Fluid flow in channels, minichannels and microchannels is driven due to presence of electric field, magnetic field or pressure driven flow and some other effects [1].

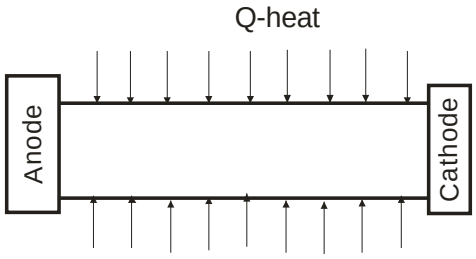
Electrohydrodynamics (EHD) known as electrokinetics, is the theory of the fluid dynamics of electrically charged fluids. It is the study of the motions of ionised particles or molecules and their interactions with electric fields and the surrounding fluid. For EHD flow we need low electrical conducting fluids such as organic fluids or alcohols [1]. Electrokinetic flow could be classified into the next different types: electrophoresis, electroosmosis, streaming potential and sedimental potential [1-5].

The fundamental concept for magnetohydrodynamics (MHD) is that magnetic fields can induce currents in a moving conductive fluid, which in turn creates forces on the fluid and also changes the magnetic field itself [2]. For MHD flows we need highly conductive fluids like plasmas, liquid metals, electrolytes and salt water.

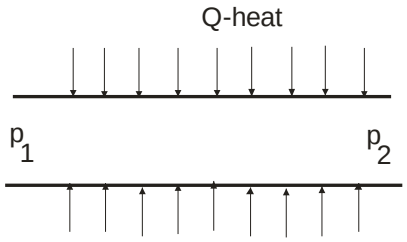
In the case if flow is transported by pressure differential we could call such phenomena like pressure hydrodynamics (PHD).

Ferrohydrodynamics (FHD) is the theory of magnetic fluid flow.

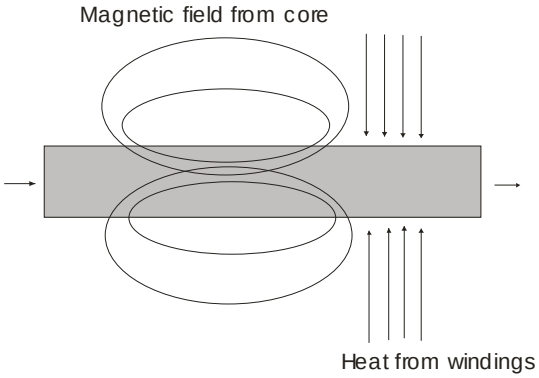
The term nanofluid is envisioned to describe a solid-liquid mixture which consists of a nanoparticles and a base liquid and this is one of new challenges for thermo-sciences provided by the nano-technology. The possible application area of nanofluids is in advanced cooling systems, in micro/nano elecctromechanical systems… The investigation of the effective thermal conductivity of liquid with nanoparticles attract much more interest experimentally and theoretically. The effective thermal conductivity of nanoparticle suspension can be much higher than for the fluid without nanoparticles.



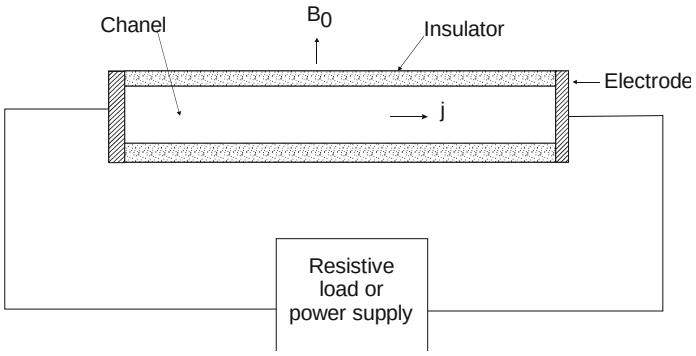
a) Electrokinetic flow



b) Pressure driven flow



c) Ferrofluid flow



d) MHD flow

Figure 1: Types of fluid flow

Calculation of properties for nanofluids for real substances is possible by the classical and statistical mechanics. Classical mechanics has no insight into the microstructure of the substance. Statistical mechanics, on the other hand, calculates the properties of state on the basis of molecular motions in a space, and on the basis of the intermolecular interactions. The equations obtained by means of classical thermodynamics are empirical and apply only in the region under observation. The main drawback of classical thermodynamics is that it lacks the

insight into the substance of microstructure. Contrary to classical mechanics, statistical mechanics calculates the thermomechanic properties of state on the basis of intermolecular and intramolecular interactions between particles in the same system of molecules. It deals with the systems composed of a very large number of particles.
In this paper are determined new constants for fluids. The results of the analysis are compared with experimental data and show a relatively good agreement.

THE CALCULATION OF THERMAOPHYSICAL PROPERTIES FOR SOLIDS AND FLUIDS

For the calculation of transport properties for polyatomic molecules in principle, a quantum mechanical treatment of processes is necessary to account for the changes of internal state. In the presented paper will be presented Chung-Lee-Starling model (CLS) [3-4]. Equations for the viscosity and the thermal conductivity are developed based on kinetic gas theories and correlated with the experimental data. The low-pressure transport properties are extended to fluids at high densities by introducing empirically correlated, density dependent functions. These correlations use acentric factor ω, dimensionless dipole moment μ_r and an empirically determined association parameters to characterize molecular structure effect of polyatomic molecules κ, the polar effect and the hydrogen bonding effect. In this paper are determined new constants for fluids.
The dilute gas viscosity for CLS model is written as:

$$\eta_0(T) = 26.69579 \cdot 10^{-1} \frac{\sqrt{MT}}{\Omega^{(2,2)*}\sigma^2} F_c \tag{1}$$

where η is in Pa s, M is the molecular mass in gmol^{-1}, T is in K, $\Omega^{(2,2)}$ is a collision integral and σ is the Lennard-Jones parameter. To make computerized calculations more convenient and to improve on the accuracy obtainable by linear interpolation of the tables we used Neufeld[2] at al. empirical formulation, obtained on the basis of numerical simulations and interpolation procedure.

$$\Omega^{(l,s)^*} = \frac{A}{T^{*B}} + \frac{C}{\exp(DT^*)} + \frac{E}{\exp(FT^*)} + \frac{G}{\exp(HT^*)} + RT^{*B}\sin(ST^{*W} - P) \tag{2}$$

The factor F_c has been empirically found to be:[14]

$$F_c = 1 - 0.2756\omega + 0.059035\mu_r{}^4 + \kappa \tag{3}$$

where ω is the acentric factor, μ_r relative dipole moment and κ is a correction factor for hydrogen-bonding effect of associating substances such as alcohols, ethers, acids and water. For dense fluids Eq. (1) is extended to account for the effects of temperature and pressure by developing an empirically correlated function of density and temperature as shown below:

$$\eta = \eta_k + \eta_p \tag{4}$$

$$\eta_k = \eta_0\left(\frac{1}{G_2} + A_6 Y\right) \tag{5}$$

$$\eta_p = \left[36.344 \cdot 10^{-6} - (MT_c)^{.5} / V_C{}^{2/3}\right] A_7 Y^2 G_2 \exp(A_8 + A_9/T^* + A_{10}/T^{*2}) \tag{6}$$

$$Y = \rho V_c / 6, \quad G_1 = \frac{1.0 - 0.5Y}{(1.0 - Y)^3} \tag{7}$$

$$T_c = \frac{1.2593\varepsilon}{k}, \; V_c = (0.809\sigma(\dot{A}))^3 \tag{8}$$

$$G_2 = \frac{\{A_1(1 - \exp(-A_4 Y)) + A_2 G_1 \exp(A_5 Y) + A_3 G_1\}}{(A_1 A_4 + A_2 + A_3)} \tag{9}$$

The constants A_1-A_{10} are linear functions of acentric factor, reduced dipole moment and the association factor

$$A_i = a_0(i) + a_1(i)\omega + a_2(i)\mu_r{}^4 + a_3(i)\kappa \text{ ,i=1,10} \quad (10)$$

where the coefficients a_0, a_1, a_2 and a_3 are presented in the work of Chung at al. [3,4]

THE CALCULATION OF EFFECTIVE VISCOSITY FOR NANOFLUIDS

In nanoparticle fluid mixtures, other effects such as microscopic motion of particles, particle structures and surface properties may cause additional heat transfer in nanofluids. Nanofluids also exhibit superior heat transfer characteristics to conventional heat transfer fluids. One of the main reasons is that suspended particles remarkably increase thermal conductivity of nanofluids. The thermal conductivity of nanofluid is strongly dependent on the nano-particle volume fraction. So far it has been an unsolved problem to develop a sophisticated theory to predict thermal conductivity of nanofluids. The presented paper is the attempt to calculate thermal conductivity of nanofluid analytically. Hamilton and Crosser developed the model for the effective thermal conductivity of two-component mixtures as a function of the conductivity of the pure materials, the composition and shape of dispersed particles. The thermal conductivity can be calculated then with the next expression:

$$\lambda = \lambda_0 \left\{ \frac{\lambda_p + (n-1)\lambda_0 - (n-1)\alpha(\lambda_0 - \lambda_p)}{\lambda_p + (n-1)\lambda_0 + \alpha(\lambda_0 - \lambda_p)} \right\} \quad (11)$$

where λ is the mixture thermal conductivity, λ_0 is the liquid thermal conductivity, λ_p is the liquid thermal conductivity of solid particles, α is the volume fraction and n is the empirical shape factor given by,

$$n = \frac{3}{\Psi} \quad (12)$$

where ψ is sphericity, defined as the ratio of the surface area of a sphere (with a volume equal to that of a particle) to the area of the particle.. The volume fraction Φ of the particles is defined as:

$$\phi = \frac{V_p}{V_0 + V_p} = n\frac{\pi}{6} d_p{}^3 \quad (13)$$

where n is the number of the particles per unit volume and d_p is the average diameter of particles. An alternative expression for calculating the effective thermal conductivity of solid-liquid mixtures was introduced by Wasp [6]:

$$\lambda = \lambda_0 \left\{ \frac{\lambda_p + 2\lambda_0 - 2\phi(\lambda_0 - \lambda_p)}{\lambda_p + 2\lambda_0 + \phi(\lambda_0 - \lambda_p)} \right\} \quad (14)$$

Comparison between Eq. (38) and Eq. (41) shows that Wasp model is a special case with the sphericity of 1.0 of the Hamilton and Crosser model.

In nanoparticle fluid mixtures, other effects such as microscopic motion of particles, particle structures and surface properties may cause additional heat transfer in nanofluids. Nanofluids also exhibit superior heat transfer characteristics to conventional heat transfer fluids. One of the main reasons is that suspended particles remarkably increase thermal conductivity of nanofluids. The viscosity of nanofluid is strongly dependent on the nano-particle volume fraction. So far it has been an unsolved problem to develop a sophisticated theory to predict thermal conductivity of nanofluids. The presented paper is the attempt to calculate thermal conductivity of nanofluid analytically. Hamilton and Crosser developed the model for the effective thermal conductivity of two-component mixtures as a function of the conductivity of

the pure materials, the composition and shape of dispersed particles. It is well known that the earliest theoretical work on the effective viscosity was due to Einstein whose derivation led to the effective viscosity to be linearly related to the particle concentration:

$$\mu_r = 1 + 2.5\phi \tag{15}$$

where μ_r is the relative viscosity defined as the ratio of the effective viscosity of the particle fluid-mixture to the viscosity of the fluid and ϕ is the volumetric concentration of the particles. Equation (15) is applicable to suspensions with low particle concentrations (less then 2%). With help of exponential model we can obtain the next expression for the relative viscosity:

$$\mu_r = 1 + 2.5\phi + \left(\frac{35}{8} + \frac{5}{4}\beta\right)\phi^2 + \left(\frac{105}{16} + \frac{35}{8}\beta + \frac{5}{12}\beta^2\right)\phi^3 + \left(\frac{1155}{128} + \frac{935}{96}\beta + \frac{235}{96}\beta^2 + \frac{5}{48}\beta^3\right) + \ldots\ldots\ldots \tag{16}$$

where β is called the exponent. If we choose β=2, we obtain the result is very close to the result obtained by Ward (W), who suggested the following expression for spherical particles:

$$\mu_r = 1 + (2.5\alpha) + (2.5\alpha)^2 + (2.5\alpha)^3 + (2.5\alpha)^4 \ldots\ldots\ldots \tag{17}$$

The presented equation is fitted with the experimental data for the concentration up to 35%.

The viscosity of nanofluid is strongly dependent on the nanoparticle volume fraction. So far it has been an unsolved problem to develop a sophisticated theory to predict viscosity of nanofluids. The presented paper is the attempt how to calculate thermal conductivity of nanofluid analytically. Cheng and Law [6] developed the model for the effective thermal conductivity of two-component mixtures as a function of the viscosity of the pure fluid and the composition of particles and and exponent factor.

The Cheng and Law or Ward models give very good results for two-phase flow with particles larger than 100 nm. For smaller particles the presented theory give wrong results with the deviation more than 100% in comparison with experimental results. The presented theoretical models for the calculation of the viscosity for nanofluids are only dependent on the viscosity of the liquid and their relative volume fraction, but not on particle size and the interaction between particles and the fluid.

In convection heat transfer in nanofluids not only on the thermal conductivity but also on the other properties such as specific heat, dynamic viscosity, … are important for analytical prediction. We can mention the factors discussed in the literature [1-10] as possible mechanisms for the anomalous enhancement of viscosity: the motion of nanoparticle, molecular level layering of the liquid at the liquid-particle interface and ballistic phenomena in nanoparticles, the effects of clustering in nanoparticles.

As in the case of analytical calculation of thermal conductivity, for the calculation of viscosity of nanofluids we have made the hypothesis that the most important additional contribution is liquid layering. With help of Eq. (9) we can express the renewed Ward model (RW):

$$\mu_r = 1 + (2.5\alpha_e) + (2.5\alpha_e)^2 + (2.5\alpha_e)^3 + (2.5\alpha_e)^4 \tag{18}$$

For the further development of mathematical model I have used Eqs. (6) and (10) for the calculation of thermal conductivity and viscosity. In the presented model we have studied nanofluid with Al_2O_3 nanoparticles and ethylene glycol as the reference fluid. The described nanofluid is very perspective for refrigeration application.

ONE DIMENSIONAL FLUID FLOW DUE TO PRESSURE AND MAGNETIC EFFECTS

Consider electromagnetic flow in rectangular and circular microchannels (Figure 1). The charged surface of a microchannel wall may attract ions of the opposite charge in the surrounding fluid. The general form of the momentum equation for electrohydrodynamic flow is:

$$\rho\frac{\partial\vec{v}}{\partial t}+\rho\vec{v}\nabla\vec{v}=-\nabla p+\nabla\left(\mu\nabla\vec{v}\right)+\vec{i}\times\vec{B}, \tag{19}$$

where the last term presents the electromagnetic force and i and B refer to the current density and magnetic field strength, respectively. For steady-state flow in a microchannel at small Reynolds numbers, the transient and inertia terms can be neglected, so Eq. (11) is simplified in the next equation:

$$0=-\nabla p+\nabla\left(\mu\nabla\vec{v}\right)+\vec{i}\times\vec{B}, \tag{20}$$

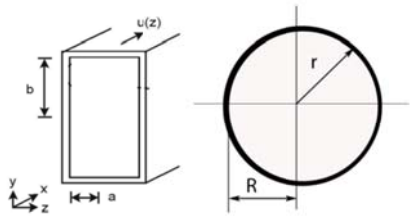


Figure 2: Rectangular and circular microchannels

During elektrokinetic flow in microchannels, the charged surface of a microchannel wall may attract ions of the opposite charge in the surrounding fluid. The resulting spatial gradient of ions near the wall leads to an Electric Double Layer (EDL). The EDL contains an immobile inner layer and mobile outer layer, which can be appreciably affected by an externally applied electric field. Assuming the fluid velocity, magnetic field and current density are orthogonal, the reduced momentum equation becomes:

$$0=-\frac{dp}{dx}+\mu\frac{d^2u}{dz^2}+i_yB_z \tag{21}$$

The terms represent pressure, viscous and electromagnetic forces in the liquid. Using Ohm's Law to express the current density in terms of fluid velocity:

$$\mu\frac{d^2u}{dz^2}+\sigma_eB_z^2u=\frac{dp}{dx}, \tag{22}$$

where σ_e and B_z refer to the electrical conductivity and magnetic field strength. For fully developed flow in a microchannnel, the pressure gradient becomes constant and independent of the magnetic field strength. In terms of the Hartman number, M_H ($M_H=aB_z\sqrt{\sigma_e/\eta}$),

$$\mu\frac{d^2u}{dz^2}-\left(\frac{M_H{}^2\eta}{a^2}\right)u=\frac{dp}{dx} \tag{23}$$

Applying the no-slip boundary conditions at z=0 and z=-2a, the analytical solution of Eq. (5) becomes:

$$u=-\frac{a^2\left(\frac{dp}{dx}\right)}{M_H{}^2\mu}+\frac{a^2\left(\frac{dp}{dx}\right)}{\left(1+2e^{2M_H}\right)M_H{}^2\mu}e^{\frac{z\cdot M_H}{a}}+\frac{a^2\left(\frac{dp}{dx}\right)e^{2M_H}}{\left(1+2e^{2M_H}\right)M_H{}^2\mu}e^{\frac{-z\cdot M_H}{a}} \tag{24}$$

The mean velocity within the microchannel becomes:

$$u_b = \frac{1}{2a}\int_0^{2a} u(z)dz \qquad u_b = \frac{a^2\left(\frac{dp}{dx}\right)(-M_H + Tanh(M_H))}{{M_H}^3 \mu} \tag{25}$$

Non-dimensionlizing this result (z^*=z/a, u^*=u/u_b) we obtain the next equation:

$$u^* = \frac{M_H\left(-1+\frac{1}{\left(1+2e^{2M_H}\right)}e^{z^*\cdot M_H}+\frac{e^{2M_H}}{\left(1+2e^{2M_H}\right)}e^{-z^*\cdot M_H}\right)}{(-M_H+Tanh(M_H))} \tag{26}$$

Without electromagnetic effects, equations (24-26) transforms into the following expressions:

$$u(z) = \frac{\left(\frac{dp}{dx}\right)}{2\mu}\left(-2az+z^2\right) \qquad u_b = \frac{1}{2a}\int_{-a}^{a} u(z)dz = \frac{a^2\left(\frac{dp}{dx}\right)}{3\mu} \qquad u^* = \frac{3}{2}\left(2-z^*\right)z^* \tag{27}$$

For the circular micro channel without electromagnetic forces, the governing equation is

$$\frac{dp}{dx} = \mu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right) \tag{28}$$

Solving the differential equation subject to boundary conditions,

$$u = \frac{\left(\frac{dp}{dx}\right)}{4\mu}(r^2-R^2), \quad u_b = \frac{1}{\pi R^2}\int_0^R u2\pi\, r\cdot dr = -\frac{\left(\frac{dp}{dx}\right)R^2}{8\mu} \quad u^* = 2-2r^{*2} \tag{29}$$

If we wish to calculate the velocity profile for MHD flow in circular channel we have to solve the next differential equation:

$$\frac{dp}{dx} = \mu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right) - \mu\frac{{M_H}^2}{R^2}u \tag{30}$$

The analytical solution of equation (12) is slightly more complicated. We have obtained the next solution of differential equation with the boundary conditions (u(R)=0, u′(0)=0):

$$u[r] = \frac{R^2\left(\frac{dp}{dx}\right)(-\text{BesselI}[0,M]+\text{BesselI}[0,\frac{Mr}{R}])}{M^2\mu\,\text{BesselI}[0,M]} \tag{31}$$

$$\mathrm{u_b} = \frac{1}{\pi R^2}\int_0^R u2\pi\, r\cdot dr = \frac{R^2\left(\frac{dp}{dx}\right)\text{BesselI}[2,M]}{M^2\mu\,\text{BesselI}[0,M]} \tag{32}$$

Previous investigations of the pressure gradient for electro-osmotic liquid flow in microchannels have generally used no-slip conditions, while for gas flow in microchannels, the slip boundary condition is taken into account. Some previous experimental investigations have demonstrated the existence of liquid slip on a microchannel wall [18,19]. In addition some previous studies were numerically performed considering a slip velocity for a liquid flow in a microchannel made from hydrophobic surfaces, taking into accountelectric field and pressure gradient [8,10]. Also, some previous studies predicted numerically a slip velocity for liquid flow in a microchannel made from hydrophobic surfaces, taking into account an imposed electric field and a pressure gradient without heat transfer. On the basis of a slip velocity profile and boundary conditions in liquid microchannel flow, the velocity profile can be determined by

$$u(0) = u_{s1}, u(2a) = u_{s2} \tag{33}$$

$$u_{s1} = \beta\left.\frac{\partial u}{\partial z}\right|_{z=0}, \; u_{s2} = \beta\left.\frac{\partial u}{\partial z}\right|_{z=2a} \tag{34}$$

The coefficient β is called the slip coefficient. If we define a reduced slip coefficient as $\beta^* = \beta/a$, we obtain the following equations:

1) Rectangular channel with electromagnetic forces;

$$u^* = \frac{M_H(cosh(M_H) - cosh(M_H - M_H z^*)) - M_H \beta^* sinh(M_H)}{M_H cosh(M_H) + (-1 + M_H \beta^*) sinh(M_H)} \quad (35)$$

2) Rectangular channel without electromagnetic forces;

$$u^* = \frac{6z^* \ -3z^{*2} + 6\beta^*}{2 + 6\beta^*} \quad (36)$$

3) Circular channel without electromagnetic forces;

$$\beta^* = \frac{\beta}{R}, r^* = r/R \qquad u^* = 1 + \frac{-2r^{*2} + 1}{1 + 4\beta^*} \quad (37)$$

4) Circular channel with electromagnetic forces

$$\mathrm{u}^* = \frac{\mathrm{BesselI}[0, M] - \mathrm{BesselI}[0, \frac{Mr}{R}]}{\mathrm{BesselI}[2, M]} \quad (38)$$

5) Circular channel with electromagnetic forces and slip

$$\mathrm{u}^* = \frac{\mathrm{M}_H(\mathrm{BesselI}[0, M_H] - \mathrm{BesselI}[0, \mathrm{M}_H \mathrm{r}^*] + M_H \beta^* BesselI\,[1, M_H]}{\mathrm{M}_H \mathrm{BesselI}[0, M_H] + (-2 + {M_H}^2 \beta^*) BesselI[1, M_H]} \quad (39)$$

RESULTS AND DISCUSSIOM

In the presented paper we will show analytical results for the aluminum oxide nanoparticles and ethylene glycol as reference fluid. The aluminum nanoparticles dispersed in the fluid are very interesting for nanofluid industrial application due to very high thermal conductivity in comparison with copper or aluminum oxides. In our case we have used experimental results from the literature where copper average nanoparticles diameter are smaller than 35 nm.

The presented article concentrated the analysis for velocity distribution inside of microchannel. Figures 3 and 4 show the analytical results for viscosity and thermal conductivity for Al_2O3 in dependence of temperature. The analytical model to calculate transport properties is not completely developed in the presented article and the details is possibly to find in literature [10].

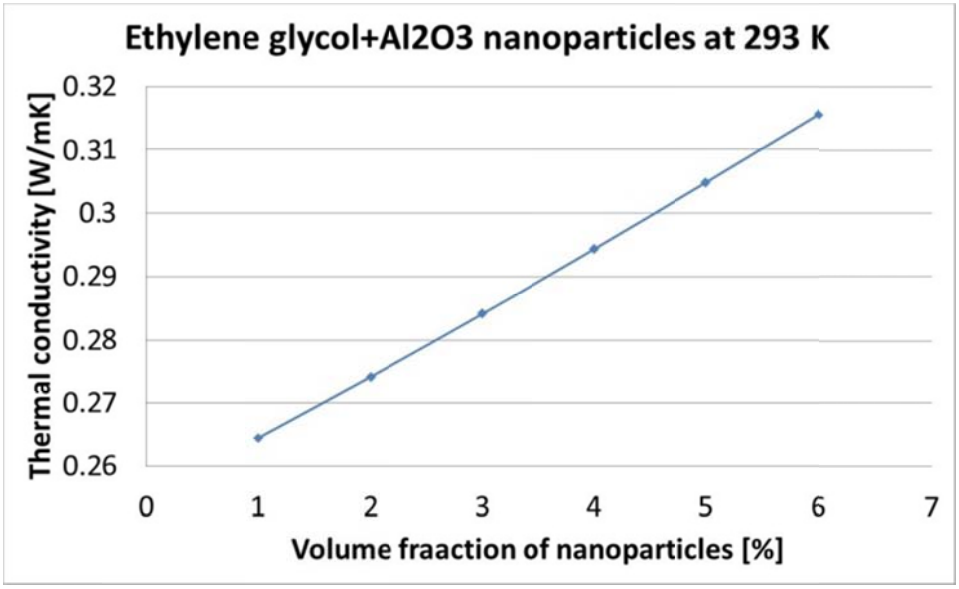


Fig. 3: Thermal conductivity of mixture between ethylene glycol and Al_2O_3 nanoparticles

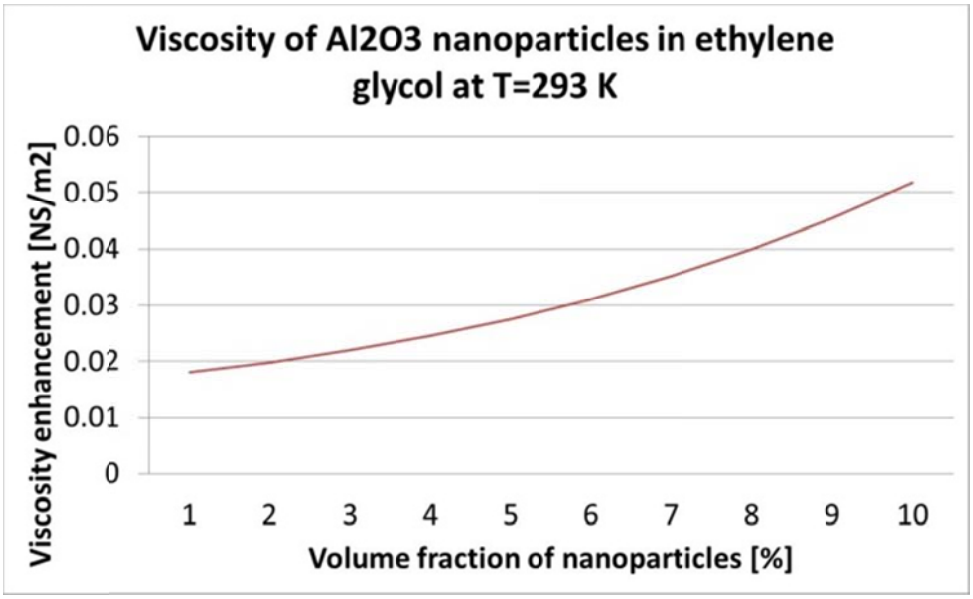


Fig. 4. Thermal conductivity of mixture between ethylene glycol and Al_2O_3 nanoparticles

Figures 5-9 show the analytical results for velocity and temperature profile in fluids in dependance of slip conditions, type of microchannel and type of fluid flow. In the presented article is for the first time in scientific literature presented the slip magnetic flow in circular channel on the basis of equation (39).

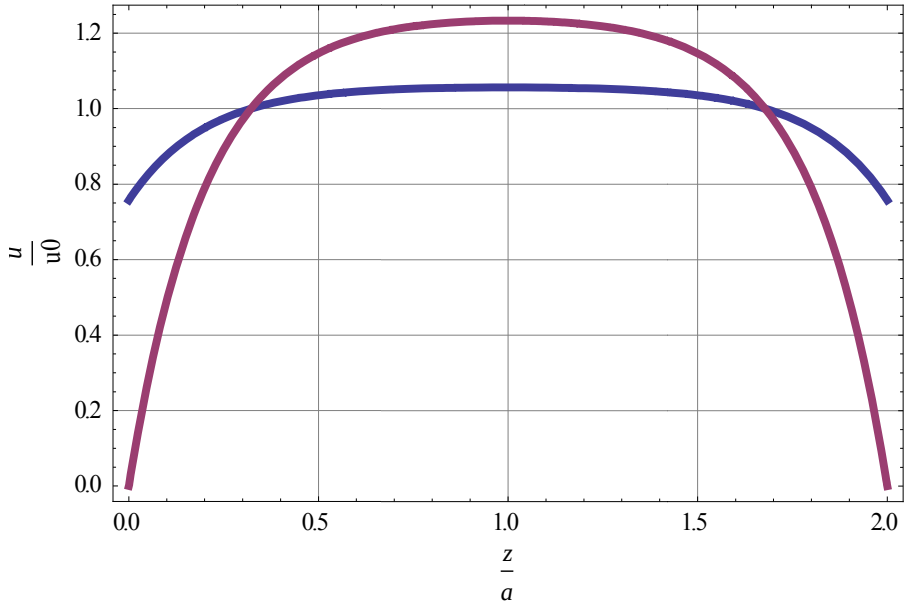


Fig. 5: Velocity profile (a) without slip (red line) and with slip β^*=0.5 (blue line) at M_H=5 in rectangular minichannel.

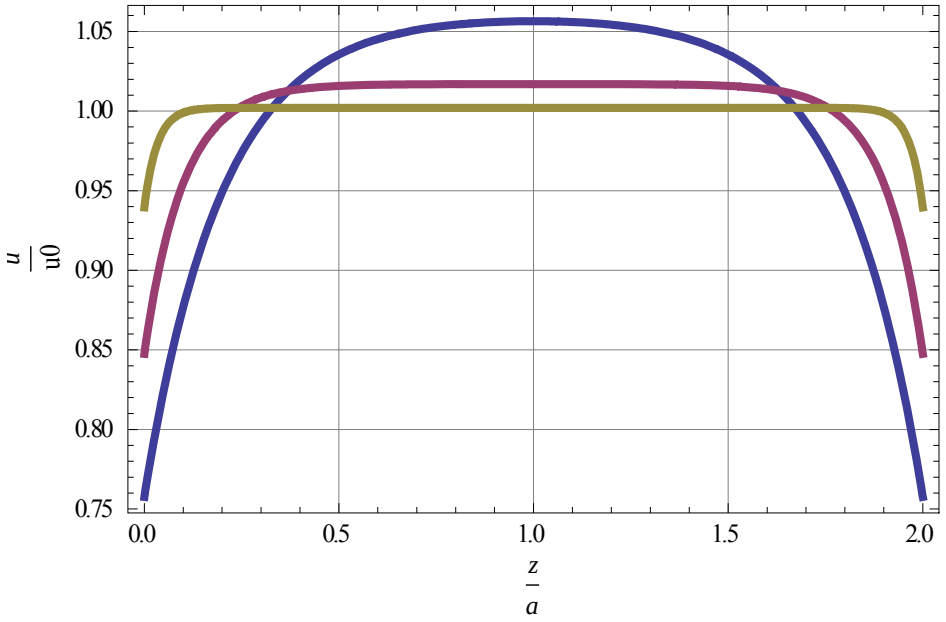


Fig. 6: Velocity profile with slip (β^*=0.5) at M_H=5 (blue line), M_H=10 (red line) and M_H=30 (green line)in rectangular minichannel with slip β*=0.5.

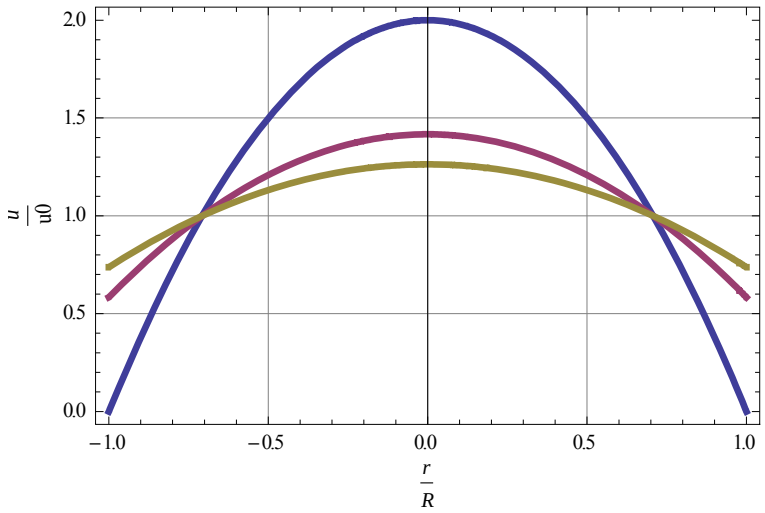


Fig. 7: Velocity profile in circular microchannel without electromagnetic effects (blue line), with slip with slip β^*=0.35 (red line) and with slip β^*=0.7 (green line).

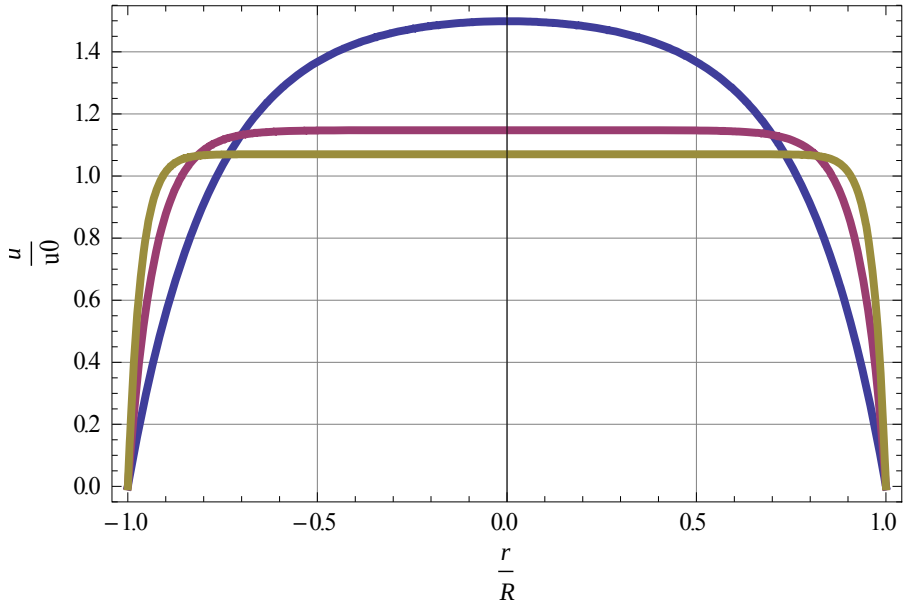


Fig. 8: Velocity profile with slip (β^*=0.5) at M_H=5 (blue line), M_H=10 (red line) and M_H=30 (green line)in circular microchannel minichannel without slip.

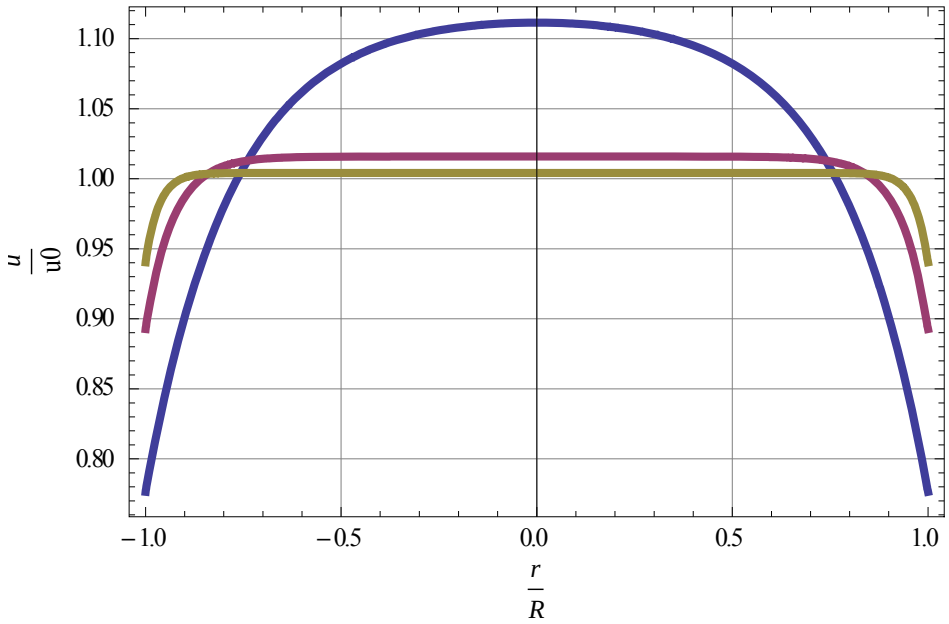


Fig. 9: Velocity profile with slip β^*=0.5 at M_H=5 (blue line), M_H=10 (red line) and M_H=30 (green line)in circular microchannel.

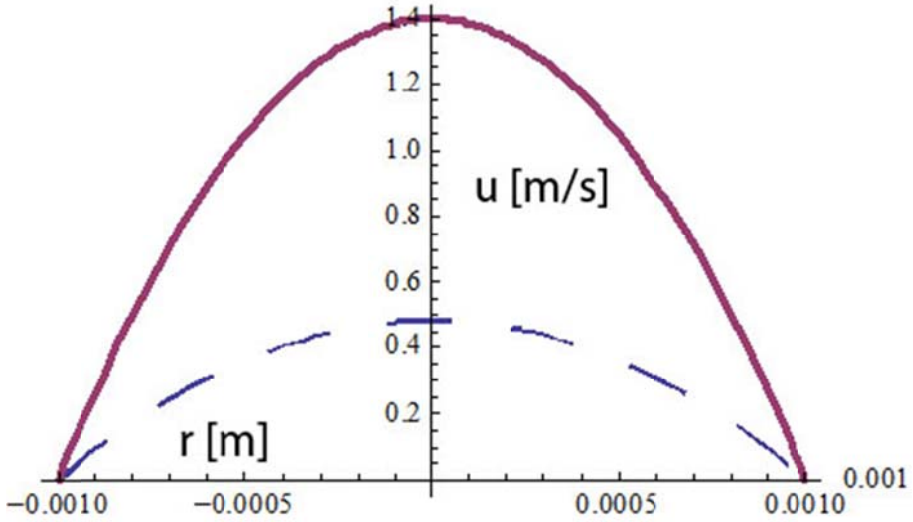


Fig.10: Velocity profile for ethylene glycol for 1% of Al2O3 (dashed line) and 10 % of Al_2O_3 nanoparticles (solid line).

CONCLUSION

The presented article shows the velocity profiles in MHD and PHD flows. The mathematical model was developed for slip flow in circular and rectangular minichannels and microchannels.

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