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DEFLECTION DUCTILITY INDEX IN CONTINUOUS REINFORCED CONCRETE BEAMS AND SLABS

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ABSTRACT

The behaviour of continuous reinforced concrete elements subjected to bending is characterized by a non-linear response whose extent depends on the available rotation capacity. In this paper the rotation capacity is reviewed and a numerical study for calculating the deflection ductility index in continuous reinforced concrete beams or one way spanning slabs is presented. The results indicate comfortable values between 1.0 and 2.0 for current beams used on building construction, while for slabs with high redistribution the limit values of slenderness and mechanical reinforcement ratios require some attention because slabs may not reach its total loading capacity.

Keywords: ductility, rotation capacity, deflection, Eurocode 2.

1. INTRODUCTION

The ductility of reinforced concrete elements is an essential property to accommodate the change occurred on deformation field during its service lifetime. This change can be caused by external loads and displacements, or by modification of the properties of materials. The ductility can be evaluated at different levels: materials, cross sections and elements, being the respective number of influential parameters successively growing.

Drift limits are generally used for reinforced concrete columns. Many results can be achieved in the literature (Erduran & Yakut, 2004; Lam et al., 2003; Lu, Gu, & Guan, 2005; Matamoros & Sozen, 2003). This visual measure is usually presented as a function of damage limit, slenderness, class and quantity of reinforcement and axial load level.

Based on numerical investigations, Erduran and Yakut (2004) developed damage curves for reinforced concrete column members as a function of the drift ratio. These drift limits can be used in the evaluation and vulnerability assessment of reinforced concrete frame building. They pointed out the most significant parameters such as axial load level, amount of transverse reinforcement, slenderness of the column, and the yield strength of the longitudinal reinforcement.

Matamoros and Sozen (2003) carried out a series of experimental tests to investigate the behaviour of columns with high-strength concrete. The tests demonstrated that an increase in the concrete strength led to an increase in the limit drift, but the increase on the axial load had an adverse effect on this limit drift.

Lam et al. (2003) tested various square-shaped reinforced concrete column specimens with low lateral confinement and high axial load. An empirical equation to describe the ultimate drift ratio was proposed. They also concluded that performance of columns may not be accurately reflected by the section ductility factor, but a better representation could be achieved by using the ultimate drift ratio.

Lu et al. (2005) evaluated the probabilistic drift limits of reinforced concrete columns for distinctive performance levels by means of statistical simulation. In general, the drift limits for reinforced concrete columns follow the normal distribution.

The study of ductility on beams is normally restricted to the calculation of the rotation capacity in the plastic hinges in order to estimate the allowable degree of moment redistribution. Numerous authors have been presented their results (Bernardo & Lopes, 2004; Carmo, 2004; CEB, 1993, 1998; Erduran & Yakut, 2007; Lopes & Carmo, 2006; Shin, Kamara, & Ghosh, 1990; Sin, Huan, Islam, & Mansur, 2011; Woods, Kiousis, Ehsani, Saadatmanesh, & Fritz, 2007) for studying the available rotation capacity, the required rotation capacity and the moment redistribution.

CEB Bulletins 218 (1993) and 242 (1998) summarize the research results on the non linear behaviour of reinforced concrete beams in previous decades. They include the results of several experimental tests that were performed to evaluate the available rotation capacity. The influence of geometry, materials and static system was identified and quantified. Values for redistribution factor were obtained from numerical results and experimental tests.

Moreover, new materials were tested. Shin et al. (1990) tested some simply supported beams with reinforced high strength concrete under monotonic and reversed cyclic loading. The relation between deflection ductility index and the reinforcement ratio was clearly observed.

Bernardo and Lopes (2004) shown that when the relative depth of compression zone in failure rises, the deflection ductility index decreases. Good correlation between both is attained if theoretical relative depth of compression zone value is used instead of experimental value.

Woods et al. (2007) studied the effects of the spacing and volumetric ratio of transverse reinforcement on the deflection ductility. The influence of these effects was clearly observed.

Sin et al. (2011) studied the flexural response of simply supported beams with normal strength concrete and reinforced lightweight aggregate concrete. Curvature ductility index was evaluated as a function of reinforcement ratio, compression reinforcement ratio and transverse reinforcement ratio.

Carmo (2004) performed a large number of experimental tests on continuous beams with reinforced high strength concrete. The available rotation capacity obtained from these experimental tests was lower than the values predicted by Model Code 90 or Eurocode 2. To explain these differences, a deformable strut and tie model was proposed for the calculation of the available rotation capacity (Lopes & Carmo, 2006).

Erduran and Yakut (2007) carried out a finite element analysis on reinforced concrete frames to establish the displacement-based damage functions. This parameter depends on drift ratio (columns) or rotation capacity (beams).

The deflection ductility index on beams (or drift limits on columns) can be a useful tool for designers because it represents how much a beam can deform after maximum load or, what is the potential of a beam for deflecting under a pre-defined load. For this reason it could be a good tool for the structural pre-design. At the same time, this is the unique global ductility parameter that includes all beam parameters.

As mentioned above, several authors reported results for deflection ductility index on beams, however their results were only correlated with sectional properties or applied on simply supported beams.

Here, besides the relationship between deflection ductility index in continuous beams with a sectional parameter (ω_t), we want also to establish the relationship with a geometry parameter (l/d) and a design parameter (δ). Therefore, this paper presents the results of a numerical study for evaluating the deflection ductility index in continuous reinforced concrete beams or one way spanning slabs as a function of: sectional, geometry, and design parameters, for two types of concrete.

The available rotation capacity defined by codes can be understood as the resume of all performed experimental tests, with a suitable margin of safety. Then, it can be applied to estimate the deflection ductility index with good agreement with experimental tests. Thus, in this numerical study, it was applied with the Non-Linear Finite Element Method.

2. BEHAVIOUR OF CONTINUOUS REINFORCED CONCRETE BEAMS

The ductility of reinforced concrete elements is an essential property to accommodate the change occurred on deformation field during its service lifetime. This change can be caused by external loads and displacements, or by modification of the properties of materials. The ductility can be evaluated at different levels: materials, cross sections and elements, being the respective number of influential parameters successively growing.

Reinforced concrete beams show a marked non-linear behaviour with the increase of actions. Fig. 1 presents a part of a continuous reinforced concrete beam with rectangular cross section submitted to a uniformly distributed load up to rupture. In case of beams, where the degree of moment redistribution is limited, the deflection response of beam under loading can be explained approximately by the graph illustrated in Fig. 2 with four different lines delimitated by the origin and four characteristic zones.

For low actions all beam sections remain uncracked with the full stiffness. Cracking at the continuous supports takes place on Zone 1, followed by cracking at the span. The load continues to increase considerably until reinforced bars at the supports reach the yielding (Zone 2). At the supports, plastic hinges are formed and they start to rotate under more loading. On Zone 3 new yielding occur on reinforced bars at the mid-span and a new plastic hinge appears at this section. With three plastic hinges a collapse mechanism is formed, it means that load capacity of beam was reached. From this zone small load increments cause large deflections. When the available rotation capacity of plastic hinges located at the supports is attained (Zone 4) the beam collapses and the load capacity is lost.

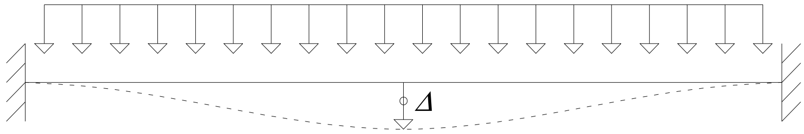

Fig. 1: Continuous beam under uniformly distributed load and deflection.

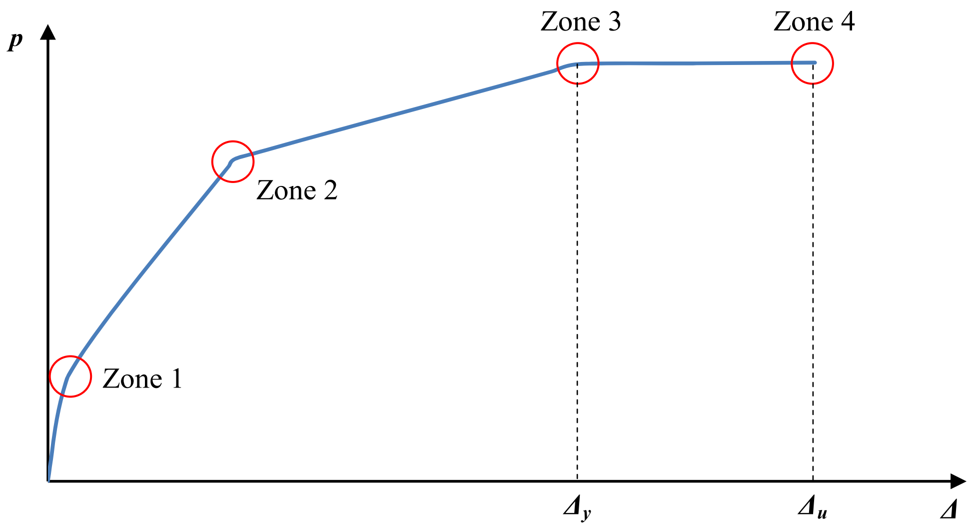

Fig. 2: Typical mid-span beam deflection.

The first two lines are justified by the linear behaviour of concrete (before and after cracking) and of reinforcing bar before yielding. After first yielding (Zone 2) the behaviour is essentially defined by other two lines assuming that reinforcing bars at the supports and mid-span are hot rolled steel and therefore show a large elongation at the beginning of hardening.

Creep is a concrete time-dependent property and its effect should be generally taken into account for the verification of serviceability limit states (EN 1992-1-1, 2004). In this paper it was considered that the creep does not influence considerably the values of yield deflection Δ_y and ultimate deflection Δ_u because the ultimate limit states occur for short periods of time and far from the serviceability limit states, consequently without significant creep effects. On the other hand, the ratio between permanent and variable loads (creep weight) are changeable for each structure, consequently it is not appropriate to take into account in this study.

2.1 Definition of Deflection Ductility Index

The deflection ductility index μ_Δ is defined as the ratio between ultimate deflection Δ_u and yield deflection Δ_y of beams (see Eq. (1)). Others ductility indexes could be used to measure the ductility of reinforced concrete beams. In deflection category, the ratio between ultimate deflection (Zone 4) and first yield (Zone 2) is commonly used. In rotation category, the ratios between available rotation capacity and yield rotation capacity at supports or mid-span are also common. Lee and Pan (2003) presented an algorithm and simplified formulas for estimating the relationship between the tension steel ratio and the rotation ductility in reinforced concrete beams. In energy category (area under deflection curve on Fig. 2), the ratio between ultimate energy and the energy up to first yield on supports (Zone 2) is used in columns. In load category, the redistribution factor δ (given in Eq. (2)) or the degree of moment redistribution η (given in Eq. (3)) are the most known parameters, specially for the designers, but other parameters can be applied such as the plastic adaptation ratios developed by Tichý and Rákosník (1977), Arenas (1986) or Moucessian (1986).

$$\mu_\Delta = \frac{\Delta_u}{\Delta_y} \tag{1}$$

$$\delta = \frac{M_{sup,red}}{M_{sup,el}} \tag{2}$$

$$\eta=(1-\delta) \tag{3}$$

where $M_{sup,red}$ and $M_{sup,el}$ refers to the support moment after redistribution and to the support moment calculated according to the theory of elasticity, respectively.

2.2 Definition of Rotation Capacity

As mentioned before, after the mechanism being formed, the deformation of beams stops when one plastic hinge reaches its available rotation capacity preventing the mechanism to rotate moreover.

The total rotation θ is equal to the sum of curvature along the member length at failure, while the elastic rotation θ_{el} is defined at the onset of yielding of reinforcement (CEB, 1998). The rotation capacity is understood as the maximum plastic rotation θ_{pl} of two sections and it is calculated as the difference between the total rotation at failure θ and the elastic rotation at the onset of yielding of reinforcement θ_{el}. Fig. 3 is shows the definition of plastic rotation according to the CEB (CEB, 1998).

The available rotation capacity is the maximum rotation capacity achieved by the materials (concrete and reinforcing steel) on the member length at failure, while the required rotation capacity is the minimum rotation capacity needed on the member length at failure by the element or structure to form a mechanism.

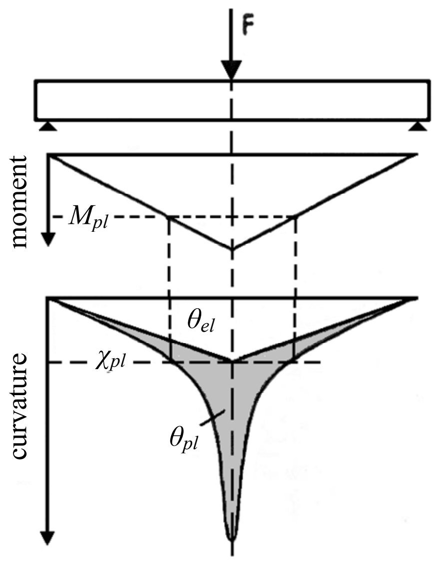

Fig. 3: Definition of plastic rotation according to the CEB (CEB, 1998).

Available and required rotation capacity depends on various factors related to the following topics: materials, geometry and loading. Some factors as concrete classes or slenderness ratio influence both rotation capacities, but many others only prevail in available or in required rotation capacity. Because of these reasons available and required rotation capacities are associated for the same structure, however it is possible to study separately each event.

Available rotation capacity is dependent on: i) mechanical properties of concrete in tension and compression, strength and ductility of reinforcing steel and bond properties of reinforcement; ii) shape of section, geometrical and mechanical reinforcement ratio, percentage of transverse reinforcement, slenderness ratio and member size; iii) static system, shear effects and load (type, application, duration, repetition and cycling) (CEB, 1998).

Required rotation capacity depends on flexural stiffness (along the beam at every instant of the loading increase), slenderness ratio, member size and loading type. Flexural stiffness is concerned to mechanical properties of concrete and reinforcing steel and bond properties of reinforcement.

Available rotation capacity is estimated by analytical models based on physical-mechanical properties of steel, concrete and their interaction, while required rotation capacity is calculated by non-linear analysis of typical structures with numerical models as Non-Linear Finite Element Method.

2.2.1 Models for available rotation capacity and Eurocode 2 procedure

There are various models recognized by international scientific community of structural engineering that present good results. Some of them are: Stuttgart Model, Naples Model, Darmstadt-Leipzig Model, Zürich Model and Delft Model (CEB, 1998). Recently Haskett et al. (2009) presented a novel rigid body displacement mechanism which can quantify not only the moment-rotation relationship of the hinge but also the limits to the rotation due to either wedge sliding (concrete fracture), reinforcing bar fracture or reinforcing bar debonding.

Although these models show different results and the experimental results point out some dispersion, in general the values of available rotation capacity predicted by the different models are similar and they compare well with the experimental results (CEB, 1998). Considering these models and existing experimental results the available rotation capacities were established on several codes.

In this paper it was applied the simplified procedure defined by Eurocode 2 (EN 1992-1-1, 2004) because it represents the most recent knowledge on this topic and it is applicable on actual existing materials. This procedure defines not only the available rotation capacity of continuous beams or one way spanning slabs, but also the conditions for its application.

The simplified procedure is based on the rotation capacity of a beam or one way spanning slab zone over a length of approximately 1.2 times the section depth (Fig. 4). It is assumed that these zones are subjected to a plastic deformation (formation of plastic hinges) under the relevant combination of actions (EN 1992-1-1, 2004). The basic value θpl for available rotation capacity according this procedure is shown in Fig. 5 for reinforcing steel class C. To take into account the shear slenderness, the basic value θ_{pl} should be adjusted according to Eq. (4). Therefore, the main parameters for available rotation capacity $\theta_{pl,av}$ in this procedure are the relative depth of compression zone x/d and the shear slenderness $M_{Sd}/V_{Sd}.d$.

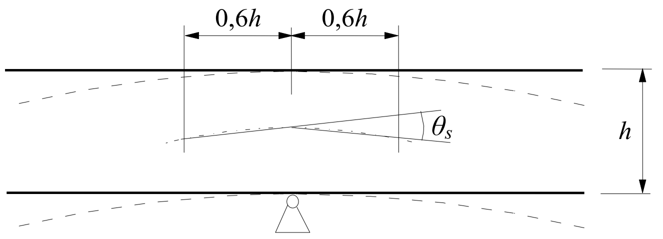

Fig. 4: Geometry definition of a plastic hinge in EC2 (EN 1992-1-1, 2004).

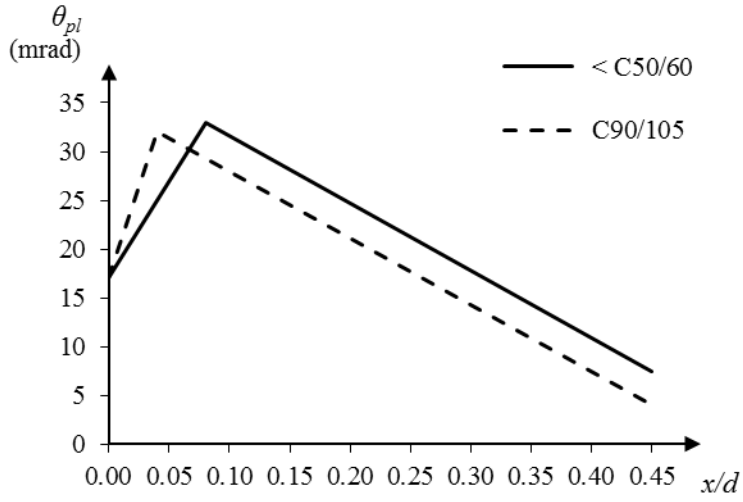

Fig. 5: Basic value for available rotation capacity in EC2 (EN 1992-1-1, 2004) for reinforcing steel class C.

$$\theta_{pl,\,av} = \theta_{p\,l} \cdot \sqrt{\frac{M_{Sd}}{3 \cdot V_{Sd} \cdot d}} \tag{4}$$

2.2.2 Required rotation capacity

The required rotation capacity may be estimated by calculating typical statically indeterminate structures up to the collapse mechanism being formed. At the critical cross section, where the first plastic hinge starts, the total rotation difference between the instant of the first yield (on this cross section) and the instant of first yield (on other cross section) that induce a collapse mechanism is the required rotation capacity.

The models used to compute required rotation capacity that satisfies the equilibrium and compatibility relations have as main parameters: flexural stiffening, geometry, external supports and loading type. Suitable variation of flexural stiffening along the members for each load increment is essential to get real results. For this aim, adequate models for concrete, reinforcing steel and tension-stiffening effect need to be introduced.

In opposite to the observations stated for the available rotation capacity, results obtained by various authors for the required rotation capacity are similar. It has been found that required plastic rotation is directly proportional to the member slenderness ratio for typical beams. Fig. 6 shows the results obtained by Eligehausen and Fabritius (1993) for a continuous beam with several equal spans under uniform distributed load. In this paper required rotation capacity is going to be automatically computed by the numerical model presented in Section 3.2.

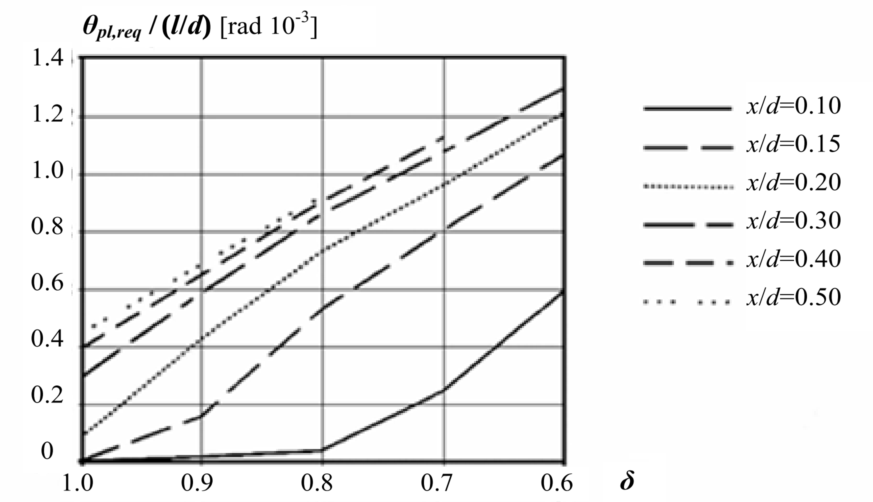

Fig. 6: Required plastic rotation in a continuous beam (Eligehausen & Fabritius, 1993).

3. METHODOLOGY

3.1 Parameters Studied

A systematic analysis was developed with the objective to quantify the deflection ductility index. The variation of the deflection ductility index was obtained by taking into account the variation of the six parameters defined in Table 1. Each parameter could have different values according to Table 1. All 324 combinations of these values were tested.

Table 1: Parameters studied and their values.

Parameter	Values
Span of the beam - l [m]	5 / 10 / 20
Effective depth of the cross-section - d [m]	0.25 / 0.45 / 0.95
Total mechanical reinforcement ratio - ω_t	0.2 / 0.4 / 0.6
Redistribution factor - δ	1.0 / 0.875 / 0.75
Static system	Fixed / Free
Concrete	C20 / C70

Initially the section width was introduced as a parameter, but it was observed that this parameter did not change the results because mechanical reinforcement ratio ω_t includes this parameter. The chosen values for span l, effective depth d and mechanical reinforcement ratio ω_t include most common beams used in building construction. For redistribution factor δ the three most typical situations were selected: δ=1.0 represents the elastic design, δ=0.875 represents the most common case for beams in buildings due to the arrangements of variable actions and, δ=0.75 represents the common limit of redistribution factor with equal quantities of reinforcement in span and supports. The two static systems were selected, fixed and free, typically the internal and the external spans respectively for continuous beams. Two concrete types were chosen to cover the normal (NSC) and high (HSC) strength types of concrete. Fig. 7 exhibits the supports and reinforcement distribution.

The total mechanical reinforcement ratio is equal to the sum of mechanical reinforcement ratios on the span (sp) and half of the sum of two supports (sup) (given in Eq. (5) and Eq. (6)). Only considering this sum as constant, it is possible, with a small error, to change the redistribution factor without modifying the load capacity of beam, when redistribution factor is the changeable parameter. On Table 2 is shown the distribution of steel reinforcement along the beam for every combination between redistribution factor δ and total mechanical reinforcement ratio ω_t. For all combinations and cross sections, the mechanical compressive reinforcement ratio was taken 50% of mechanical tensile reinforcement ratio.

$$\text{Fixed: } \omega^t = \omega_{sp} + \frac{1}{2}\cdot\left(\omega_{sup} + \omega_{sup}\right) = \frac{A_s^{sp}\cdot f_y}{b\cdot d\cdot f_c} + \frac{A_s^{sup}\cdot f_y}{b\cdot d\cdot f_c} \tag{5}$$

$$\text{Free: } \omega^t = \omega_{sp} + \frac{1}{2}\cdot\left(\omega_{sup} + 0\right) = \frac{A_s^{sp}\cdot f_y}{b\cdot d\cdot f_c} + \frac{1}{2}\cdot\frac{A_s^{sup}\cdot f_y}{b\cdot d\cdot f_c} \tag{6}$$

Fig. 7: Static systems and distribution of reinforcement.

Table 2: Distribution of reinforcing steel.

δ	ω_t	Fixed		Free	
		ω^{sp}	ω^{sup}	ω^{sp}	ω^{sup}
1.0	0.2	0.066	0.133	0.106	0.188
	0.4	0.133	0.267	0.212	0.376
	0.6	0.200	0.400	0.318	0.564
0.875	0.2	0.083	0.117	0.117	0.166
	0.4	0.167	0.233	0.234	0.332
	0.6	0.250	0.350	0.351	0.498
0.75	0.2	0.100	0.100	0.128	0.145
	0.4	0.200	0.200	0.256	0.290
	0.6	0.300	0.300	0.384	0.435

3.2 Numerical Model

The numerical model uses the Non Linear Finite Element Method to compute strains and stresses along the beams. Each tested beam was divided on various Bernoulli's beam type elements with a similar length of 0.6*h*, considering the definition of plastic hinge length in EC2 (Fig. 4) . One integration point by element was used. The performed computations showed the formation of plastic hinges on the expected elements. In Fig. 8 is illustrated the finite element meshes used for the analysis of the two static systems.

a) Fixed b) Free

Fig. 8: Geometry meshes and supports.

Current software calculate the stiffness of the elements for each loading increment, however in this case the stiffness was previously evaluated considering an accurate cross section analysis. The moment-curvature relationship is calculated for each cross section and the stiffness is evaluated by integrating in the element domain. Fig. 9 and Fig. 10 exhibit the stress-strain relationships for reinforcing steel B500C and concrete C20 and C70, respectively, used on the cross section analysis considering mean values for parameters. For concrete in compression it was applied EC2 model and for concrete in tension it was adopted a Tension-Stiffening law developed by Figueiras (1983).

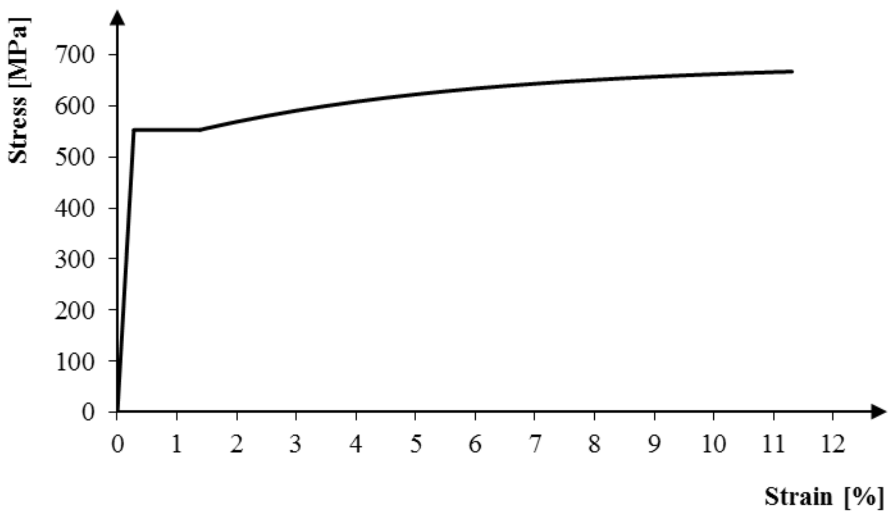

Fig. 9: Stress-strain relationship for reinforcing steel B500C (ARCER, 2003) by using mean values.

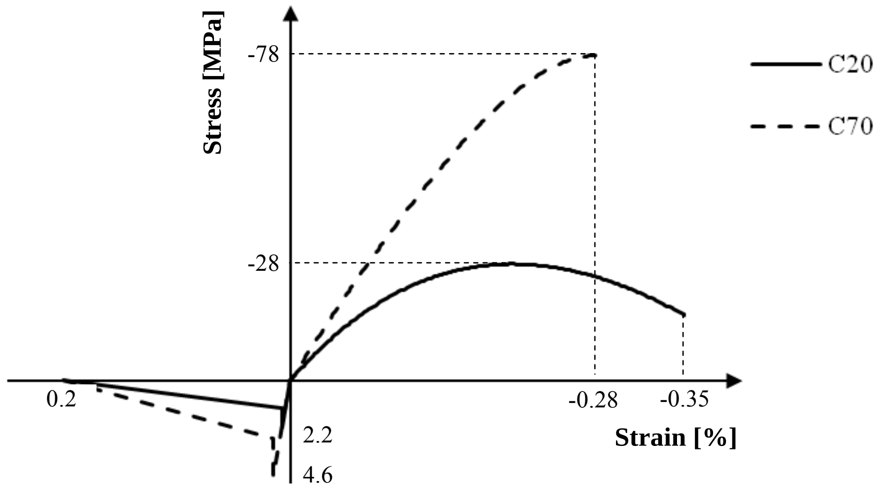

Fig. 10: Stress-strain relationship for two used concretes by using mean values.

There is a small incongruence between this stress-strain relationship with mean values and the basic value for available rotation capacity defined in Fig. 5 for design values. This option was taken mainly due to the following reasons: i) the non-existence of statistical characterization of the available rotation capacity in EC2, ii) the great accuracy gain with the real values of stress-strain relationships of materials instead of design values during the computation of the deformations and, iii) the need to present reliable values. However this option has not a significant influence because the required rotation capacity increases significantly with loading while the available rotation capacity remains constant.

The analysis was implemented by increasing the load up to the rotation capacity on a plastic hinge reaches its available rotation capacity, according to the simplified procedure defined by Eurocode 2, exposed on Section 2.2.1.

4. RESULTS AND DISCUSSION

4.1 General results

In Fig. 11 is shown the typical results obtained by this numerical study for a beam or one way spanning slab with l=5 m, d=0.25 m, δ=1.0, C20 and various mechanical reinforcement ratio. As it was expected Δ_u decreases while Δ_y increases with the increase of total mechanical reinforcement ratio ω_t.

The deflection ductility indexes were computed for the studied 324 combinations. Figs. 12 to 15 show the deflection ductility index as a function of slenderness ratio l/d and total mechanical reinforcement ratio ω_t for fixed and free static systems, respectively, for normal and high strength concretes. In these Figures the plane surface for μ_Δ = 1.0 represents the limit from which the structures reach the ultimate deflection Δ_u without reaching its total design loading capacity.

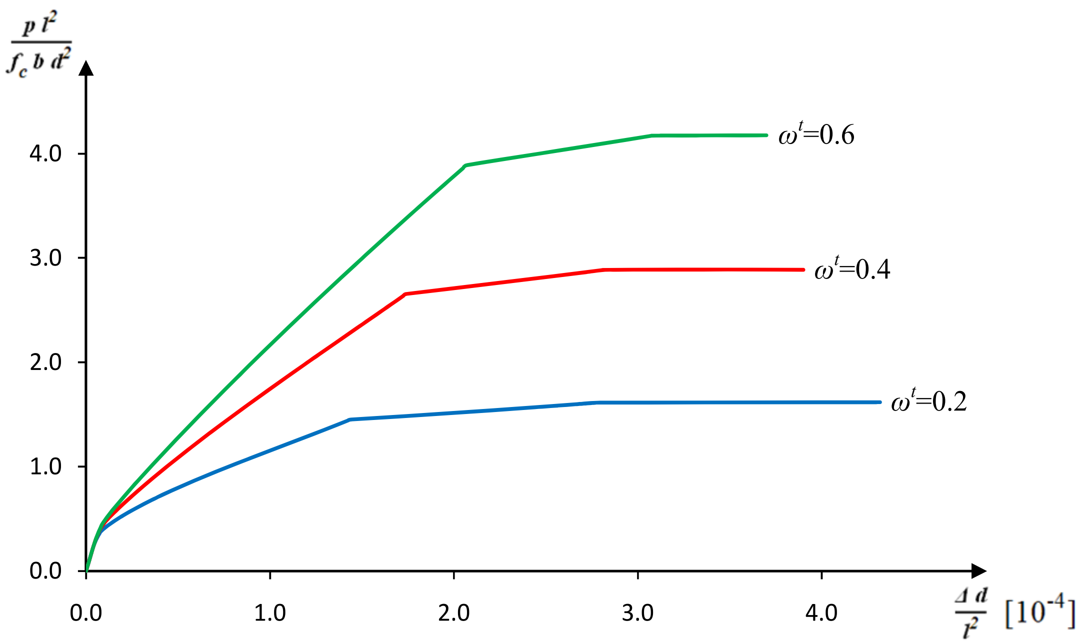

Fig. 11: Results for fixed static system with l=5 m, d=0.25 m, δ=1.0 and C20.

To obtain the final results in Figs. 12 to 15 the Method of Least Squares was applied to smooth out some divergent values resulting from the computing process and to interpolate and to extrapolate for others values of the parameters. During the early analysis of ductility index achieved from the variation of different parameters, it was verified that the influence of the effective depth d and span *l* could be considered by one parameter, the slenderness ratio *l/d*. Therefore, one parameter was removed and this fact allowed viewing all the results in a 3D type diagrams, as in Figs. 12 to 15.

The shape of results for the two static systems was similar for the same type of concrete. In fact, it was verified that the shape of deflection ductility index μ_Δ practically does not depends on the static system, on the other hand the slenderness ratio l/d, the redistribution factor δ, the type of concrete of concrete (NSC or HSC) and the total mechanical reinforcement ratio ω_t show relevant influence.

The influence of concrete type was more marked for high values of total mechanical reinforcement ratio ω_t and for free static system. This helps to clarify some controversy existent on literature about the influence of concrete strength on ductility indexes (Ahmad & Barker, 1991; Bernardo & Lopes, 2004).

These numerical results were obtained with the formulation of Eurocode 2 for available rotation capacity that includes a margin of safety. For this reason, it is expected that experimental results could be slightly higher. In fact, the results obtained here for deflection ductility index, approximately between 0.5 and 3.5, are smaller than experimental results encountered on literature (Carmo, 2004; CEB, 1998; McCarty, 2008; Ventorini, 2003). Taking this into account, the numerical results calculated here should be understood as the minimum characteristic values for the deflection ductility index expected for beams.

The great majority of experimental tests on literature are referring to simply supported beams, where deflection ductility index can be considerably larger than the values obtained for continuous beams. In addition, many of these old experimental tests used reinforcing bars different from actuality, which means that the confrontation of these experimental tests with this numerical study should be cautious.

After some tests it was verified that for a variation of the available rotation capacity $\theta_{pl,av}$, approximately an equal amount of variation is expected on the deflection ductility index μ_Δ. For a variation of the redistribution factor δ, approximately an equal amount of variation is expected on the deflection ductility index μ_Δ. For a variation of the total mechanical reinforcement ratio ω_t, only approximately 15% negative variation is expected on the deflection ductility index μ_Δ.

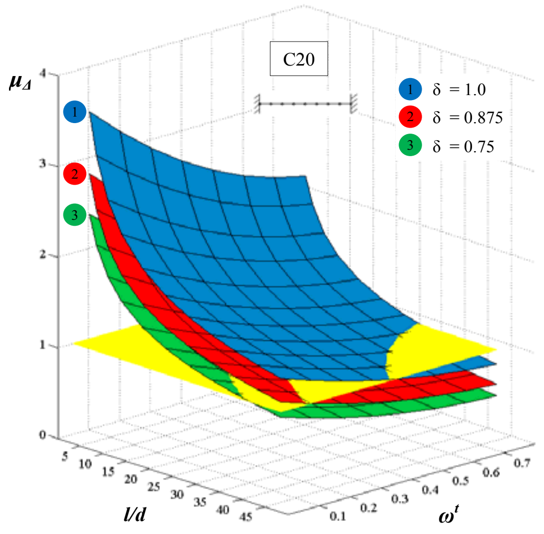

Fig. 12: Deflection ductility index for C20 on fixed static system.

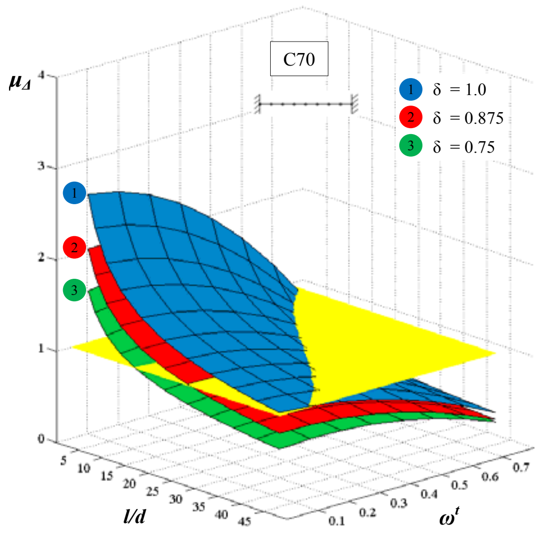

Fig. 13: Deflection ductility index for C20 on fixed static system.

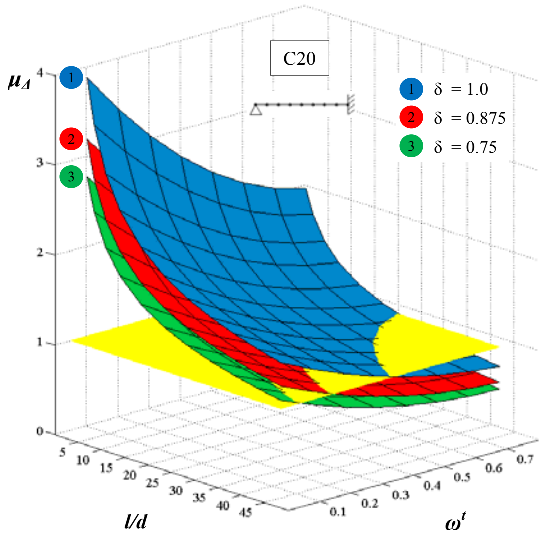

Fig. 14: Deflection ductility index for C20 on free static system.

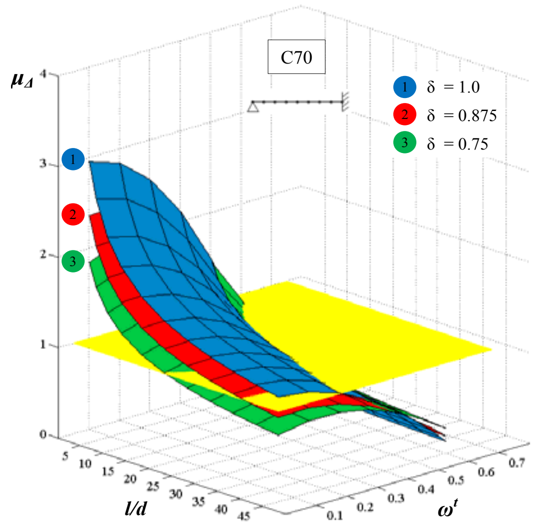

Fig. 15: Deflection ductility index for C70 on free static system.

4.2 Parametric results

For helping the analysis of the Figs. 12 to 15 horizontal cuts for $\mu_\Delta = 1$ are represented in Figs. 16 and 17, and vertical cuts for $\omega_t = 0.4$ are represented in Figs. 18 and 19. In Figs. 16 and 17 is easy to get the maximum slenderness ratio for a beam.

For typical slabs ($\delta = 0.75$, $\omega_t < 0.3$) the deflection ductility index need to be verified if very large slenderness or high reinforcement ratios are used, because slabs may not reach their total loading capacity. For the current beams in buildings ($\delta \leq 0.875$, $10 < l/d < 20$) the deflection ductility index shows values between 1.0 and 2.0, which is comfortable and safety.

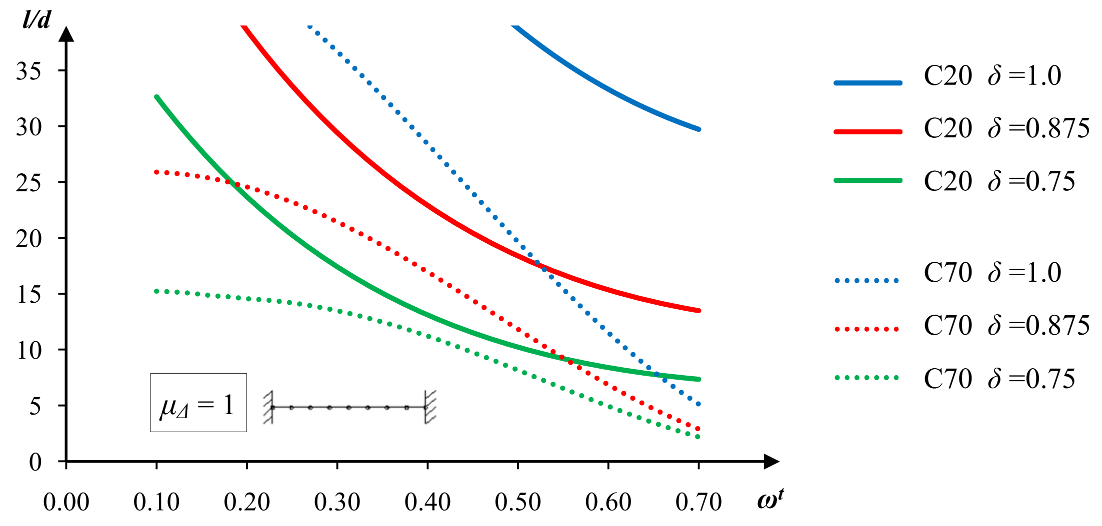

Fig. 16: Maximum l/d for μΔ =1 on fixed static system.

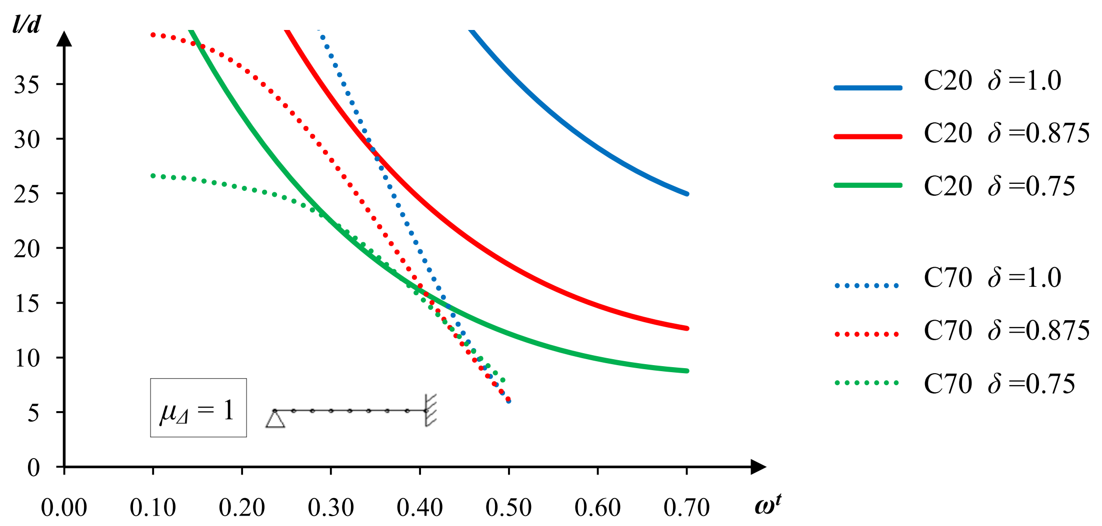

Fig. 17: Maximum l/d for μΔ =1 on free static system.

In order to apply the HSC in reinforced structures, the values obtained here for deflection ductility index could be restrictive for high values of total mechanical reinforcement ratio. However, the geometrical reinforcement ratio in HSC is two to three times greater than NSC for the same total mechanical reinforcement ratio, which means that the values of total mechanical reinforcement ratio used in HSC are generally smaller than NSC. From this standpoint, the application of HSC in reinforced structures is encouraged.

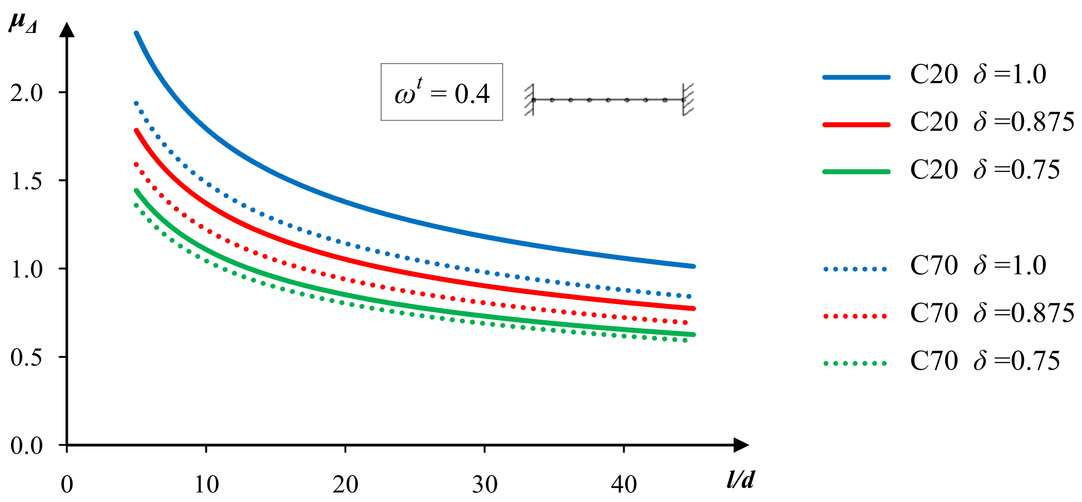

Fig. 18: Deflection ductility index for ωt = 0.4 on fixed static system.

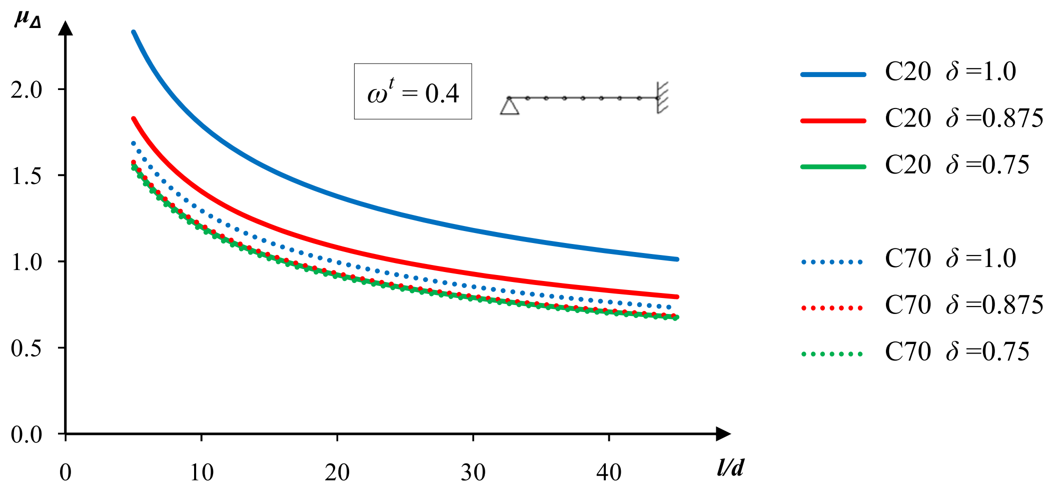

Fig. 19: Deflection ductility index for ωt = 0.4 on free static system.

5. CONCLUSIONS

The ductility in continuous beams or one way spanning slabs continues to deserve attention by the international scientific community of structural engineering because this problem is not completely clarified. The deflection ductility index can be a useful tool for designers, but in case of beams this parameter is not explicitly presented by several authors or by codes.

In this paper was presented a numerical study for the calculation of the deflection ductility index according to the Eurocode 2 as a function of three global parameters: *l/d* (geometry parameter), ωt (sectional parameter) and δ (design parameter), for two types of concrete (NSC and HSC). The numerical model was built with Bernoulli's beam type elements and the Non Linear Finite Element Method with adequate stress-strain relationships for concrete and reinforcing steel.

The results of deflection ductility index indicate, for current beams used on building construction, comfortable values between 1.0 and 2.0, while for slabs with high redistribution some attention are needed for limit values of slenderness and mechanical reinforcement ratios because slabs may not reach its total loading capacity.

The results presented here for deflection ductility index were obtained with the formulation of Eurocode 2 for available rotation capacity that includes a margin of safety. Therefore it is expected that experimental results could be slightly higher, as related on literature.

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