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ANALYTICAL AND NUMERICAL SOLUTION FOR EXPANDABLE TUBULAR

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ABSTRACT

Solid Expandable Tubular (SET) technology is widely used in petroleum industry to enhance oil production and to reach deep reservoir. One of the most important parameters in this process is to determine the force needed to run the expansion as well as the expansion limitations. The objective here is to explore an analytical solution for the expansion process based on shell membrane theory. The results will then be compared against that of finite element using a commercial package.

Keywords: Solid Expandable Tubular, Membrane stresses, axisymmetric shells, flow rule.

INTRODUCTION

Solid expandable tubular (SET) technology has been used over the last two decades in the petroleum industry. The working principle of this technology is somewhat simple where a tubular is expanded radial using a conical mandrel. The inlet radius of the conical mandrel is equivalent to the inner radius of the intended expandable tubular. The exit radius on the other hand represents the radius of the post expansion radius. The expansion ratio can be as large as 20% [A.G. Filippov et al.] of the tube inner radius. Due to the radial plastic deformation, the tube retains most of the expanded radius. It is important though to limit or avoid plastic deformation along the axial direction as it may cause the pipe to rupture. The expansion process can be investigated experimentally [A.C. Seibi and T. Pervez] which requires tremendous resources in term of the experimental setup. Another alternative is numerical simulation using commercial packages [T. Pervez at al.] which it is known to be computer intensive.

TUBE EXPANSION

The tube is shown in Figure 1. If we treat the tube as a membrane, the membrane stresses are related by [Intermediate mechanics of Materials by J.R. Barber]

$$\frac{\sigma_\theta}{R_2} = \frac{p}{t}$$

where

$$R_2 = \frac{r}{\cos\alpha}$$

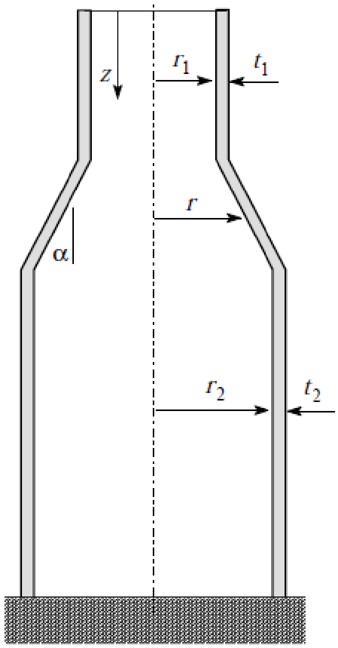

Fig. 1 Schematic of the expanded tube

in the conical region and p is the internal pressure, here caused by contact with the mandrel. Thus, we have

$$\sigma_\theta = \frac{pr}{t\cos\alpha} \tag{1}$$

Also, equilibrium in the axial direction (Figure 2) requires that

$$2\pi\cos\alpha\left(tr\sigma_z - (t+dt)(\sigma_z + d\sigma_z)(r+dr)\right) + \mu p\cos\alpha\left(\frac{2\pi r dz}{\cos\alpha}\right) + p\sin\alpha\left(\frac{2\pi r dz}{\cos\alpha}\right) = 0.$$

or

$$-\cos\alpha\left(t\sigma_z dr + trd\sigma_z + r\sigma_z dt\right) + \mu p(rdz) + p\tan\alpha(rdz) = 0$$

Divide through by dr, noting that $dz/dr = \cot\alpha$, giving

$$-\cos\alpha\left(t\sigma_z + rt\frac{d\sigma_z}{dr} + r\sigma_z\frac{dt}{dr}\right) + \mu pr\cot\alpha + pr = 0$$

or

$$\sigma_z + r\frac{d\sigma_z}{dr} + \frac{r\sigma_z}{t}\frac{dt}{dr} = \frac{\mu pr}{t\sin\alpha} + \frac{pr}{t\cos\alpha}$$

Eliminating the unknown contact pressure using (1) to get

$$\sigma_z + r\frac{d\sigma_z}{dr} + \frac{r\sigma_z}{t}\frac{dt}{dr} = \sigma_\theta(\mu\cot\alpha + 1) \qquad (2)$$

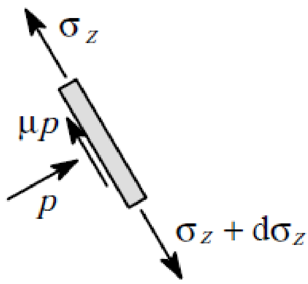

Fig. 2 Force equilibrium of an element taken from the expanded tube

With the membrane assumption, $p << \sigma_z, \sigma_\theta$ and it is clear from the geometry that both σ_z and σ_θ will be tensile. The maximum shear stress will therefore be

$$\frac{1}{2}\max(\sigma_z, \sigma_\theta)$$

And at present we have no way of knowing which is maximum. Let's tentatively assume that σ_θ is the maximum principal stress, so that the Tresca yield criterion gives

$$\sigma_\theta = S_Y$$

where S_Y is the tensile yield stress. The governing equation (6) for σ_z is then

$$\sigma_z + r\frac{d\sigma_z}{dr} + \frac{r\sigma_z}{t}\frac{dt}{dr} = S_Y(\mu\cot\alpha + 1) \qquad (3)$$

THE FLOW RULE

Suppose the tube flows at velocity *V* along some line *S*, such that the thickness *t* and the radius *r* are functions of a coordinate s. the volume of material flowing past a given point per unit time is given by

$$Q = 2\pi r V t$$

And this must be independent of s, so

$$rV\frac{dt}{ds} + rt\frac{dV}{ds} + tV\frac{dr}{ds} = 0$$

Now

$$\Delta\varepsilon_\theta = \frac{1}{r}\frac{dr}{ds}\Delta s$$

and

$$\Delta\varepsilon_z = \frac{1}{V}\frac{dV}{ds}\Delta s$$

Thus, if we assume the flow rule

$$\frac{\Delta\varepsilon_z}{\Delta\varepsilon_\theta} = \frac{\sigma_z}{\sigma_\theta},$$

we can also write

$$\frac{dV}{ds}\Delta s = V\Delta\varepsilon_z = \frac{V\sigma_z}{\sigma_\theta}\Delta\varepsilon_\theta = \frac{V\sigma_z}{r\sigma_\theta}\frac{dr}{ds}\Delta s$$

and hence

$$\frac{dV}{ds} = \frac{V\sigma_z}{r\sigma_\theta}\frac{dr}{ds}$$

Substituting into the constant volume equation, we then get

$$r\frac{dt}{ds} + t\frac{\sigma_z}{\sigma_\theta} + t\frac{dr}{ds} = 0$$

or

$$\frac{dt}{ds} = -\frac{t}{r}\left(\frac{\sigma_z}{\sigma_\theta} + 1\right)\frac{dr}{ds}$$

In the present case, we therefore have

$$\frac{r}{t}\frac{dt}{dr} = -\left(\frac{\sigma_z}{\sigma_\theta} + 1\right) = -\left(\frac{\sigma_z}{S_Y} + 1\right)$$

and hence the governing equation (7) becomes

$$r\frac{d\sigma_z}{dr} - \frac{\sigma_z^2}{S_Y} = S_Y\left(\mu\cot\alpha + 1\right) \qquad (4)$$

To solve this non-linear ODE, write it as

$$\frac{d\sigma_z}{dr} = \frac{S_Y^2\left(\mu\cot\alpha + 1\right) + \sigma_z^2}{rS_Y}$$

and hence

$$\int\frac{d\sigma_z}{S_Y^2\left(\mu\cot\alpha + 1\right) + \sigma_z^2} = \int\frac{dr}{rS_Y}$$

or writing

$$\sigma = \sigma_z / S_Y\,; \lambda^2 = \mu\cot\alpha + 1,$$

$$\int\frac{d\sigma}{\lambda^2 + \sigma^2} = \ln\left(r/A\right)$$

Where A is an arbitrary constant. The solution is

$$\frac{1}{\lambda}\arctan\left(\frac{\sigma}{\lambda}\right) = \ln(r/A)$$

or

$$\sigma = \lambda\tan(\lambda\ln(r/A)),$$

For the constant A, we have $\sigma = 0$ at $r = r_1$, so $A = r_1$ and

$$\sigma = \lambda\tan(\lambda\ln(r/r_1)),$$

$$\sigma_z = S_Y\lambda\tan(\lambda\ln(r/r_1))$$

The maximum achievable σ_z is SY which will occur when

$$\lambda\tan(\lambda\ln(r/r_1)) = 1$$

or

$$\frac{r}{r_1} = \exp\left[\frac{1}{\lambda}\arctan\left(\frac{1}{\lambda}\right)\right] \qquad (5)$$

RESULTS

The analytical results are validated using commercial finite element software. Figure 3 shows the finite element model used to simulate the expansion process. Comparison of the analytical and finite element solution of the axial stress is shown in figure 4. The comparison shows a good agreement. The element type used in the finite element model is a shell element with bending capability. It is worth mentioning that the shell membrane theory used to drive the analytical solution neglects the bending effect. Figure 4 shows the maximum expansion ratio that can be applied before the axial stress reaches the yield limit (equation 5) for different coefficients of friction μ and mandrel angles α. The plot shows that the expansion ratio converges to a steady state value after which the change in the mandrel angle has a little or no effect. It also demonstrates the significant effect of the coefficient of friction. The expansion ratio can be dramatically enhanced if a lubricant is used between the mandrel and the pipe as well as if a smooth surfaced mandrel is used.

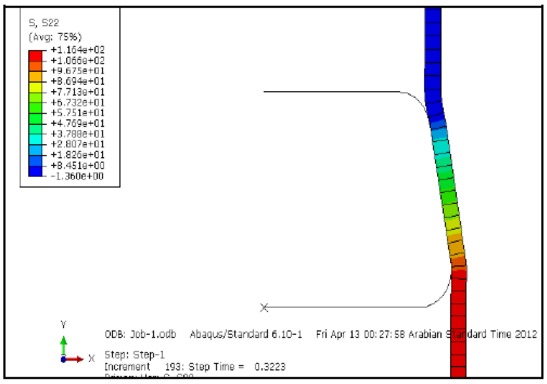

Fig. 3 Finite element (FE) model

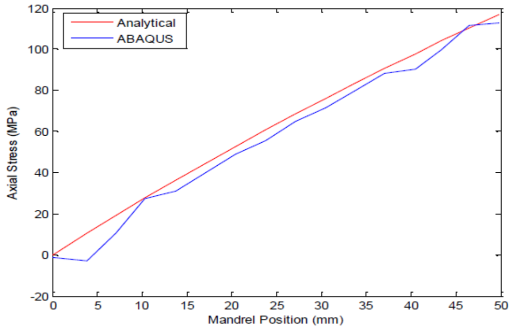

Fig. 4 comparison of analytical and FE solution of axial stress

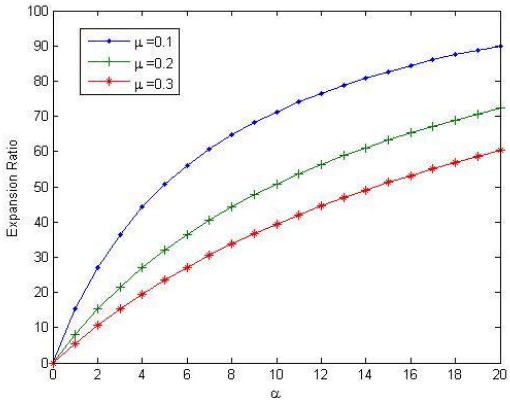

Fig 5 Maximum expansion ratio for different mandrel angle α and coefficient of friction μ

CONCLUSION

This paper investigates an analytical solution for the solid expandable tubular technology. This solution can be an alternative to computer intensive's finite element simulation. A very good agreement is found when the analytical solution is compared to finite element results of the same model. The analytical solution can be used to determine the expansion limitation in terms of the maximum expansion ratio and the force needed to apply the expansion.

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