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ADDRESSING GEOMETRICAL NONLINEARITY CONSIDERING PROBABLISTIC MATERIAL NONLINEARITY

Iffat Siddique(*)
Dipartimento di Ingegneria Strutturale, Edile e Geotecnica(DISEG), Politecnico Di Torino, Torino, Italy.
(*)*Email*: iffat.siddique@polito.it

ABSTRACT

The statistics reflecting the contribution of the material parameters to the total variability of the response parameter are presented by carrying out sensitivity analysis (SA). To accomplish SA a numerical model for the geometrical plus material nonlinear analysis of 2D structural elements is developed. The model employs corotational formulation combined with numerical integration and hence is suitable for many commonly used cross sectional shapes. The accuracy of the proposed algorithm is validated through examples from previous literature. Then material uncertainties are addressed in probabilistic fashion through Monte Carlo simulations.

Keywords: geometrical and material nonlinear structural analysis, co-rotational formulation, numerical integration, Monte Carlo Simulations, sensitivity analysis.

1. INTRODUCTION:

One of the fundamental assumptions of linear analysis is that neither the shape nor the material properties change during all the deformation process i.e. change in the stiffness is really small. Such linear analysis provides merely an approximation of the real behavior of the structures. Most of the challenging problems call for nonlinear analysis in order to have a real picture of the structural behavior. Although changing stiffness is common in all types of nonlinear analysis, nonlinearities are classified depending upon the principle origin. A lot of research work is done on the nonlinear analysis (section2.1&2.2) but considering both types of nonlinearities together is reported in very few works (Battini and Pacoste, 2002). Uncertainties are an inevitable part of analysis and it is really important to consider them in analysis. In contrast to old safety factor method new and better approach is to address these uncertainties in the probabilistic way (section 2.3). In order to check how reliable our analysis is, it is always useful to conduct sensitivity analysis for a number of important decisive results (Section 3).

In this paper a sensitivity analysis for the material properties of 2D reinforced concrete structures is presented. For it, a MATAL code addressing nonlinear geometrical and a nonlinear elasto-plastic material behavior of structures is developed. The code is then validated through examples from previous texts at various stages. Further the uncertainties in the material properties are simulated using crude Monte Carlo method. Then a detailed sensitivity analysis is carried out for compressive strength of concrete and yield strength of steel as these are the two most influencing material properties. The effect of other properties like young's modulus on the system can be easily inferred from the results of these two properties.

2. PROBABILISTIC ANALYSIS AND NONLINEARITIES:

2.1 Geometrical Nonlinearities:

Geometric nonlinearities result when the forces producing structural deformations are a nonlinear function of the displacements and produces change in the shape of the structure. Geometric nonlinearities are extremely important in collapse simulation because they capture the effects of buckling, large changes in structural shape and the changes in internal forces necessary to keep the structure in static equilibrium. Except for very simple problems, it is extremely difficult to obtain a closed form solution. Hence incremental iterative techniques are to be used based on computer simulations. Various formulations address these nonlinearities depending upon the kinematic description and the choice of the reference frame. In the context of geometrically nonlinear FEM analysis, three kinematic descriptions have been extensively used. Total Lagrangian formulation, Updated Lagrangian formulation (Yang and Leu, 1991), and corotational formulation (CR) (Crisfield, 1990). In the analysis done here CR formulation has been used because of the several advantages listed in the next section. A summary of majority of the important research works about geometrical nonlinear analysis conducted in the past is also summarized in some papers. (Yang et al., 2003).

2.1.1 Corotational Concept:

When a frame element is loaded it will deform from its original configuration. During this process each element undergoes the following three actions: translation, rotation and deformation. The rotation and translation being the rigid body motions can be removed from the beam element leaving behind the strain producing deformations. The strain producing local deformations are the ones related to the forces induced in the beam elements.

A corotational formulation separates these two components at the local element level by attaching a local element reference coordinate system, which rotates and translates with the beam element. The rigid body rotations and translations are zero with respect to this local co-rotating coordinate system. The derivation of corotational formulation to get relationship between global and local variables, the angle of rotation and a variationally consistent tangent stiffness matrix can be found in literature (Yaw, 2008). Many of the structural materials experience large rotations but small strains. CR formulation can effectively treat such problems. It can decouple small-strain material nonlinearities from geometric nonlinearities. CR is very well suited to the treatment of structural elements having rotational degree of freedom for arbitrary large rotations e.g.; beams, plates and shells etc. It is extremely difficult to treat with such problems with TL description which is the main competitor of CR formulation (Felippa, 2000). Keeping in view all these advantages this formulation has been used in our MATLAB code.

2.2 Material Nonlinearities:

Large deformations lead to post-elastic response in the structures. In order to simulate these large deformations it is generally necessary to account for material nonlinearities. In literature there are a lot of ways of considering these nonlinearities in analysis. Among them noticeable ones include: through the development of concentrated plastic hinges (Li and Li, 2007), studying gradual development of inelasticity across the beam depth referred to as distributed plasticity approach (Liew et al., 1993). Sivaselvan and Reinhorn (Sivaselvan and Reinhorn, 2002) presented a flexibility based approach to the collapse of plane frames in contrast to the previous mentioned displacement based approach. One of the key benefits of flexibility based approach is the ability to use single frame member compared to multiple element discretization used in the displacement-based approach. We can also find in literature fiber-beam element using flexibility (Torkamani and Sonmez, 2001).The fiber beam element during its incremental global analysis, adopts integration of fiber across the beam depth and hence the designer can keep track of the state of the distributed plasticity.

By combining material and geometrical nonlinearities it becomes possible to model plastic and geometrical instabilities, which can be found in many of the previously cited works (Battini and Pacoste, 2002). In our analysis material nonlinearities are incorporated through numerical integration across the volume of the element along with the geometrical nonlinearities.

2.3 Probabilistic Analysis and Its Need:

Presence of uncertainties in the analysis and design due to measurement, physical, mechanical or statistical constraints has been recognized by the engineers since years. In the past, the tradition was to simplify the problem by considering the uncertain parameters as deterministic ones and then accounting for the uncertainties by using empirical safety factors. As these factors are determined by considering past experience and hence do not fully guarantee the safety of structures. Also, they do not give any idea about how different parameters influence the structural safety as they do not take into account the underlying distribution of the random variables involved in the system. Also, the deterministic safety factors approach do not provide adequate information to achieve optimal use of the available resources while maximizing safety at the same time. A new increasing fashion of addressing these uncertainties is the probabilistic analysis which takes into account the parameter variability along with its underlying distribution. Hence it provides more information about the system behavior, the influence of various parameters on system performance along with their interaction among themselves.

Material uncertainties due to skill and experience of workmanship, various manufacturing procedures and plants, environmental impact, existing structures etc contribute a significant part to the overall uncertainties of the system. In our analysis only material uncertainties have been addressed. The main source used for the probabilistic input data was JCSS model code. Table-1 summarizes all the data recommended in the code.

3. SENSITIVITY ANALYSIS:

Sensitivity analysis is the study of how uncertainty in the input affects the uncertainty in the output of a mathematical model or a system. Before stepping into the detailed reliability analysis it is always fruitful to conduct sensitivity analysis for a number of important decisive results. Among them includes: which parameter require additional research for concrete knowledge about the system and hence reducing output uncertainty, parameters that have a minor impact on the system output and therefore can be eliminated resulting in model simplification, correlations between the input parameters and output and many other.

3.1 Methods of Sensitivity Analysis:

There are several ways of undertaking sensitivity analysis. These are generally grouped as: *one way sensitivity analysis* which allows reviewer to assess the impact of changes in one specific parameter on the model output. Then we have *multiway sensitivity analysis* in which it is necessary to examine the relationship of two or more different parameters changing simultaneously. However, the presentation and interpretation of multiway sensitivity analysis becomes increasingly difficult and complex as the number of parameters involved increases (Hamby, 1994). Also there are *probabilistic sensitivity analyses* that provide a useful technique to quantify the level of confidence that a designer has in his decisions. In probabilistic sensitivity analysis, rather than assigning a single value to each parameter, a probabilistic distribution is assigned to all the parameters of the mode. Hence a range of the data is assigned through mean value, standard deviation and ‘shape’ of the data spread.

In this work several methods have been selected covering two important broad categories of sensitivity analysis mentioned above. Among the one way sensitivity analysis sensitivity index and parameter uncertainty factor were calculated (Hamby, 1994). Also Pearson correlation coefficient has been determined. Thereafter probabilistic SA has been performed by random sampling following probabilistic distribution of the parameters.

3.2 Presenting Sensitivity Analysis:

Sensitivity Analysis methods can be classified in a variety of ways and accordingly results can be presented. *Mathematical methods* are used for the validation and verification purposes. Then there are *statistical methods* which involve running simulations in which inputs are assigned probability distributions and assessing the effect of variance in inputs on the output distributions (Hamby, 1994). They allow identifying the effect of interactions among the individual inputs. Also we have some *graphical methods* which give representation of sensitivity in the form of graphs, charts or surfaces(Christopher Frey and Patil, 2002). They can be used as a screening method before the further analysis of a model to represent strong dependencies among input and output variables (Christopher Frey and Patil, 2002).

The mathematical results of the sensitivity analysis is presented herein Table-3. Also graphical representation of the results in the form of histograms and scatter plots are included in the Section-6 (Results and Discussion).

4. COMPUTATIONAL METHODOLOGY:

The work was started by taking a model problem of a beam and developing a MATLAB code based on corotational formulation for the geometrical nonlinear analysis. The code was based on some mathematical expressions for calculating the internal force vector taken from lecture notes of Yaw and the results in the form of load deflection curve were compared with the curve given by Yaw (Yaw, 2008). The code so far could perform geometrical non linear analysis for the linear elastic 2D elements only. After verifying that the results were an exact match the code was then modified. These expressions were afterwards replaced by generalized expressions in the form of integrals over the volume of the element (Battini, 2002) and the code was modified to compute internal force vector by numerical integration. This modified code results for the linear elastic material model was then verified against the previous results. The Load deflection curves for both the codes are shown in fig-1. Afterwards the constitutive law was changed from linear elastic to elasto-plastic to incorporate material non linearity along with geometrical non linearity for the analysis of the line elements. Uncertainties are always an inevitable part of the data also of material characteristics so to use nominal strength values in analysis becomes questionable. Hence probabilistic approach was adopted. A small code was written to simulate the probabilistic characteristics of the material properties (compressive strength of concrete and yield strength of steel) using Monte Carlo simulation method (Pengelly, 2002). And finally sensitivity analysis was carried out for the two random parameters i.e. compressive strength of concrete and yield strength of steel.

4.1 Sensitivity Analysis:

4.1.1 Sensitivity Index (SI):

It is a simple method to calculate %age difference when varying one input parameter from its minimum to its maximum value. For this, we vary the parameter, whose sensitivity is to be evaluated by a percentage of its standard deviation while keeping all the other parameters constant at its mean value. This helps us to calculate output %age difference (Table-3) as:

$$Sensitivity\,Index(SI) = \frac{Dmax - Dmin}{Dmax}$$

$D_{max/min}$= maximum/minimum value of ultimate load.

4.1.2 Parameter Uncertainty factor (PUF):

Another statistical method of evaluating the importance of parameters is the PUF and is given as:

$$Parameter\,Uncertainty\,Factor(PUF) = 2Std\frac{change\quad output}{Change\quad input}$$

4.1.3 Pearson correlation coefficient (Rx,y):

In order to have a better idea of the parameter randomness on the system output, random samples of one of the parameters according to its probabilistic distribution were simulated using Monte Carlo simulation technique while the other sample taken as a constant value(either equal to its Mean, Mean ±Std, Mean±2Std). The Pearson correlation coefficient is then computed to give an estimate of the correlation between the input and output and is given by:

$$\mathrm{Rx,y} = \frac{\sum_{i=1}^{N}(\mathrm{Xi}-\mathrm{X}')(\mathrm{Yi}-\mathrm{Y}')}{\sqrt{(\sum_{i=1}^{n}(\mathrm{Xi}-\mathrm{X}')^2} - \sum_{i=1}^{n}(\mathrm{Yi}-\mathrm{Y}')^2)}$$

X’ and Y’ represent the mean values of the input and output parameters. The results are shown in Table-3 along with the associated graphical representation in the section-6 (results and discussion).

4.1.4 Probabilistic Sensitivity Analysis:

To understand the overall system behavior we consider the randomness of both the variables involved in our system. According to the probabilistic distribution of both the parameters random samples were obtained and used in the calculation of the ultimate load. This is the most important of all the analysis as it is very close to the reality problems and takes into account the interaction of both parameters as well. The results of this analysis are shown in Fig-4.

5. EXAMPLE PROBLEM:

5.1 Configuration:

As an example problem, in order to validate the authenticity of the developed code, a cantilever column with a fixed vertical compressive load of 1280 KN and a variable horizontal load (CEB/FIP manual of Buckling and instability; pg. 29) was selected. The sectional and material properties are given in Table-2. All the probabilities data used in the analysis is provided in Table-1.

5.2 Computer Modeling:

The modeling has been done using MATLAB by dividing the column into eight equal finite elements along with its length with nine nodes. A constant compressive force of 1280KN acts on it and a variable horizontal load is applied in increasing incremental manner to get the maximum horizontal load that can be applied.

For carrying out numerical integration; the section was divided into varying no. of strips to achieve integration along the cross section. Further to get integration along the entire volume of the element, Gauss quadrature was adopted for the integration along the length of the element. Here we have taken four gauss points.

Table 1 Statistical parameters for compressive and yield strengths

Quantity	Probabilistic Data			
	Distribution	Mean	Std	Parameters
f_{co}	LN	3.85		S'=0.009,v'=10, n'=3.0
Y_1	LN	1.0	0.06	-
f_y	Gaussian	S_{nom} + 2Std	30	S_{nom}=500

Table 2 Sectional and material properties of example problem

Sectional Properties						
Length	**Width**	**Height**	**D**	**Ac**	**Steel layers**	**As**
mm	mm	Mm	Mm	mm^2	#	mm^2
4000	400	400	40	160000	2	3020
Material Properties						
fc	**Ɛc1**	**Ɛcu**	**Fy**	**Es**	**Ɛel**	**Ɛy**
MPa	-	-	MPa	Mpa	-	-
200	0.0020	0.0035	420	200000	0.0021	0.0021

6. RESULTS & DISCUSSION:

The response of the structure (ultimate load) obtained from both analytical method and numerical integration techniques are in close agreement showing the accuracy of the developed base code (Fig.1).

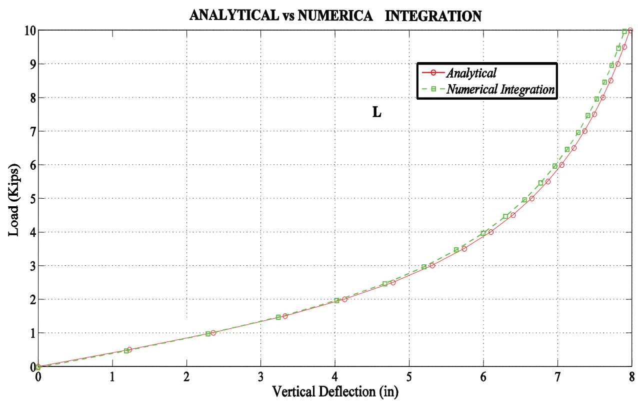

Fig.1 Comparison of Analytical & Numerical Integration

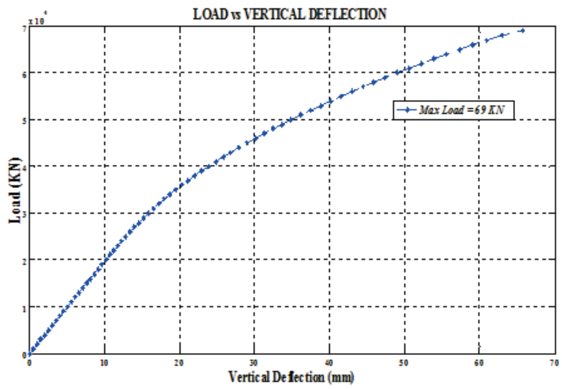

Fig.2 Load Deflection Curve

The Fig.2 shows the load deflection curve of the example problem and is found to in exact synchronization with the one given in the CEB/FIP manual. The results of the sensitivity analysis are presented in the table below:

Table 3: Statistics of Sensitivity Analysis

Quantity	**SI**	**PUF**	**Rx,y**
f_c	0.23	6	-0.984
f_y	0.12	3	-0.993

Though all these are one way sensitivity analysis procedures. The first two methods compare output variability at some specified points of the input parameter. Both of them give quite similar results. That compressive strength of concrete has a much larger contribution to the variability of the response variable (i.e. ultimate load). The Pearson correlation coefficient of both the variables is close to -1. This means that the there is a strong linear relationship between either of the input variables and the ultimate load capacity of the structural element being investigated. And hence ANOVA methods have not been investigated in this study so far. The scatter of the data with respect to two variables considering one of them as variable is shown in Fig.3. The scatter plots show that due to randomness of the compressive strength the data is scattered evenly within 80 to 105 KN (for this specified problem) while in case of yield strength the data is clustered at some specific values, though this clustering is in the same range of 80 to 105 KN. Hence it may be concluded that the statistics will quiet be similar despite the fact that a slightly different distribution may fit the results of both the variables. The results of the probabilistic sensitivity analysis considering both the variables as random along with the effect of their interaction are plotted as a histogram. Various distributions were tried to fit in the ultimate load data and it was found that lognormal distribution is the one that comes out to be the best fit (Fig. 4).

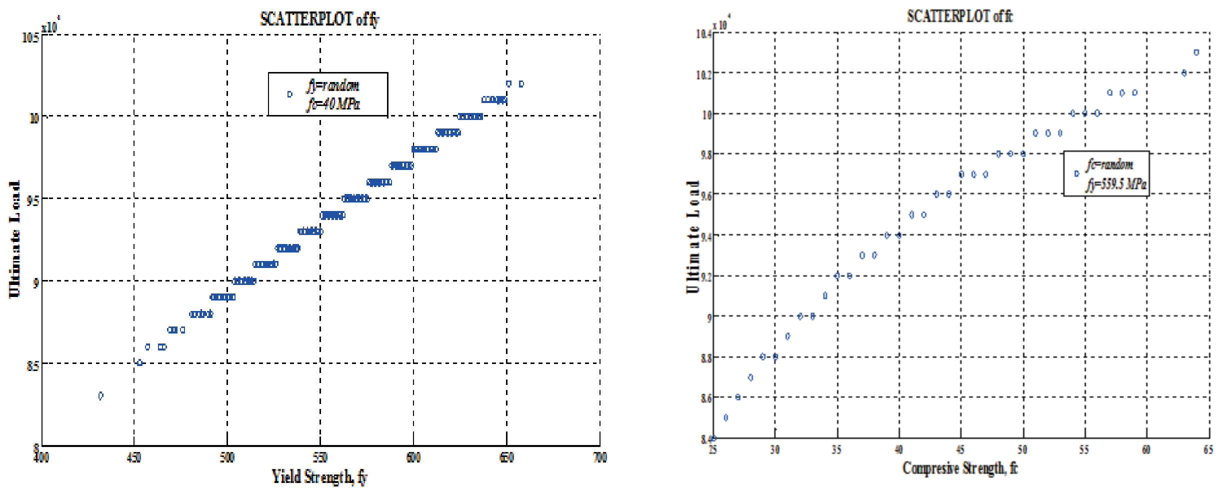

Fig.3 Scatterplots showing effect of parameter variability on the distribution of structural response

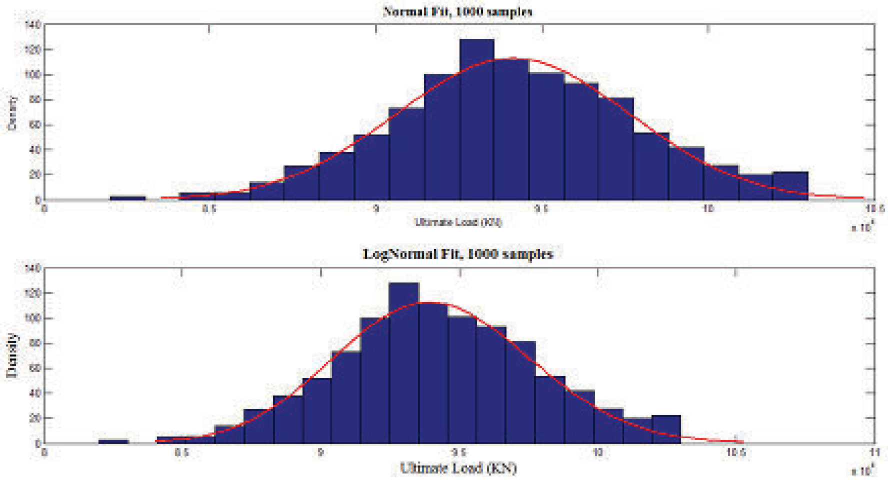

Fig.4 Probabilistic SA and best fit PDF for ultimate Load

7. CONCLUSIONS:

A sensitivity analysis for the material properties has been conducted. For this first of all, a MATLAB code for the geometrical nonlinear analysis based on corotational formulation was developed. In the model, elasto-plastic nonlinear material behavior has been considered. Uncertainty in the material properties are addressed in a probabilistic fashion simulated using Monte Carlo simulations. And finally sensitivity analysis has been performed and it is concluded that: both the yield strength of steel and compressive strength bear a strong linear relationship with the failure load. And ultimate load is found to have a log normal probabilistic distribution.

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