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A MAGNETIC APPROACH TO THE IDENTIFICATION OF EFFECTIVE CHARACTERISTICS OF METAL FIBRE COMPOSITES USED IN CIVIL ENGINEERING

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ABSTRACT

Composite materials used in civil engineering typically contain short metal fibres to reduce cracking and deterioration of their mechanical properties, thus efficient approaches to the validation of their effective engineering characteristics, as volume fraction of fibres, some measure of macroscopic homogeneity, orientation of fibres, etc., are needed. This paper pays attention to a magnetic approach, using the Hall effect and the properties of solutions of the Laplace equation together with the advanced computational homogenization analysis, coupled with the least-square based optimization technique.

Keywords: building materials, metal fibre composites, non-destructive testing, inverse problems, computational homogenization.

INTRODUCTION

Advanced building structures, as discussed in (Cunha, 2011), as well as other structures of civil engineering, frequently use materials as silicate composites, reinforced by metal particles, typically short fibres, preventing the tension stresses and strains as sources of undesirable micro- an macro-cracking. Mechanical behaviour of such composites is determined by the choice of fibre properties and their volume fraction, location and orientation in the matrix, sensitive to the technological procedures (as special compaction) and to the early-age treatment – cf. (Yilmaz, 2010) and (Soulioti, 2011). Since the technological requirements are rather demanding, some reliable a posteriori validation of expected solid material structure is needed. The employment of the destructive approach relies usually on the separation of particles, taken from the early-age matrix, alternatively obtained from the crushed part of the existing structure, in the laboratory; consequently the volume fraction of particles can be evaluated accurately, but any information related to the original orientation of particles is missing. Moreover, such experiments are not allowed by national and European technical standards for some types of structures. This is the motivation for the development of non- or lower-invasive approaches to the analysis of material characteristics of metal fibre composites, relying on some (geometrically, physically, etc.) distinguishable properties of a matrix and particles.

One possible choice for the detection of volume fraction, location and orientation of particles in a matrix without any damage is offered by radiographic methods, discussed in (Hobst, 2013), supplied with image processing techniques, applying edge detection algorithms, fast Fourier transform, etc., by (Vala, 2012). However, such approach is available only to material specimens of limited thickness (not to massive structures), under rather strict safety provisions. Several alternative methods have been presented in the last decade: namely (Van Damme, 2004) estimates the effective material permittivity employing a coaxial probe together with microwave reflectometry techniques, (Ozyurt, 2006) makes use of the AC-impedance spectroscopy, (Lataste, 2008) performs special low-frequency electrical resistance measurements and (Faifer, 2009) develops a method based on impedance-over-frequency measurements, employing certain two-electrode probe, supported by the numerical FFT (fast Fourier transform) computations. Recently (Faifer, 2011) and (Wichmann, 2013) exploit the ferromagnetic behaviour of metal particles to evaluate their volume fraction; the deviation of measurement values gives basic information to the required homogeneity and isotropy.

We shall pay attention namely to analysis of behaviour of a material specimen or a whole massive structure in the artificial magnetic field, thanks to quite different (relative) magnetic permeability of a non-metal matrix and metal particles. Let us notice that, under some physical simplifications, neglecting the effect of other physical, chemical, etc. processes and external influences, a very similar approach is available for the electric field where the (relative) electric permittivity replaces the magnetic permeability in the analogous boundary value problem for a Laplace-type equation. However, to be able to exploit the database of measurement results from the Department of Building Testing at BUT (Brno University of Technology), obtained from the Hall probe with permanent magnets, as evident from the left-hand part of Fig. 1, detecting intensity of magnetic field in the classical fibre concrete, we shall assume the presence of a (nearly) stationary magnetic field in the specimen, whose radial symmetric geometric model (neglecting the processes inside the probe) is presented on the right-hand part of Fig. 1. We shall show (at least for a good experimental arrangement):

i) that the numerical solution of a boundary value problem for the stationary distribution of a magnetic field (caused by the presence of a permanent magnet in the Hall probe) with some effective (macroscopic) value of magnetic permeability can be avoided at all,

ii) how such effective (homogenized) values can be obtained under the assumption that the (deterministic or stochastic) distribution of particles in a matrix and their (relative) permeabilities are known in advance,

iii) how the analysis i) helps us to solve the inverse problem of identification of the volume fraction of particles and some material homogeneity and isotropy characteristics.

The problems connected with i), ii), iii) will be discussed in three following sections of this paper, accompanied by an illustrative example.

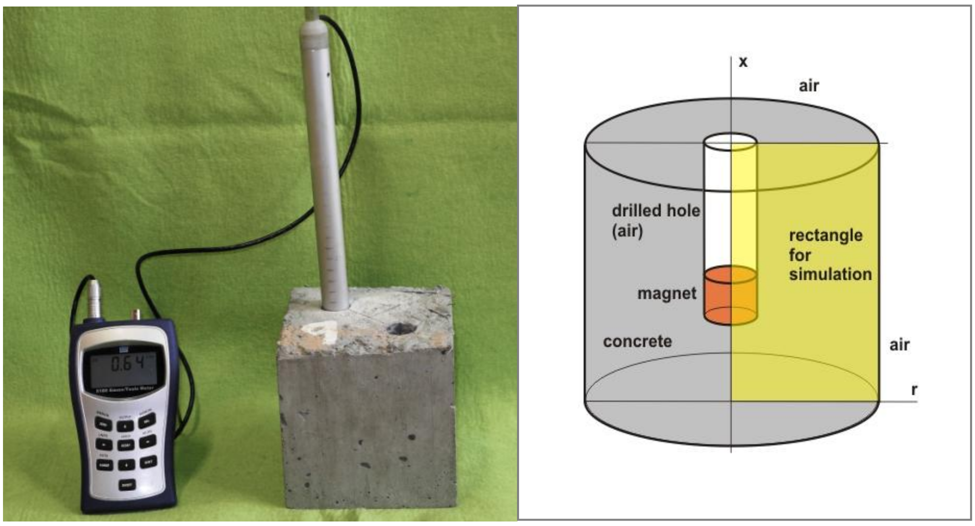

Fig. 1 The Hall probe (left image) and the related computational scheme (right image).

PHYSICAL AND MATHEMATICAL BACKGROUND

Following the experimental setting by Fig. 1, let us consider the magnetic field strength H [A/m], the magnetic potential B [N/(A·m)] and the scalar magnetic potential ψ [A] on certain (rather special) domain Ω in the 3-dimensional Euclidean space R^3, supplied by the Cartesian coordinate system $x=(x_1,x_2,x_3)$; all symbols $\nabla := (\partial/\partial x_1, \partial/\partial x_2, \partial/\partial x_3)$ and $\Delta := \nabla\cdot\nabla$ will express standard Hamilton and Laplace differential operators. The system of Maxwell equations can be reduced to

$$H = -\nabla\psi, \quad \Delta\psi = 0 \quad \text{on } \Omega \ ; \tag{1}$$

for all details see (Hobst, 2011). Only one constitutive relation is needed:

$$B = \mu H \quad \text{on } \Omega \ , \tag{2}$$

containing some effective permeability μ [N/A^2]; in this section we shall suppose that μ (from the macroscopic point of view) is a given positive constant. The natural Neumann boundary condition is $B\cdot\nu = b$ on a suitable part of the boundary Γ of Ω, later referred as Θ, supplied locally by the unit outward normal $\nu := (\nu_1,\nu_2,\nu_3)$; non-zero prescribed b can occur thanks to the magnetization of the permanent magnet. Applying (1) and (2), we have $\nabla\psi\cdot\nu = -H\cdot\nu = -B\cdot\nu/\mu$, which gives

$$\nabla\psi\cdot\nu = -b/\mu \quad \text{on } \Theta \ . \tag{3}$$

It is reasonable to prescribe the homogeneous Dirichlet boundary condition on the remaining part of Γ (among others, to force the unique setting of ψ).

In the simplest case, b can be identified piecewise with some real constant b_* and piecewise with zero on Θ. In particular, for the cylindrical magnet located in the cylindrical hole in a cubic material sample, we can consider the boundary condition (3) with $b = b_*$ on the contact with the magnet, otherwise with $b = 0$. In the case of 2-axes symmetrical geometrical configuration we are allowed to set $b = 0$ also on symmetry axes (the magnetic fluxes perpendicular to such axes are not taken into consideration). Motivated by Fig. 1, showing the cylindrical hole of the same (positive) radius r_0 as that of the magnet, considering the cylindrical coordinate system (x_1, r, ϑ) with $x_2 = r\cos\vartheta$, $x_3 = r\sin\vartheta$, for $r \ge r_0$ we consequently obtain $\Delta\psi = \partial^2\psi/\partial x_1^2 + \partial^2\psi/\partial r^2 + (\partial\psi/\partial r)/r$; this seems to be a good starting point for non-expensive two-dimensional computational simulations, at least for sufficiently big specimens (the index 1 of x_1 on Fig. 1 is omitted).

Fig. 2 shows the results of such simulations, performed in the COMSOL environment, applying the finite and infinite element technique with triangular meshing. The upper left scheme shows the initial mesh and applied boundary conditions. Thanks to the linearity of (1), (2), (3), the value of b_* does not affect the isolines on the remaining color graphs. The graph under the scheme highlights large values of ψ near the magnet only; this justifies the application of simplified boundary conditions (far from such phenomena) to computational simulations, but restricts all reasonable measurements of b_* to certain area close to the magnet. The remaining upper graphs demonstrate that the distribution of the size of H (in the standard Euclidean norm) is more transparent in the logarithmic scale. The remaining lower graphs, both in the logarithmic scale, present the particular components of H.

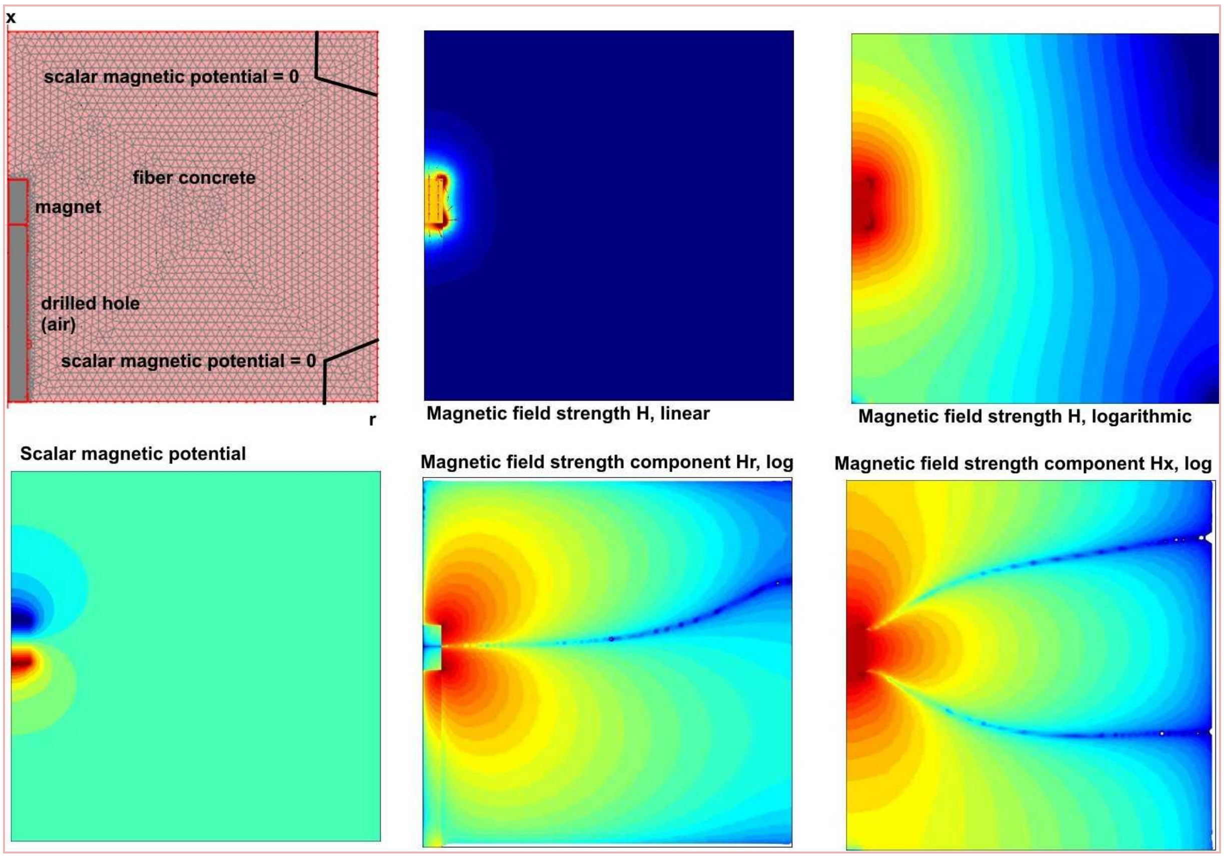

Fig. 2 Results of the computational simulation of the Hall probe in COMSOL.

Although some irregularities visible on Fig. 2 seem to need more detailed explanation (namely of their physical or numerical sources), the most important conclusion is that the Hall probe, located in the drilled hole, as an invasive element in the measurement systems, should not influence the measured quantity b_* substantially. Moreover, these results are in good correlation with those obtained from the original software code referring to several functions taken from the *pde* toolbox of MATLAB, although some slightly modified boundary conditions have been applied, as presented in (Hobst, 2011).

The general differential formulation of the boundary value problem (1), (3) admits an easy conversion to the variational one. Applying the standard notation of Lebesque and Sobolev spaces, for the function space V, introduced as the subspace of $W_1^2(\Omega)$ satisfying all needed homogeneous Dirichlet boundary conditions, thanks to the Green-Ostrogradskiĭ theorem (on integration by parts) we are allowed to rewrite (1) in the form

$$(\nabla\phi,\nabla\psi)-[\phi,\nabla\psi\cdot\nu]=0 \quad \text{for all } \psi\in V\,; \tag{4}$$

$(.,.)$ here denotes the scalar product in the Lebesgue space $L^2(\Omega)$ or $L^2(\Omega)^3$ and $[.,.]$ the scalar product in the Lebesgue space $L^2(\Gamma)$. Inserting (3) into (4), we receive

$$\mu(\nabla\phi,\nabla\psi)=[\phi,b] \quad \text{for all } \psi\in V\,. \tag{5}$$

Working with the well-known magnetic constant $\mu_0=4\pi\cdot10^{-7}$ N/(A·m) together with the new quantity $\gamma:=b/\mu_0$ and with the constant $\lambda:=\mu/\mu_0$ (which is the dimensionless relative effective permeability), we can replace (5) by

$$(\nabla\phi,\nabla\psi) = [\phi,\gamma]/\lambda \quad \text{for all } \psi \in V\,. \tag{6}$$

For some reference (calibration) values (λ_*,γ_*), e.g. those corresponding to the pure matrix without any particles, under the same experimental setting, we have then the very simple relation $\lambda/\lambda_* = \gamma_*/\gamma$ between our values (λ,γ) and (λ_*,γ_*), with no duty to solve (6) numerically at all.

COMPUTATIONAL HOMOGENIZATION

To evaluate the relative permeability λ from that of the matrix λ_c (most frequently concrete and similar materials) and that of the particles λ_s (e.g. steel fibres) is in general a complicated problem, whose solution (without additional simplifications) is not available. For simplicity, let us now suppose that our composite has an exactly an Y-periodic microstructure for the unit cube $Y = \{(x_1,x_2,x_3)\in R^3 : |x_i - 1/2| \le 1/2 \text{ for all } i\in\{1,2,3\}\}$ (the obvious generalization to Y introduced with $a/2$ instead of $1/2$ here for an arbitrary positive a can be left to the reader) and belongs to $L^\infty(\Omega)$, i.e. $\lambda(y)$ is measurable (in the Lebesgue sense) and bounded almost everywhere on Ω and $\lambda(y) = \lambda(y+e_i)$ for every $y\in R^3$ and any $i\in\{1,2,3\}$ where e_i denotes the unit vector corresponding to the i-th Cartesian coordinate. Following Chap. 6.1 of (Cioranescu, 1999), we have to find all solutions $\chi_i \in W_\#^{1,2}(Y)$ (with $i\in\{1,2,3\}$, an index $\#$ highlights the periodicity) of the local problems

$$\int_Y (\nabla\omega(y), \lambda(y)(e_i - \nabla\chi_i(y)))\ \mathrm{d}y = 0 \quad \text{for all } \omega \in W_\#^{1,2}(Y)\,. \tag{7}$$

The result of the homogenization procedure, applied to (7), i.e. that using the two-scale convergence arguments by Chap. 9.2 of (Cioranescu, 1999), is then a matrix with $i,j\in\{1,2,3\}$, expressed, using the notation of mean values $\langle.\rangle$ on Y (whose volume, i.e. the mean value of 1, is equal to 1) and the Kronecker symbol $\delta_{..}$, as

$$\lambda_{ij} = \langle\lambda(y)\rangle\delta_{ij} - \langle\lambda(y)\ \partial\chi_j(y)/\partial y_i\rangle\ . \tag{8}$$

Nevertheless, instead of (5) we have the direction-sensitive limit equation

$$\sum_{i,j=1}^{3} \lambda_{ij}(\partial\phi_i,\partial\psi_j) = [\phi,\beta] \quad \text{for all } \psi\in V\,. \tag{9}$$

The smallest difference between λ_{ij} from (8) and some $\bar\lambda_{ij} = \delta_{ij}\lambda_*$ (with an a priori unknown value λ_*) in an appropriate (usually spectral) matrix norm is then a natural measure of material inhomogeneity.

Since (7) is a linear partial differential equation with periodic boundary conditions, it is seemingly not difficult to obtain a sufficiently accurate numerical approximation of its solution. However, we have simple functions $\lambda(y)$ with quite different values λ_c and λ_s, which can force unpleasant numerical oscillations. (Kristenson, 2003) derives, coming from the equation of the type (9), certain (rather complicated) semi-analytical results for regular spherical inclusions, under some additional assumptions degenerating to the classical Maxwell-Garnett mixing formulae. Other physically motivated approaches to the improvement of this formula, to handle the case of multiple scattering, even for other then spherical particles, have been developed in (Whites, 2000), (Wu, 2001), (Whites, 2002) and (Mallet, 2005). Recently, as a direct generalization of (Kristenson, 2003), (Pieper, 2012) tries to determine a suitable unit cell to represent a non-periodic medium. From the point of view of mathematical homogenization theories, this requires to replace the Lebesgue measures in (7), (8) by some much more general (Borel, Young, Radon, etc.) measures, including probabilistic ones: namely (Nguetseng, 2011) applies the Gelfand representation theory and the deep analysis of spectra in Banach algebras to introduce the so-called weak and strong σ-convergence on (rather abstract) homogenization structures and to derive their properties beyond the periodic and deterministic assumptions, as a (not very transparent) substantial generalization of the two-scale convergence, studied in (Cioranescu, 1999).

Nevertheless, the above mentioned general approaches are not ready to be implemented in efficient numerical calculations, even as elements of more complicated identification algorithms. Such calculations should apply e.g. stochastic finite elements, or, alternatively, Sobol sensitivity analysis with Monte Carlo simulations – cf. (Kala, 2011). Fortunately, for sufficiently low volume fractions ξ of particles, a more simple approximation by (Giordano, 2003) is available, based on the generalization of the Maxwell-Garnett formula for spheres to ellipsoids, making use of the mixture theory for randomly oriented particles, including some limit cases. Just in our case the description of metal fibres as ellipsoids of rotation (to avoid non-smooth boundaries) can be acceptable. The ideal aim seems to be, under some additional assumption on the orientation of particles, to find some explicit monotone and continuous dependence between the effective permeability λ and the volume fraction ξ (which substitutes both the exact description of a periodic structure in (7) and its hypothetical stochastic generalization).

Let σ be some probability density function defined on a unit sphere S (usually a function of an azimuthal and a polar angle). Let us suppose that all ellipsoidal particles have the same size and shape. The crucial relation of Part 3 of (Giordano, 2003) (in our notation, for magnetic fields instead of electric ones), recommended for $\xi << 1$, is

$$\lambda_{ik} = \lambda_c + \xi(\lambda_s - \lambda_c)\hat{\omega}_{ik} \quad \text{for all} \quad i,k \in \{1,2,3\} \tag{10}$$

where $\omega_{ik} := \sum_{j=1}^{3} \gamma_c n_{ji} n_{jk} \,/\, (\lambda_c + L_j(\lambda_s - \lambda_c))$, $\hat{\omega}_{ik}$ is the mean value of $\sigma\omega_{ik}$ evaluated on S, (a_1, a_2, a_3) is the triple of non-increasing positive lengths of axes of all ellipsoidal particles, (n_{j1}, n_{j2}, n_{j3}) for $j \in \{1,2,3\}$ are the components of unit vectors corresponding to these 3 axes (principal directions) and $L_j := (a_1 a_2 a_3 / 2)\int_0^\infty \left((s + a_j)\sqrt{(s+a_1^2)(s+a_2^2)(s+a_3^2)} \right)^{-1} \mathrm{d}s$. To obtain reasonable results for more realistic small values of ξ, some Bruggeman-type approximation procedure should be applied to (10). (Giordano, 2003) works (after more general introductory considerations) with the uniform distribution of σ only, thus the mean value of $n_{ji} n_{jk}$ is equal to $1/3$ for any $j \in \{1,2,3\}$, λ can be used instead of $\lambda_{11} = \lambda_{22} = \lambda_{33}$ (and $\lambda_{12} = \lambda_{13} = \lambda_{23} = 0$), both indices of $\hat{\omega}_{ik}$ can be omitted, too, and the Bruggeman-type relation generates a simple form of one ordinary differential equation

$$\mathrm{d}\lambda / (\lambda_s - \lambda) = \hat{\omega}\, \mathrm{d}\xi / (1 - \xi) \,. \tag{11}$$

Especially for $a_2 = a_3$, $\varsigma := a_1 / a_2$ the analysis of (11) gives

$$\xi = 1 - \frac{\lambda_s - \lambda}{\lambda_s - \lambda_c}\left(\frac{\lambda_c}{\lambda_s}\right)^{3L(1-2L)(2-3L)}\left(\frac{M_1}{M_2}\right)^{2(3L-1)^2/((2-3L)(1+3L))} \qquad (12)$$

where $M_1 := (1+3L)\lambda_c + (2-3L)\lambda_s$ and $M_2 := (1+3L)\lambda + (2-3L)\lambda_s$ contain a factor L, determined from the relation $L := \varsigma(2\varsigma\vartheta + \ln((\varsigma - \vartheta)/(\varsigma + \vartheta))/(4\vartheta^3)$ with $\vartheta := (\varsigma^2 - 1)^{1/2}$. In particular, for a (theoretically) infinite length and zero diameter of particles we receive $L = 1/3$.

However, no simple generalizations of (12) are available. (Giordano, 2003) removes the assumption $a_2 = a_3$; the resulting formula is much more complicated than (12). (Giordano, 2008) extends these results to very special types of coated and graded particles. (Sushko, 2009) generalizes (11) to anisotropic media, even for an arbitrary finite number of types of particles, and sketches the derivation of certain semi-analytical results similar to (12); for non-uniform distributions of σ unpleasant elliptic (and even more complicated) integrals cannot be avoided, which obstructs to get transparent formulae suitable to practical calculations.

IDENTIFICATION OF MATERIAL CHARACTERISTICS

The main requirement from experimental research is to identify basic material characteristics from sufficiently simple measurements. For the sake of simplicity, let us now consider a (nearly) macroscopically homogeneous material with a scalar characteristic λ (and constant σ, not needed here) with $\lambda_c \approx 1$ (no magnetic properties in the matrix can be observed). Let us mention that a similar assumption on λ_s is not reasonable: its value, namely for steel, varies substantially and may be not guaranteed by the producer, although some recommended values like $\lambda_s \approx 1000$ can be found in the literature. Following the discussion under (6), we are able to set λ from γ (thanks to some calibration values) in a very simple way, thus we can interpret λ as measurement results. Thus (12) introduces a real function f of two real variables, namely $\xi = f(\lambda_s, \lambda)$, for a set of couples (ξ_*, λ_*) available from magnetic measurements, corresponding to (ξ, λ). Consequently f can be understood as a function of just one still unknown variable λ_s; in the following text the prime symbol denotes a derivative with respect to λ_s everywhere.

Let H be an appropriate Hilbert space, supplied by a scalar product $\prec .,. \succ$, e.g. $H = R^m$ for some integer m (finite number of measurements) or $H = L^2_w(I)$, introduced as a weighted Lebesgue space on a real interval I, equipped with some weight function w. Then, following the least squares approach, it is reasonable to minimize an error

$$E(\lambda_s) = \prec \xi(\lambda_s) - \xi_*, \xi(\lambda_s) - \xi_* \succ / 2\,. \qquad (13)$$

Differentiating (13), we have easily $E'(\lambda_s) = \prec \xi(\lambda_s) - \xi_*, \xi'(\lambda_s) \succ$; the second differentiation then yields $E''(\lambda_s) = \prec \xi'(\lambda_s), \varepsilon_* \xi'(\lambda_s) \succ + \prec \xi(\lambda_s) - \xi_*, \xi''(\lambda_s) \succ$ where (quite formally) all values (or components) of ε_* in H are equal to 1. The iterative procedure, introduced in general as $\lambda_s^{k+1} = \lambda_s^k - E'(\lambda_s^k)/G^k$ for $k \in \{0,1,2,\dots\}$, is then applicable with the aim $\lambda_s^k \to \lambda_s$ in

R for $k \to \infty$: here $G^k := (F'(\lambda_s^k) - F'(\lambda_s^{k-1}))/(\lambda_s^k - \lambda_s^{k-1})$ for $k \in \{1, 2, \dots\}$ assuming that $F'(\lambda_s^k)F'(\lambda_s^{k-1}) < 0$, applying the regula falsi method (two initial estimates are needed), alternatively $G^k := F''(\lambda_s^k)$ for $k \in \{0, 1, 2, \dots\}$, applying the Newton method (one initial estimate is needed).

If no calibration data are available, at least one additional unknown parameter must be considered in the minimization problem analogous to (13). The more general case $\xi = f(\lambda_c, \lambda_s, \lambda, p)$ for symmetric square matrices λ of order 3 (although λ_c and λ_s are still scalars) and some vectors p of real parameters, needed to introduce σ uniquely, involves several types of difficulties. Firstly, the calibration argument following (6) is not valid, at least in such trivial form: more measurements related to various directions are clearly required. Moreover, ξ depends on several (mutually independent) real variables, thus the global minimization of (13) is a more delicate problem. Finally, a serious obstacle comes from the impracticability of efficient evaluation of f from some algebraic formula like (13).

IILLUSTRATIVE EXAMPLE

As an illustrative example, let us study the identification of volume fraction ξ for the fibre glass composite, whose matrix is prepared from glass crumble compound with acrylate resin; its production technology should guarantee random distribution of applied steel particles of (nearly) cylindrical particles of (approximate) length 0.1 m and radius 0.5 mm. Apart from mechanical, thermal, etc. properties of such composite, one its non-negligible advantage of presentation is the transparency (and certain aesthetic level) of cube specimens, documented by the small photo included in Fig. 3. The mimimization of a least squares error (13) from particular experiments should determine λ_s (neglecting a potential non-zero term $\lambda_c - 1$) and identify the dependency between ξ and γ / γ_* by (12). The residual error by (13) should be explainable from random dispersion of measured values by statistical means, or, alternatively, refer to violation of our assumption, as the macroscopic inhomogeneity in the first place.

Three series of 26 experiments with various distances between permanent magnets in the Hall probe and measurement points (contained in the sensitive region detected by Fig. 2) have been made for 3 cases with guaranteed ξ equal to 0.5%, 1% and 1.5%; γ_* was taken from another experiment with the same material without metal reinforcement. Consequently our choice for (13) is just $H = R^{3\times 26}$ (without any special weights). The original software have been developed in the MATLAB environment to construct the formal (first and second) derivatives E' and E'', whose evaluation is needed in the iterative procedure, using the *symbolic* toolbox of MATLAB (referring to the core of MAPLE); this makes it possible to prepare the software code quite independently of the choice of the specific formula (13), without any additional functions from MATLAB (or other) optimization toolboxes. The method of regula falsi is implemented as a starting one, with the adaptive switch to the Newton method, whose quadratic convergence rate can be expected. The result of an identification procedure is evident from the graph on Fig. 3.

Thanks to the implementation of rather thin and long particles, only a slight nonlinearity in (12), understood as a relation between ξ and λ, occurs here; even the simplification $L = 1/3$ leads to very similar results. However, the Hall probe experiment has not distinguished all results between 3 classes of volume fractions sharply, which is reflected by the numerical identification procedure, too. It is not easy to decide what part of such dispersion of results can be explained by probabilistic considerations and what is caused by macroscopic anisotropy, inhomogeneity or violation of other simplifying assumptions.

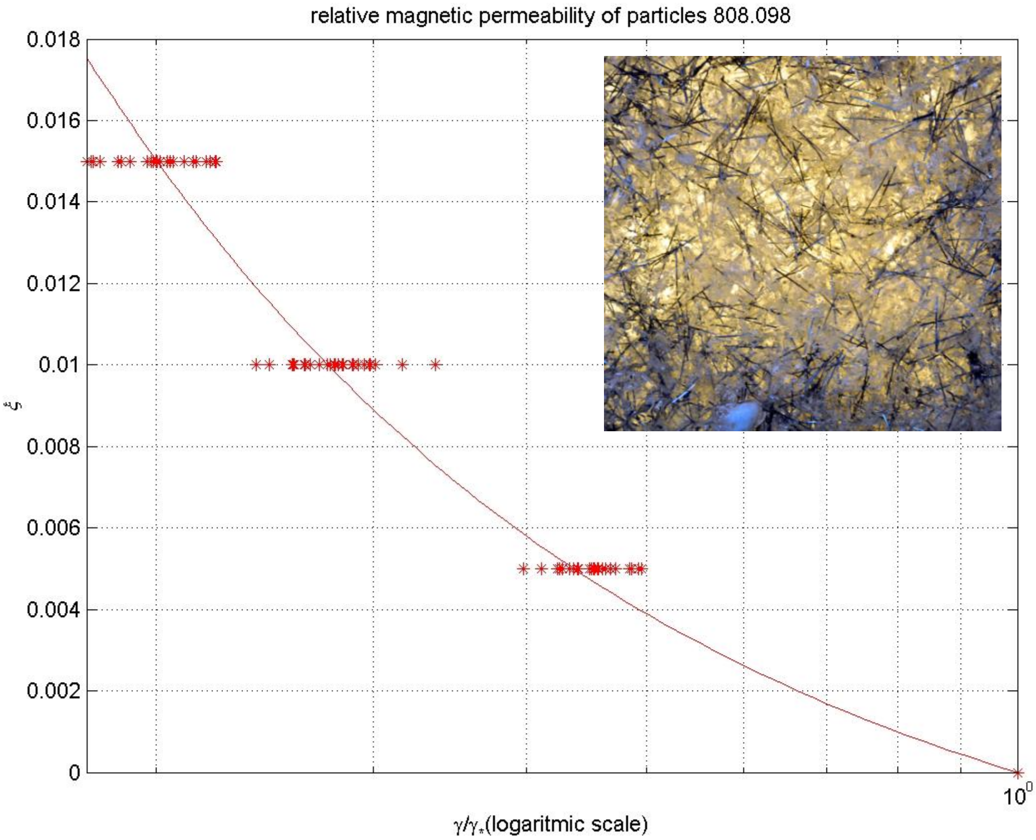

Fig. 3 Volume fraction ξ as a function of γ / γ_* from the identification procedure based on the least squares approach and on the regula falsi and Newton iteration methods in MATLAB. The small photo shows the surface of the tested fibre glass specimen.

CONCLUSIONS

This paper should be understood as an introductory study to the non-destructive or low-invasive approach to the macroscopic identification of content and randomness of location and orientation of small particles in the structure of building materials, making use of their magnetic properties. The crucial point of all such considerations is the development of a homogenization procedure, specific to the analyzed class of materials, including its formal verification and its validity range. This leads to non-trivial problems of both physical and mathematical analysis, uncovered by commercial software tools, whose deeper study is very desirable.

Still other difficulties consist in the needed separation of errors of various kinds, whose rough classification due to their origin could be:

i) random errors generated by the measurement device,

ii) errors caused by the violation of macroscopic material homogeneity and / or isotropy,

iii) errors coming from various physical and mathematical simplifications, connected with the inexact validity of the Laplace equation (5), supplied by very special boundary conditions, as well as with the (above mentioned) conditions for proper (two-scale convergence, mixture theory, etc. motivated) homogenization,

iv) errors brought from disturbing physical, chemical, etc. processes, neglected in our considerations at all, e.g. those needed for the scale bridging by (Steinhauser, 2008) or (Kozák, 2011),

v) numerical errors of the computational tools, due to their finite precision.

Moreover, some requirements to measurement conditions seem to be in contradiction: we need to obtain large values of the magnetic field strength, but sufficiently far from the permanent magnets, moreover no significant magnetic fluxes from the specimen or the real structure into the environment are allowed, etc. The analysis of this type belongs to research priorities of the authors for the near future, namely as a part of the project of the specific university research in the Czech Republic, referenced below.

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