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ANALYSIS OF THE STABILITY AND DESIGN OF CANTILEVERED SIGN SUPPORTS IN HIGHWAYS

Fábio Paiva, Rui C. Barros(*)
Department of Civil Engineering (DEC), University of Porto, Portugal
(*)*Email:* rcb@fe.up.pt

ABSTRACT

This work intends to contribute to a better understanding of overhead sign structures used in highways; moreover similar structures have been designed and constructed by the metallic company Metalogalva (Trofa, Portugal) with which the authors presently collaborate. Therefore the main objective of the present work consists in the analysis of the stability and design of a cantilever support structure in a Highway. The frame studied tends to emphasize one of the more common structures used in Highways, the cantilever support structure.

The overall methodology used when designing to the ultimate limit state (STR) is described. The definition of the actions on the structure and the assumptions adopted in the structural analysis at material and geometrical level are exposed. Simplified methods for checking the cross-section and for insuring the buckling resistance of the members are also appropriately characterized, as defined in the Eurocode 3. It is highlighted specific wind-dynamic problems related with this type of structure (fatigue effects). A design example details all the previous analysis and design methodologies addressed in this paper.

In the conclusions a general review of the work that was carried out is addressed, as well as suggestions for future developments.

Keywords: stability, design, Eurocode 3, overhead sign frames, Highways

INTRODUCTION

This paper specifies criteria and advice according with the Eurocode 3, for the ultimate limit state design of sign cantilever structures, used on highways and all purpose roads, where any part of the sign frames and their supporting structures is mounted over the carriageway and on the hard shoulder (BD 51/98, 1998). Cantilever or non-cantilever structures that contain such highway signs are provided to adequately support directional signs; usually of fixed legend, but occasionally for variable message signs (BD 51/98, 1998).

Minimum requirements are provided or referenced for aesthetics, clearances, constructability, inspection ability, and maintainability of structural supports, as clearly stated at AASHTO 2009 Standard Specifications for Structural Supports for Highways Signs, Luminaires and Traffic Signals (AASHTO, 2009):

- Clear Zone Distance - Where the practical limits of structure costs, type of structures, volume and design speed of through-traffic, and structure arrangement make conformance with the specific national standard impractical, the structural support should be provided with a breakaway device or protected by the use of a guardrail or other barrier;

- Aesthetics - The structural support should complement its surroundings, be graceful yet functional in form, and present an appearance of adequate strength. The support should have a pleasing appearance that is consistent with the aesthetic effect of the highway's other physical features. Supports should have clean, simple lines, which will present minimum hazard to motorists;
- Maintenance - All structural supports should be inspected for the effects of corrosion and fatigue (AASHTO, 2009).

In the structural design of sign supports, there should not only be considered the mechanical strength, stiffness and stability, but also should be taken into account the highway view-scene. If possible, there is a need to coordinate the traffic signs along the highway. Usually, supporting methods of traffic signs divide them into three categories: road-side, cantilever and gantry (Dai et al., 2011). This paper will focus on the design of a cantilever support sign (Fig.1).

The analysis must meet the particularity of the cantilever sign structure being subjected simultaneously to in-plane actions (gravity) and out-of-the-plane actions (wind). That situation originates members (beam-columns) subjected to bi-axial bending and high torsion effects. There is therefore a set of secondary objectives addressed in this paper, related with: the modeling of the structure, the type of analysis performed, the definition of different actions on the structure, and the procedures for checking the structural safety (Paiva, 2009).

Fig.1 Cantilever sign support to be analyzed

BASIS OF STRUCTURAL DESIGN

Requirements

The basic requirements of a structure are to sustain all likely actions and influences, to remain fit for purpose, and to have adequate structural resistance, durability, and serviceability. These requirements must be met for the structure's entire design working life, including construction.

The design working life is the assumed period for which a structure (or part of it) is to be used for its intended purpose with anticipated maintenance, but without major repair being necessary (Eurocode 1990, 2002) as can be seen in Fig. 2.

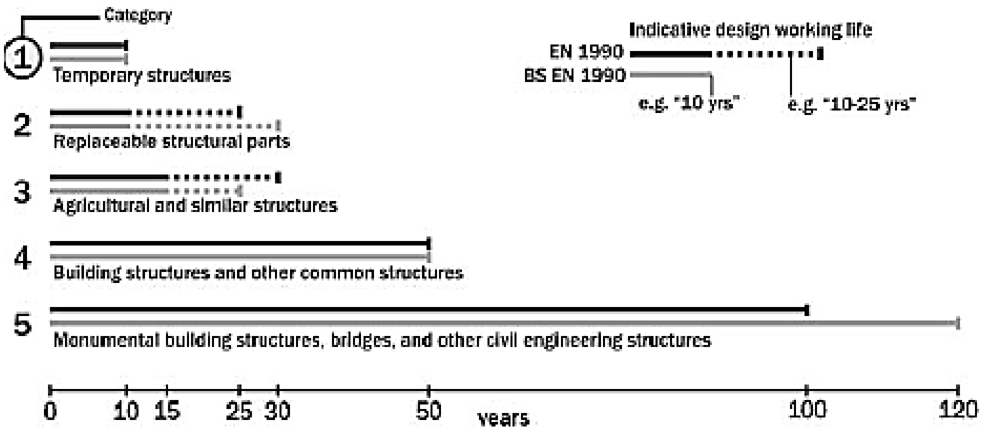


Fig.2 Design working life of various structures (Bond and Harris, 2008)

For the cantilever support a design life of 50 years is considered to be enough for the subsequent structural design.

Principles of limit state design

Limit state design involves verifying that relevant limit states are not exceeded in any specified design situation. Verifications are performed using structural and load models, the details of which are established from three basic variables: actions, material properties, and geometrical data. Actions are classified according to their duration and combined in different proportions for each design situation (Bond and Harris, 2008). Only ultimate limit states will be considered in this paper.

Design situations

Design situations are conditions in which the structure finds itself at different moments in its working life.

In normal use, the structure is in a persistent situation; under temporary conditions, such as when it is being built or repaired, the structure is in a transient situation; under exceptional conditions, such as during a fire, explosion and vehicle collision the structure is in an accidental situation or (if caused by an earthquake) a seismic situation (EN 1990, 2002).

Only a persistent design situation shall be considered, all others design situations are not considered in this work. A reference is made to Part 4 of BD 51/98 (1998) mentioning that when any part of the sign or structure is over the carriageway, or over hard shoulder or hard strip supports within 4.5 m of an edge of the carriageway, these shall be designed to withstand the vehicle collision loads (accidental limit state).

Ultimate limit states

Ultimate limit states (ULS) are concerned with the safety of people and the structure (EN 1990, 2002). The EN 1990 identifies three ULS that must be verified where relevant: loss of equilibrium (EQU); failure by excessive deformation, transformation into a mechanism, rupture, or loss of stability (STR); and failure caused by fatigue or other time-related effects (FAT). As said earlier only the ultimate limit state for the (STR) state is verified in this paper.

Some considerations related with the fatigue limit state in cantilevered and non-cantilevered supports are made. These structures are exposed to several wind phenomena that can produce cyclic loads. Vibrations associated with these cyclic forces can become significant. In fact the NCHRP Report 412 identifies galloping, vortex shedding, natural wind gusts and truck-induced gusts as wind-loading mechanisms that can induce large-amplitude vibrations and/or fatigue damage in cantilevered sign support structures. The amplitude of vibration and resulting stress ranges are increased by the low levels of stiffness and damping possessed by many of these structures.

In some cases, the vibration is only a serviceability problem because motorists cannot clearly see the mast arm attachments or are concerned about passing under the structures. In other cases, where deflections may or may not be considered excessive, the magnitudes of stress ranges induced in these structures have resulted in the development of fatigue cracks at various connection details including the anchor bolts. The wind-loading phenomena possess the greatest potential for creating large-amplitude vibrations in cantilevered support structures. In particular, galloping and vortex shedding are aeroelastic instabilities that typically induce vibrations at the natural frequency of the structure (i.e., resonance). These conditions can lead to fatigue failures in a relatively short period of time (AASHTO, 2009).

ACTIONS

This section specifies only loads considered in the design example present at the end of this paper. Load combinations that are used for the design or structural evaluation of supports for highway signs are also defined.

Dead Load

The dead load shall consist of the weight of the structural support, signs, and any other appurtenances permanently attached to and supported by the structure. No temporary loads during maintenance were considered, as only persistent design situation is studied. The points of application of the weights of the individual items shall be their respective centers of gravity.

Snow Load

This type of action is not considered in this work.

Wind Load

Wind load is the pressure of the wind acting horizontally on the supports, signs, and other attachments computed in accordance with the EN 1991-1-4 (2005).

The wind force $\boldsymbol{F_W}$ acting on a structure or on a structural component may be determined directly by using

$$F_w = c_s c_d * c_f * q_p(z_e) * A_{ref} \tag{1}$$

where

$\boldsymbol{c_s c_d}$ is the structural factor, for simplification will be considered equal to 1.0;

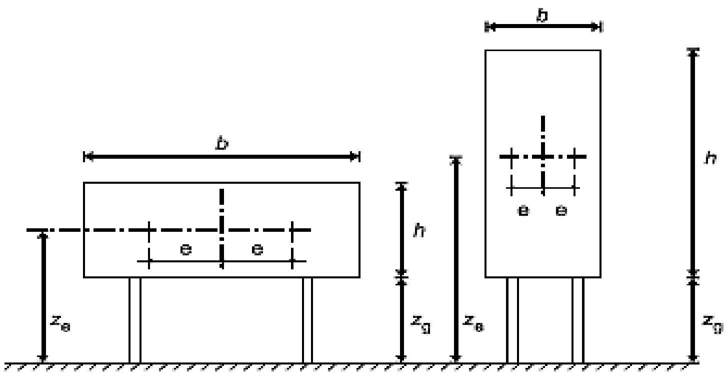
$\boldsymbol{c_f}$ is the force coefficient for the structure or structural element;

$\boldsymbol{q_p(z_e)}$ is the peak velocity pressure defined in 4.5 EN1991-1-4 (2005) at reference height $\boldsymbol{z_e}$;

$\boldsymbol{A_{ref}}$ is the reference area of the structure or structural element.

The determination of the force coefficient for signboards will be detailed accordingly to section 7 of EN 1991-1-4 (2005).

For signboards separated from the ground by a height $\boldsymbol{z_g}$ greater than *h/4* (Fig. 3), the force coefficients are given by $\boldsymbol{c_f = 1.80}$.



NOTE 1 reference height: $z_e = z_g + h/2$

NOTE 2 reference area: $A_{ref} = b \cdot h$

Fig.3 Key for signboards from Eurocode 1991-1-4 (2005)

The resultant force normal to the signboard should be taken to act at the height of the centre of the signboard with a horizontal eccentricity $e = \pm 0{,}25\, b$.

Signboards mounted on slender legs or poles are susceptible to divergence or stall flutter instabilities and should be checked using the rules in Annex E of EN 1991-1-4 (2005). Divergence is a static torsional instability which occurs when the wind moment increases faster than the torsional resistance.

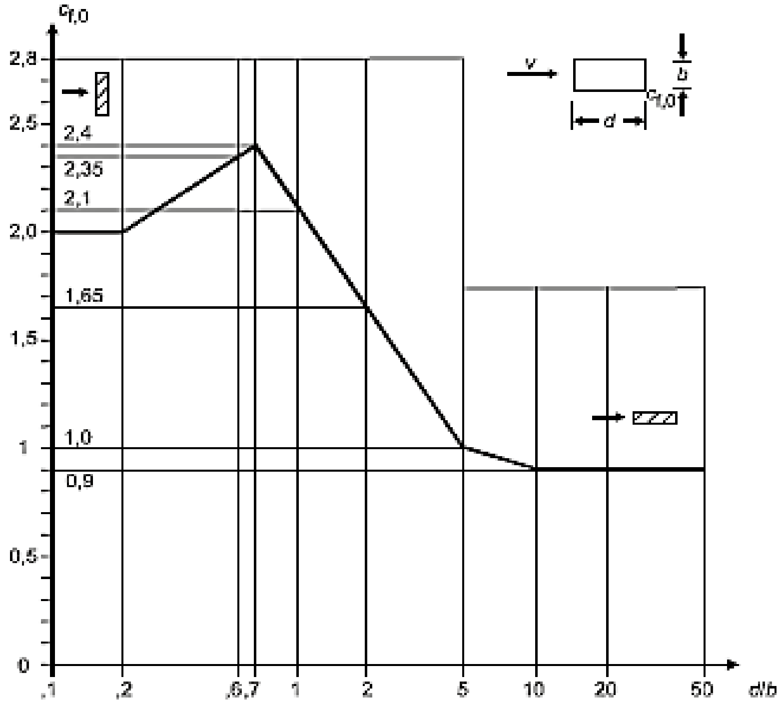
The force coefficient c_f of structural elements of rectangular section, with the wind blowing normally to a face should be determined by:

$$c_f = c_{f,0} \psi_r \psi_\lambda \qquad (2)$$

$c_{f,0}$ is the force coefficient of rectangular sections with sharp corners and without free-end flow as given in figure 4;

ψ_r is the reduction factor for square sections with rounded corners; it depends on Reynolds number;

ψ_λ is the end-effect factor for elements with free-end flow.



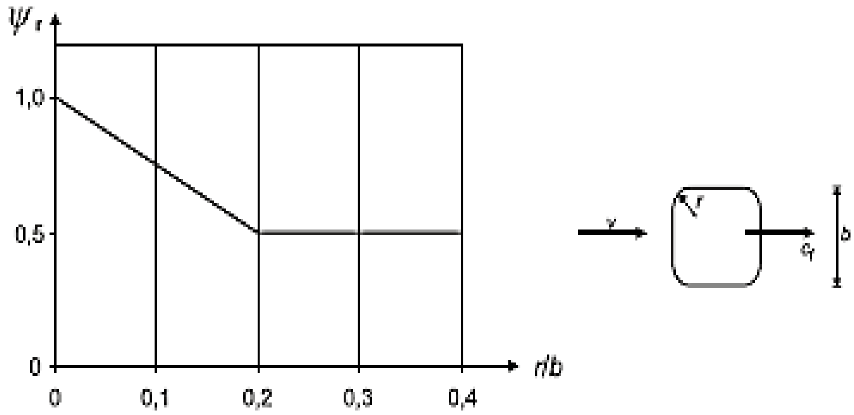


Fig.4 Force coefficients $c_{f,0}$ of rectangular sections with sharp corners and without free end flow (top) and reduction factor ψ_r for a square cross-section with rounded corners (bottom) from Eurocode 1991-1-4 (2005)

The reference area $A_{ref} = l.b$ should be determined, where l is the length of the structural element being considered. The determination of ψ_λ depends on slenderness ratio λ. The value of $\psi_\lambda = 1$ will be considered for safety.

Horizontal supports of sign structures and traffic signal structures shall be designed for wind loads (Fig. 5), W_h (on the structure) and W_p(on the panel), applied normal to the support at the centers of pressure of the respective areas (AASHTO, 2009). The load eccentricity relatively to the center of the signboard is also considered.

The vertical supports for highway sign cantilevered frame structures shall be designed for the effects of wind from any direction. An acceptable method to design such frames for the effects of wind from any direction is by applying the following two load cases of normal and transverse wind loads acting simultaneously.

This method is applicable where all signs are approximately in one plane and is not applicable for structures with arms in two or more planes (Fig. 5).

Table 1 Wind Load cases

Load Case	**Normal component (n_c)**	**Transverse component (t_c)**
1	1.0	0.2
2	0.6	0.3

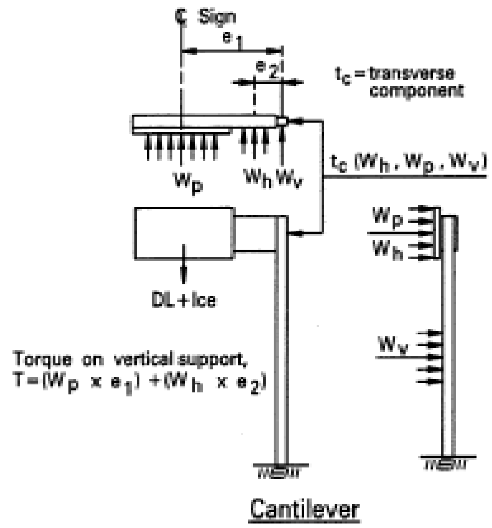


Fig.5 Loads on Sign Cantilever structures from AASHTO (2009)

COMBINATIONS

All ULS load combinations must include the effect (if any) of imperfections from any direction. All actions which can occur at the same time are applied together, as follows:

$$\sum_{j} \gamma_{G,j} G_{k,j} + \gamma_{Q,1} Q_{k,1} + \sum_{i>1} \gamma_{Q,i} \psi_{0,i} Q_{k,i} \tag{3}$$

where $\boldsymbol{\psi_{0,i}}$ is the “combination factor” for the i-th load. The “combination factor” reduces the intensity of loading when several variable loads are considered to be acting together.

Verification by the partial safety factor method

EC3 uses a partial safety factor format for checking ULS, in which the partial safety factors allow for the appropriate uncertainties of resistance, analysis and loads. Expressing this in a simplified way, the partial safety factor format checks that:

$$\left(\frac{specified\ resistance}{\gamma_M}\right) \geq effect\ of\ \sum (specified\ actions \times \gamma_F) \tag{4}$$

The partial safety factors γ_M and γ_F are given as "boxed values", which are currently specified for each country in the National Application Document (Fig. 6).

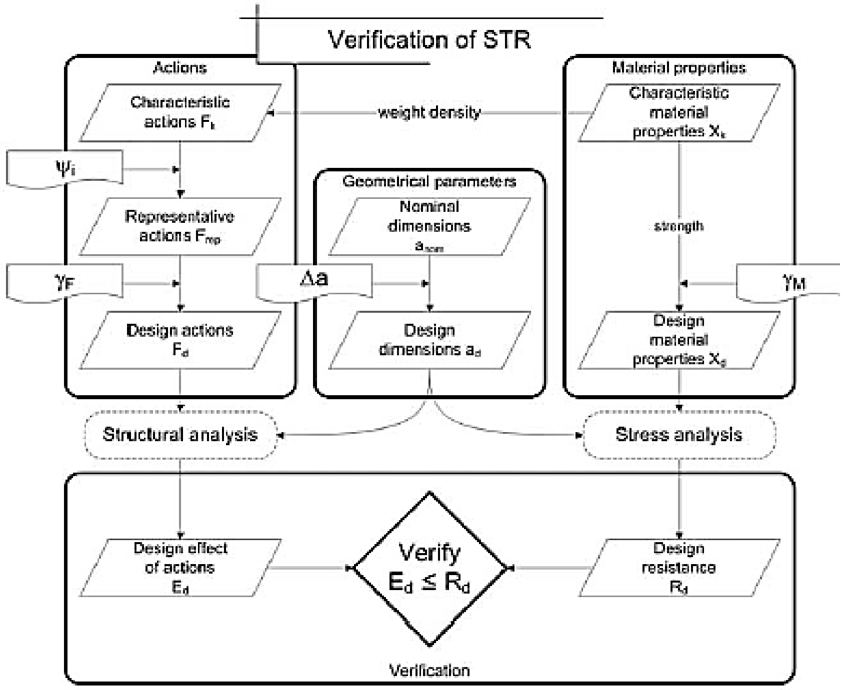


Fig.6 Overview of verification of strength from Bond and Harris (2008)

METHODS OF ANALYSIS

A global analysis is necessary for calculation of the internal forces in the whole structure. The most important distinctions between the various possible analysis methods are:

- Elastic or inelastic
- Geometrically linear or non-linear

Plasticity may be represented either as discrete plastic hinges or as gradually spreading plastic zones. Geometrical non-linearity — often referred to as second order analysis — arises from both the influence of compressive forces in reducing the effective flexural stiffness of individual members ($P-\delta$effect) and the effect of overall frame deformations leading to a magnification of the internal moments ($P-\Delta$effect). Many analyses combine both material and geometrical non-linear effects as a way of more nearly approximating the true ultimate strength (Nethercot, 2000).

Table 2 Main assumptions/behavioral features incorporated in frame analysis (Nethercot, 2000)

Geometry	**Material**	**Joints**	**Additional Effects**
Linear	Linear elastic	Pinned	Column bases
$P-\Delta$	Rigid plastic	Rigid	3-D behaviour
Non-linear	Elastoplastic	Semi-rigid	
Finite displacements	Nonlinear (time, temperature)	Partial strength	

Fig. 7 reproduces the well-known comparison between several different structural analysis approaches. Generally speaking, the more complex the approach the greater the computing power required and the more "fragile" the associated numerical procedures necessary to achieve the solution. Associated with linear analysis is the great benefit that the effects of multiple load cases may be handled using the principle of superposition (Nethercot, 2000).

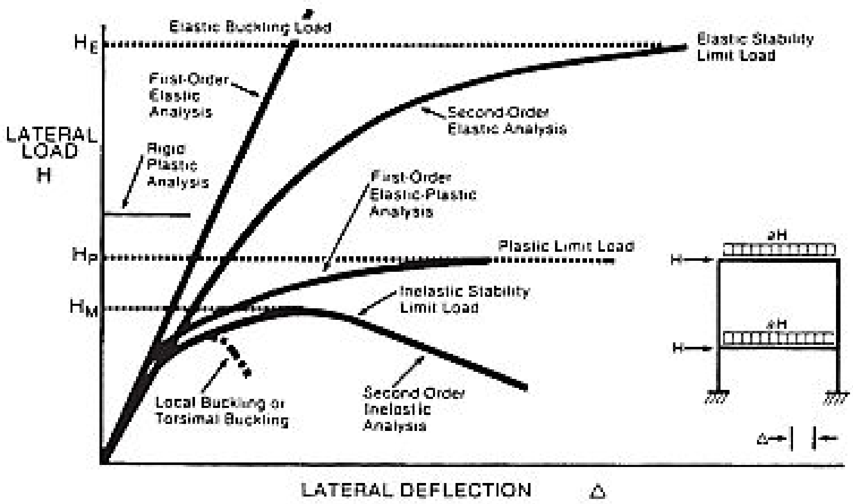


Fig.7 Comparison of analytical models from Nethercot (2000)

Classification — overall response

Availability of the variety of analysis methods outlined in the previous section means that selection of the most appropriate for use in any given situation becomes a serious issue. A guiding principle should be to select the simplest that is consistent with ensuring that all important effects are properly represented. Thus, whilst the full inelastic non-linear analysis might well appear necessary, considerable research effort has been spent in recent years in developing simpler approaches that capture the main behavioral features and/or in defining structural limits within which certain effects are sufficiently subdued that they may either be neglected or their influence considerably simplified (Nethercot, 2000).

Therefore some criteria are required. EC3 deals with this, at the level of the principle to be satisfied, on the basis of the following statement: A frame may be classified as non-sway if its response to in-plane horizontal forces is sufficiently stiff for it to be acceptably accurate to neglect any additional internal forces or moments arising from horizontal displacements of its nodes (Nethercot, 2000). This statement mathematically results in the criteria (Paiva, 2009):

$$\alpha_{cr} = \frac{F_{cr}}{F_{Ed}} \geq 10 \text{ for elastic analysis} \tag{5}$$

$$\alpha_{cr} = \frac{F_{cr}}{F_{Ed}} \geq 15 \text{ for plastic analysis} \tag{6}$$

where,

α_{cr} is the factor by which the design loading would have to be increased to cause elastic instability in a global mode;

F_{Ed} is the design loading on the structure;

F_{cr} is the elastic critical bucking load for global instability mode based on initial elastic stiffnesses.

The problem of linear buckling analysis of a structure is formulated and solved through the problem of eigenvalues. For each required buckling mode, critical load coefficients (eigenvalues) and eigenvectors are determined. The methods of solving the problem are the subspace iteration or the block subspace iteration (Autodesk Robot Structural Analysis, 2013).

The axial buckling load $N_{b,Rd}$ than can be sustained may be considerably lower than the theoretical Euler buckling load N_{cr}. In fact, as an example, the following cases may occur:

- Residual stresses (self-equilibrating) in the member due to the way it has been manufactured can result in first yield, and therefore lateral instability, occurring at a lower axial load.
- Local buckling of the plates that make up the member might occur; it can result in a reduction in the effective stiffness of the member and therefore a reduction in the buckling load. This effect is usually very small.
- If the member forms part of a larger structure, it is possible that some global buckling instability (flexural, lateral-torsional) will occur, before the member reaches its buckling load (Hendy et al., 2011).

Therefore F_{cr} is an entirely theoretical value, which an ordinary structure could not reach because its failure load would be reduced by the onset of yield, which reduces the stiffness of the structure. But F_{cr} is a very useful value for four reasons, even though it is only theoretical:

- It shows the sensitivity to second-order effects by the ratio F_{cr}/F_{Ed};
- It can be calculated with fewer steps than the one's required for a second-order analysis (which must be incremental);
- It allows the calculation of a sufficiently accurate magnification factor to be applied to normal first-order analysis in the majority of cases.
- It reflects the sensitivity to second-order effects for each load case, because the value of F_{cr} depends on the distribution of load on the structure for each load case, i.e. each load case has its own F_{cr} (SCI-P164, 2001).

Classification — connection effects

Recognition of the potentially very important role of connection behavior in influencing the overall response of frames means that the approach taken to determining the distribution of internal moments and forces in the structure should be consistent with the type of connections employed (Nethercot, 2000). The analysis basis procedure for the cantilever support joints was a continuous one (full moment capacity and rotationally rigid).

STRUCTURAL MEMBERS BEHAVIOUR

Structural members may be present as individual members or as part of the planar sub-assemblage of a structure. Individual members are rarely encountered; in practice, they may occur as actual pin-ended columns or simple supported beams only (Lindner, 2000).

They interact with the other members of the structure only in the case that they are loaded by them or transfer their support reaction to them. Therefore, normally all members build part of the overall structure, and a great number of them are beam-columns which are loaded by axial forces and bending moments at the same time. The borderline cases are columns that are loaded by axial forces only and beams that are loaded by bending moments only (Lindner, 2000).

The elements of the frame are treated as beam-columns. Here the influence of the overall system on the internal forces must be accounted for properly. As a result of the loading, the following internal forces and failure modes (Table 3) can be present:

Table 3 Internal forces and possible failure modes for the cantilever support

Internal Forces	**Failure mode**
N axial compression	in-plane flexural buckling
M_y in-plane bending	out-of-plane flexural buckling
M_z out- plane bending	torsional and torsional-flexural buckling
T Torsion	combined lateral-torsional buckling

In the possibility that a hollow section constitutes the cantilever support members, lateral-torsional and torsional-flexural failure modes are prevented by the bi-symmetrically section and the high torsion constant.

Stability verification

According to the type of frame and the global analysis, second order effects and imperfections may be accounted for by the following method:

- Partially by the global analysis and partially through individual stability checks of members according to section 6.3 of EN 1993-1-1 (EC3, 2005).

In the EC3 is provided more than one approach for assessing stability of the elements. The method chosen by the designer relates to various aspects, such as the complexity of the problem to analyze, the formulation of the problem, or the accuracy of results.

These approaches can be divided into three main groups (EC3 paragraph 5.2.2), as shown in Fig. 8, although there is flexibility and intersection between the various types of analysis.

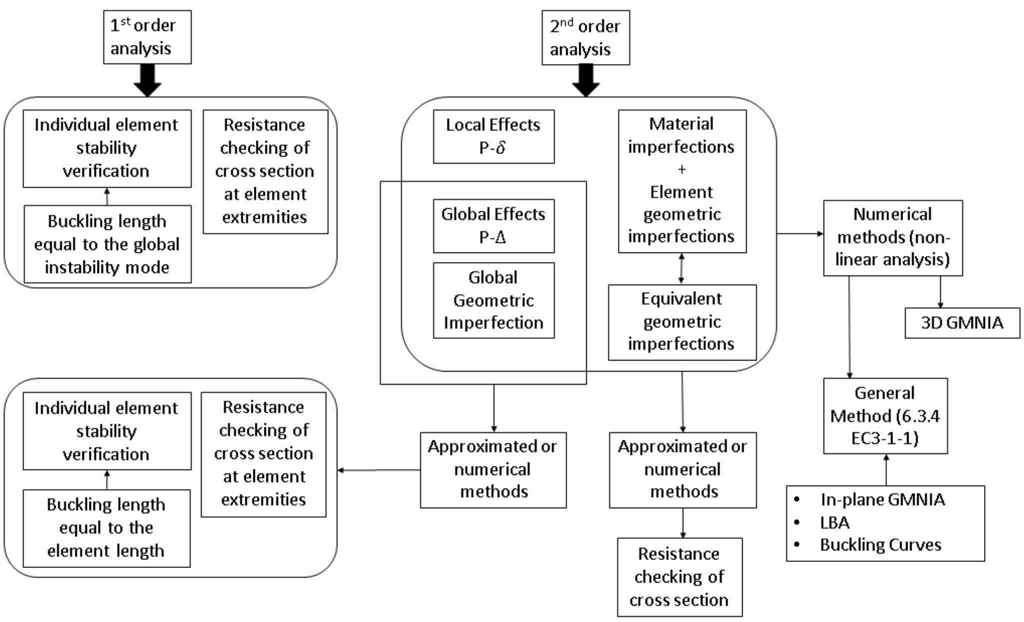


Fig. 8 Schematization of different methods available for element stability verification by EC3 (Marques et al., 2009)

The approach followed in this paper is based in the buckling curves of the EC3:

- Stability checks based upon interaction formulas (15) and (16) in EC3 (EN 1993-1-1, 2005): the most used methodology, but however, applies to cases of simpler structures. This type of verification can be done to first or second order analysis, varying the interaction formulas to be used depending on the type of analysis adopted.

If not otherwise mentioned, it is assumed that: all sections are compact sections; all cross-sections are doubly symmetrical; and the cross-section is constant along the length of the member. The first assumption implies that the full plastic capacity can be reached by the cross-section and that no local plate buckling will occur. Therefore, thin-walled cross-sections need additional or alternative rules. In some cases special rules are necessary if no doubly symmetrical cross-sections are present, like mono-symmetrical sections or angle sections (for which flexural-torsional mode of failure can occur). If the third assumption is not valid, then additional considerations must be taken into account (Lindner, 2000) as indicated in paragraph 6.3.4 of EC3 (EN 1993-1-1, 2005).

If the internal forces are calculated with regard to second-order elastic theory, all unfavorable effects must be taken into account in the analysis. Imperfections will increase the internal forces and must therefore be accounted for. There are two types of imperfection: residual stresses and geometric imperfections, e.g. out-of-straightness, but for reasons of easy application all kinds of imperfection should be taken into consideration by equivalent geometric imperfections. These equivalent geometrical imperfections are calculated from the results of ultimate load calculations, where residual stresses, geometrical imperfections and the effect of plastification along the beam length are included (Lindner, 2003).

Simplified design method

As an example the strength check for a rectangular structural hollow section of uniform thickness, and for welded box sections with equal flanges and equal webs without fastener holes, under axial compression N and bending moments M_y and M_z is described. If full plasticity (class 1 or 2) is allowed to occur, the plastic block diagram leads to the following expressions:

$$M_{N,y,Rd} = M_{pl,y,Rd}(1-n)/(1-0{,}5a_w) \quad \text{but} \quad M_{N,y,Rd} \le M_{pl,y,Rd} \tag{7}$$

$$M_{N,z,Rd} = M_{pl,z,Rd}(1-n)/(1-0{,}5a_f) \quad \text{but} \quad M_{N,z,Rd} \le M_{pl,z,Rd} \tag{8}$$

where

$$N_{pl,Rd} = Af_y \qquad a_w = \frac{A-2bt}{A} \; but \; a_w \le 0{,}5$$

$$M_{pl,Rd} = W_{pl}f_y$$

$$n = N_{Ed}/N_{pl,Rd} \qquad a_f = \frac{A-2ht}{A} \; but \; a_f \le 0{,}5$$

For biaxial bending the following criterion may be used

$$\left[\frac{M_{y,Ed}}{M_{N,y,Rd}}\right]^{\alpha} + \left[\frac{M_{z,Ed}}{M_{N,z,Rd}}\right]^{\beta} \le 1 \tag{9}$$

where for rectangular hollow sections the powers α and β are determined by:

$$\alpha = \beta = \frac{1{,}66}{1-1{,}13n^2} \quad \text{but} \quad \alpha = \beta \le 6$$

Where shear and axial force are present, allowance should be made for the effect of both shear force and axial force on the resisting moment. Provided that the design value of the shear force V_{Ed} does not exceed 50% of the design plastic shear resistance $V_{pl,Rd}$, no reduction of the resistances defined for bending and axial force in paragraph 6.2.9 of EC3 needs to be made; except where shear buckling reduces the section resistance (EN 1993-1-5, 2006).

For members subject to torsion for which distortional deformations may be disregarded the design value of the torsional moment T_{Ed} at each cross-section should satisfy:

$$\frac{T_{Ed}}{T_{Rd}} \leq 1{,}0 \tag{10}$$

$$T_{Ed} = T_{t,Ed} + T_{w,Ed} \tag{11}$$

$T_{t,Ed}$ is the internal St. Venant torsion;

$T_{w,Ed}$ is the internal warping torsion.

As a simplification, for the case of a member with a closed hollow cross-section, such as a structural hollow section, it may be assumed that the effects of torsional warping can be neglected ($T_{w,Ed} = 0$).

For combined shear force and torsional moment, the plastic shear resistance (for structural hollow section) accounting for torsional effects should be reduced from:

$$V_{pl,T,Rd} = \left[1 - \frac{\tau_{t,Ed}}{(\frac{f_y}{\sqrt{3}})/\gamma_{M0}}\right] V_{pl,Rd}\ ;\ V_{pl,Rd} = \frac{A_v(\frac{f_y}{\sqrt{3}})}{\gamma_{M0}} \tag{12}$$

$$\tau_{t,Ed} = \frac{T_{Ed}}{2A_c t} \tag{13}$$

$$\frac{V_{Ed}}{V_{pl,T,Rd}} \leq 1{,}0 \tag{14}$$

where

t is the thickness of the cross-section at the point where the stresses are calculated.

A_c is the area delimited by a line at mid-thickness of each part of the cross-section.

A_v is the shear area.

Buckling resistance of members – Compression and biaxial bending

For members of structural systems the resistance check may be carried out on the basis of the individual single span members regarded as being cut out from the system. Second order effects of the sway system (P-Δ effects) have to be taken into account (end moments of the member).

Members which are subjected to combined bending and axial compression (not susceptible to torsional deformations) should satisfy the two following failure criteria, along the planes x0y (failure y-y) and x0z (failure z-z):

$$failure\ y-y \qquad \frac{N_{Ed}}{\frac{\chi_y N_{Rk}}{\gamma_{M1}}} + k_y \frac{C_{my} M_{y,Ed}}{\frac{\chi_{LT} M_{y,Rk}}{\gamma_{M1}}} + 0{,}6 k_z \frac{C_{mz} M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1 \tag{15}$$

$$failure\ z-z \qquad \frac{N_{Ed}}{\frac{\chi_z N_{Rk}}{\gamma_{M1}}} + 0{,}6 k_y \frac{C_{my} M_{y,Ed}}{\frac{\chi_{LT} M_{y,Rk}}{\gamma_{M1}}} + k_z \frac{C_{mz} M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1 \tag{16}$$

with

$$k_y = 1 + (\overline{\lambda_y} - 0{,}2)\eta_y \leq 1 + 0.8\eta_y \tag{17}$$

$$k_z = 1 + (\bar{\lambda}_z - 0{,}2)\eta_z \leq 1 + 0.8\eta_z \quad \text{for RHS} - \text{profiles} \tag{18}$$

$$\eta_y = \frac{N_{Ed}}{\chi_y N_{Pl,Rd}}; \eta_z = \frac{N_{Ed}}{\chi_z N_{Pl,Rd}} \tag{19}$$

Its advantage is that all failure modes mentioned before are dealt with by the same type of formula, but the parameters are used with regard to the special failure mode being investigated (Lindner, 2003). In addition, simplifications like $\chi_{LT} = 1$ are made for beams with certain types of cross-sections, such as square or circular hollow sections, fabricated circular tubes or square box sections; in fact these sections are not susceptible to lateral-torsional buckling, in face of paragraph 6.3.2.1 (2) of EC3 (EN 1993-1-1, 2005) and also $\Delta M_{Ed} = 0$ (class 1 or 2).

It should be noted that in equation (15) as well as in equation (16), the moment which does not correspond with the plane of failure is reduced by a factor of 0.6. This implies that, in the case of a short member and where no axial force is present, the plastic cross section interaction domain is approximated quite well by this bilinear equation (Lindner, 2003).

Equivalent moment factors C_m are used in order to convert a variable moment distribution into a constant equivalent moment distribution. In the case of linear moment distribution, it is given that $C_m = 0.6 + 0.4\psi \geq 0.4$ which is the Austin formula. For other cases these factors were again recalculated from the ultimate load results, and given in Table B.3 of the EC3 (EN 1993-1-1, 2005). For members with sway mode the equivalent moment factor should be taken as 0.9, if not otherwise calculated more accurately. For a member where the maximum moments of $M_{y,Ed}$ and $M_{z,Ed}$ are not located at the same point, the maximum values of both moments must be used, because variable moments are converted into a constant moment by the C_m factors (Lindner, 2003).

For slenderness $\bar{\lambda} \leq 0.2$ or for $N_{Ed}/N_{cr} \leq 0.04$, the buckling effects may be ignored and only cross sectional checks apply.

DESIGN EXAMPLE

In the following example (Paiva, 2009) a cantilever support will be designed for a persistent design situation, to the ultimate limit state (STR). With this example it is pretended to emphasize the following aspects:

- The susceptibility of these kind of structures to second order effects;
- The advantages of hollow sections inserted in a structure subjected to (low) axial force, combined bending and torsion actions.

The cantilever support represented in Fig. 9 is constituted by a square hollow section 250*250*8 (uniform member) in steel S355. The structure is located somewhere in Porto district (Portugal). The column and the beam have a length of 6.5 m and 6.0 m, respectively. The signboard dimensions are also detailed in the figure.

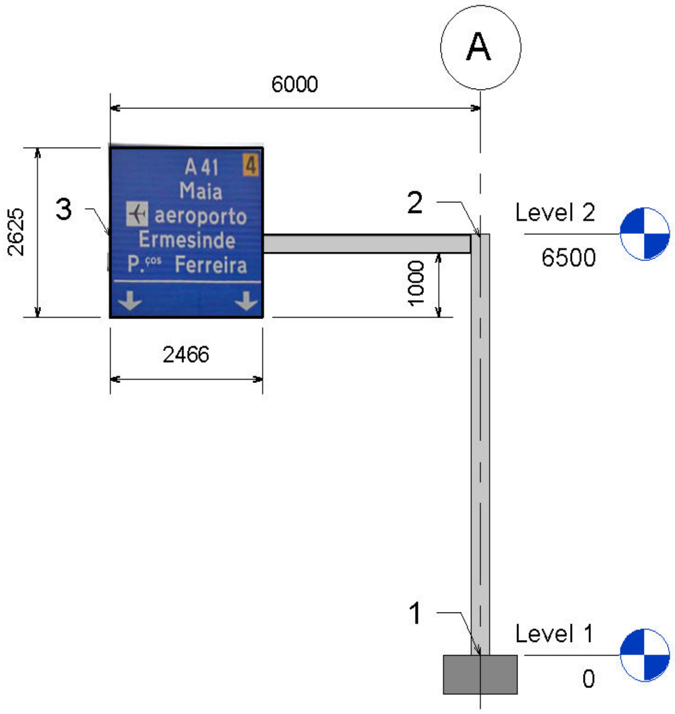


Fig.9 Geometry of the design example of cantilever sign support (dimensions in mm)

Structural analysis

The first step in the analysis is to quantify the permanent and variable actions. The signboard is modeled by vertical bars spaced 0.49 m apart and characterized by a load of 0.35 kN/m^2 that translates its self-weight and other elements connected to the cantilever support.

The wind action on the signboard results in a force of 13.6 kN with an eccentricity of 0.62 m to the left of the signboard center. The imperfections were not considered, because the criteria $H_{Ed} \geq 0.15V_{Ed}$ was verified for every combination of actions. In Table 4 a resume of actions considered in the analysis is presented.

Table 4 Actions in the cantilever sign support

Load Type	Signboard	Beam	Column
Self-weight	0.35 kN/m^2	0.58 kN/m	0.58 kN/m
Wind load	2.11 kN/m^2	0.58 kN/m	0.58 kN/m

The next step is to determine the α_{cr}, and so to check the sensitivity of the structure to second order effects. These will be checked by a simplified equation and by a buckling analysis. For the first method, a static elastic analysis is needed to determine the internal forces on the structure. Taking account the critical combination of actions (wind case 1), a summary of the forces in the nodes (1 and 2 labeled in Fig. 9) of the structure is given in Table 5.

Table 5 Internal forces in the cantilever support for the critical combination

Member	N (kN)	M_y (kN.m)	M_z (kN.m)	T (kN.m)	V_y (kN)	V_z (kN)
Beam (Node 2)	0	127.11	28.84	6.01	7.79	26.07
Column (Node 2)	7.79	6.01	28.84	127.11	6.43	26.07
Column (Node 1)	12.89	195.11	70.61	127.11	4.82	32.12

The critical elastic load of the column can be obtained through the following formula:

$$N_{cr} = \frac{\pi^2 EI}{{l_{cr}}^2} \tag{20}$$

For the present situation the axial force (only self-weight originates axial load) varies along the column from 7.79 kN to 12.89 kN, this means an increment of 60% the axial load at the top. This requires that buckling length l_{cr} has to have into consideration that axial load variation. Taking into account the boundary conditions (fixed at the base and free at the top) and the ratio of N_{min}/N_{max} equal to 0.60, the l_{cr} results in a length of 1.704*6.5=11.08 m (Arguelles et al., 2005). Finally N_{cr} can be determined from (20), which results in 1227.23 kN and α_{cr}= 95.2. The determination of α_{cr} through a buckling analysis results in a value of 91.6, very similar to the simplified method.

The current analysis does not need to take into consideration 2nd order effects, has equation (5) is verified.

Verification

The verification of the strength of the structure will start with the cross-sectional resistance and then the buckling resistance of the members.

Table 5 Resistance forces/moments in the cantilever support

Member	$N_{b,Rd}$ (kN)	$M_{N,y,Rd}$(kN.m)	$M_{N,y,Rd}$ (kN.m)	$V_{y,T,Rd}$ (kN)	$V_{z,T,Rd}$ (kN)
Beam (Node 2)	-	249.57	249.57	749.14	749.14
Column (Node 1)	2023.21	249.57	249.57	261.50	261.50

Table 6 Checking cross-section resistance

Member	$Equation\ (9)$	$Equation\ (14)$
Beam (Node 2)	0.35	0.03
Column (Node 1)	0.79	0.10

Table 7 Checking buckling resistance

Member	$Equation\ (15)$	$Equation\ (16)$
Column	0.87	0.69

As can be seen from the tables above, both the cross-sections and the members verified the conditions of the Eurocode 3 for the ultimate limit state (STR).

CONCLUSIONS

An overall methodology to design cantilever support structures to the ultimate limit state is presented in this paper. The assumptions considered in the analysis and in the design are clearly exposed.

Through the buckling analysis it was concluded that the cantilever support reveals no susceptibility to second order effects. The low axial load installed in the column will not be enough to potentiate a flexural instability mode. Also the high torsion in the column has enormous reducing effect in the shear resistance capacity of the cross section.

Regarding the full design of the structure other ultimate limit states would have to be considered. A distinctive word is given especially to the fatigue limit state and to serviceability requirements. Those limit states usually constrain the connections and members designs by demanding restricted limits to verify. In the case of cantilever structural supports that are susceptible to damaging vibrations and that were not designed for fatigue, they should be equipped with appropriate damping or energy-absorbing devices. Stresses due to fatigue actions (equivalent static wind load) on all components, mechanical fasteners, and weld details should be limited to satisfy the requirements of their respective detail categories within the constant-amplitude fatigue limits. To avoid serviceability problems galloping and truck gust-induced vibration deflections of cantilevered single-arm sign supports should not be excessive. Additionally, permanent camber is usually provided in addition to dead load camber for overhead sign structures.

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