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OPTIMAL PIEZOELECTRIC ACTUATION IN COMPOSITE STRUCTURES

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ABSTRACT

In this paper, dynamics, electromechanical couplings, and control of piezoelectric laminated cylindrical shells and rectangular plates are presented. It is assumed that the piezoelectric layers are distributed on the top and bottom surfaces of the structures. First of all the governing equations and boundary conditions including elastic and piezoelectric couplings are formulated and solutions are derived. Then control of the plate/shells deflections and natural frequencies using high control voltages are studied in order to optimize the structural response.

Keywords: composite plates, composite cylindrical shells, PZT actuators, eigenvalues.

INTRODUCTION

Piezoelectric materials are able to produce a mechanical strain when an electrical field is applied to the piezoelectric actuator – see Fig. 1.

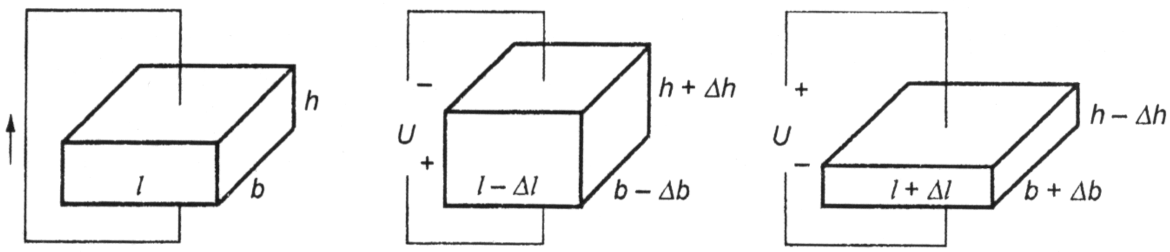

Fig.1 Piezoelectric strains

The converse effect has also been observed, which has led to their use as sensors. Sensors respond to a physical stimulus and transmit a resulting impulse. When a device is actuated by power from one system and supplies power, than it is called as piezoelectric transducer.

Piezoelectric elements are mainly made of polymer or ceramic materials. The values of their material properties are directly connected with materials used- see Table 1. Comparison of the piezoelectric materials may be made roughly using the Table 1. PI Ceramic (www.piceramic.com) provides a wide selection of piezoelectric ceramic materials based on modified Lead Zirconate Titanate (PZT) and Barium Titanate. The material properties are classified according to the EN 50324 European Standard.

Ferroelectric polymers, such as poly vinylidene fluoride (PVDF) and poly vinylidene-co-trifluorethylene (P(VDF-TrFE)) (Nalwa, 1995) are very attractive for many applications because they exhibit good piezoelectric and pyroelectric response, low acoustic impedance, which matches water and human skin, and, moreover, their properties can be tailored to meet various requirements (Zhang, 2002). For applications as dielectric materials, it is desirable to enhance the dielectric constant of these polymers substantially (Zhang, 2002).

Table 1. Materials constants (piezoelectric properties)

Piezoelectric strain constant		PZT-4 type I (piezoelectric ceramic) [4]	PZT polymer [5]	PVDF [6]
Shear mode direction 1	- d31 [pC/N]	123	20-60% vol. (parallel cut): 2-11	Uniaxial film: 22 Bi-axial film: 6 Copolymer: 7-8
Shear mode direction 2	- d32 [pC/N]	Nd	Nd	Uniaxial film: 3 Bi-axial film: 5 Copolymer: 7-8
Shear mode direction 3	d33 [pC/N]	289	20-60% vol. (parallel cut): 4-29	Uniaxial film: 30 Bi-axial film: 30 Copolymer: 33-34

Nd – not determined.

The research concerning optimal design of laminated structures with PZT layers is spread on various areas dependent on the type of structures considered (i.e. beams, plates, shells etc) and on the type of the response of structures subjected to the control (e.g. deformations, buckling, eigenfrequencies etc.). The forms of the objective functions, design variables and optimization algorithms are also different and depend on the particular engineering applications. Shape optimization of actuators / sensors location can be used in different areas, such as:

- deformation analysis,
- buckling analysis,
- free vibration
- flutter.

Crawley, de Luis (Crawley, 1987) presented static and dynamic analytical models for segmented piezoelectric actuators which are compared with the experimental test of cantilevered beams. Then Crawley, Lazarus (Crawley, 1991) extended those analytical models to isotropic and anisotropic plates.

The buckling behavior of adaptive structures requires substantial consideration, while development of new modeling and computational capabilities for analyzing the buckling and postbuckling response of smart structural components is required. Thomson and Loughlan (Thomson, 1995) used piezoelectric actuators to actively control the buckling of composite strips. De Faria and de Almeida (De Faria, 1999) reported a theoretical framework and a finite element for the buckling of beams with a pair of surface attached piezoactuators using classical beam theory, and presented the enhancement of prebuckling behavior of slender beams through piezoelectric control. Franco Correia et al. (Correia, 2000) developed the optimal design of buckling laminated composite plates with integrated piezoelectric actuators using stochastic global optimization technique where the design objective is the maximization of the buckling load of the plate.

Franco Correia et al. (Correia, 2000) presented refined (Lagrangian) finite element models based on higher order displacement fields applied to the optimal design of laminated composite plates with bonded piezoelectric actuators and sensors. The fiber orientation angle in the composite layers and electrical potentials applied to the piezoelectric actuators were treated as the design variables. Next Franco Correia et al. (Correia, 2001) analyzed a rectangular composite plate (3-ply graphite /epoxy). The objective was to optimize the size and placement of the actuator pairs, in order to maximize the performance of the actuation power for maximum deflection. The optimization of the adaptive structure was carried out with three concurrent objectives: maximization of the actuation performance; minimization of the mass of the actuators; and to keep the required actuation energy/voltage below a certain level. Ramos Loja et al. (Ramos Loja, 2002) investigated a family of laminated plate/shell B-spline finite strip models based on higher order displacement fields applied to the optimal design of laminated composite plate/shell structures with embedded and/or surface bonded piezoelectric actuators and sensors. The optimization problem consisted among others in finding the ply angle in the anti-symmetrically disposed substrate layers and the electric field applied respectively to the bottom and top piezoelectric layers, which minimize the plate mid-plane central point deflection. They compared first and higher order models of a simply supported rectangular plate made of an 8-ply substrate graphite/epoxy and two outer surface bonded piezoelectric layers which was submitted to a constant pressure loading. Adali et al. (Adali, 2005) considered a fiber composite plate with initial imperfections and under in-plane compressive loads. The aim of the study was to minimize plate deflection using the piezo actuators and the fiber orientations. Two cases of electric fields, namely, the in-phase and out-of-phase voltages were applied to the actuators bonded symmetrically on the top and bottom of the glass/epoxy composite plate. Moon (Moon, 2006) presented a general finite element formulation of an optimal control scheme such as a linear quadratic regulator (LQR) with output feedback for the nonlinear flutter suppression of a composite panel with piezoelectric actuators and sensors bonded on the top and bottom surfaces. The optimal shape and location of actuators and sensors (employed the genetic algorithm) were determined in order to achieve maximum deflection of the simply supported symmetrically laminated 8-layer square graphite/epoxy composite plate. Chee et al. (Chee, 2002) proposed a heuristic algorithm, named buildup orientation distribution, for the determination of the orientation of piezoelectric actuator patches in the application to optimization shape control of smart structures. They considered sandwich (multi-layered composite plate) made of aluminum honeycomb core sandwich between two graphite/epoxy face sheets and piezoelectric patches. Koconis et al. (Koconis,1994) developed analytical methods for determining the optimum values of the applied electrical fields to fixed rectangular-shaped actuators for achieving the specified shapes for sandwich plates and shells. They employed the basis of mathematical models using two dimensional, linear, shallow shell theory including transverse shear effects which were important in the case of sandwich construction.

Batra and Liang (Batra, 1997) analyzed the steady-state vibrations of a simply-supported rectangular laminated plate with embedded PZT layers using the three-dimensional linear theory of elasticity. Mota Soares et al. (Mota Soares, 1999) studied the mechanical and electrical behavior of laminated composite plate structures with embedded and/or surface bonded piezoelectric actuators and sensors. The objective of the optimization problem is to maximize the fundamental natural frequency of the 6-ply graphite/epoxy skewed plate with two surface bonded pairs of piezoelectric sensor strips. Simoes Moita et al. (Simoes

Moita, 2006) considered a simply-supported square laminated plate integrating piezoelectric actuator and sensor layers or patches, bonded on upper and lower surfaces. They searched for the optimal core lamination sequence, which leaded to the maximum fundamental natural frequency of the plate, by using the simulated annealing optimization method. Roy et al. (Roy, 2009) employed genetic algorithm (GA) in order to investigate optimal vibration control of smart fiber reinforced polymer (FRP) composite shell structures. The optimal placement of actuators using GA and linear quadratic regulator applied for the optimal vibration control of a simply supported smart (spherical and cylindrical) composite shell panel (with piezo-patches) subjected to an impulse applied load was solved in details. Han and Lee (Han, 1999) studied placement of piezoelectric sensors and actuators. They used genetic algorithms to find efficient locations of piezoelectric sensors and actuators of a smart composite plate. A broad review of the used performance indexes that gauge the effectiveness of the optimization process is also presented by Jin et al. (Jin, 2005) and Frecker (Frecker, 2003).

PIEZOELECTRIC SENSOR AND PIEZOCERAMIC ACTUATORS APPLICATIONS

Piezoelectric sensors represent another broad area of piezoelectric applications. They are being used in ultrasonic level measurement, in classical vibration recorders to detect imbalances of rotating machine parts or in crash detectors in the automotive industry and in flow rate measurement applications. There, for example, the propagation time of the reflected echo of an ultrasonic wave is evaluated. Flow rate measurements are based on propagation time measurements or on the Doppler effect (measurement of phase difference). Further typical applications of “soft“ piezoceramics are to be found in object identification and surveillance (e. g. surveillance sensors for cars, glass tampering detectors, etc.), sound transmitters (buzzers) and sound receivers (microphones), to their use in the sound pickups of musical instruments.

The most important applications of the “hard“ piezoceramic materials are for the generation of high-powered ultrasonic waves. The advantages of these PZT materials are piezoelectric high coupling factors, their moderate permittivity, high Q-factors and very good stability under high mechanical loads and operating field strengths. Low dielectric losses facilitate their continuous use in resonance mode with only low intrinsic warming of the component.

Practical examples of their applications can be found in the fields of the machining of materials (ultrasonic welding, bonding, drilling etc.), in ultrasonic cleaning (typically kHz range), in ultrasonic processing (e. g. liquid dispersion), in the medical field (ultrasonic dental scale removal, surgical instruments, etc.), in sonar technology.

Piezoceramic actuators use the effect of the relative change in length when an electric field is applied. They are characterized in particular by high mechanical load capacities (up to 100 [MPa]), extremely low power dissipation or very low energy loss (zero current when not moving), short response times (in the submillisecond range), extremely high motion resolution (in the subnanometer range) and high reliability (more than 1010 switching cycles).

These elements are therefore destined for use in high-tech fields (semi-conductors, telecommunications, optics, etc.), and also, to an increasing extent, in the automotive field (fuel injection systems), for pneumatic valve technology, micropumps and vibration damping.

ACTIVE VIBRATION DAMPING

The damping of undesired vibrations in mechanical structures by means of piezoelectric components can be carried out either actively or passively. These methods are characterized as follows:

- active vibration damping:
  - external power source and control electronics required,
  - application of countermovements in the control loop.
- passive vibration damping:
  - energy conversion in the material itself,
  - the electrical energy generated by the structural vibrations (mechanical energy) in the piezoelectric elements is converted into heat for example, by means of resistors.

In active vibration damping a structure exhibiting undesirable, weakly damped natural resonances is equipped with special actuators and sensors connected in a servoloop. The controller is set up so that in the vicinity of the intrinsic frequencies, the actuator behaves like a high-viscosity damper.

If one integrates piezoelectric elements (also termed adaptive materials), e. g. actuators, in the form of piezoceramic plates or disks, into a structure, it can then be equipped with sensor and actuator functions. With suitable control algorithms, it can then adapt itself to the desired conditions.

The principle consists in exciting vibrations in the piezoelectric actuator by means of an electronic amplifier. Because the actuator is closely coupled with the mass of the assembly to be damped, if the force from the vibration introduced is opposite in phase from the unwanted vibrations, they can be neutralized or minimized.

Piezoelectric actuators, including multilayered elements (e. g. PICMA multilayer actuators), can be used anywhere where precisely dosed periodic counterforces are needed in structures. The applications are currently mainly in the fields of aerospace (e. g. for saving fuel; vibration damping of lattice structures for antennas, etc.), vehicle manufacture (e. g. noise minimization), and also increasingly in mechanical engineering (rotating drives), etc.

FUNDAMENTAL RELATIONS

Let us consider governing equations describing deformations of composite multilayered structures having piezoelectric actuators or sensors. Both for 3-D or 2-D (shells or plates) structures the derivation of fundamental relations starts from the variational Hamilton's principle that can be written in the following form:

$$\delta\int_{t_1}^{t_2}\left(K - P + W_{nc} + P_e\right)dt = 0 \tag{1}$$

$$\text{where: } K = \frac{1}{2}\int_{\Xi}\rho\left[\overset{\bullet}{U}\right]^T\left[\overset{\bullet}{U}\right]d\Xi\,,\quad P = \frac{1}{2}\int_{\Xi}[\varepsilon]^T[\sigma]d\Xi\,,\quad P_e = \frac{1}{2}\int_{\Xi}[D][E]d\Xi \tag{2}$$

The work done by the body forces, surface forces, damping elements and the concentrated forces are clubbed together in the term Wnc. The terms written in the square brackets represent tensors of displacements U, strains ε and stresses σ, respectively. The dot over the symbol U means the differentiation with respect to physical time t. Ξ denotes the space occupied by the analyzed structure:

$$\Xi = \Xi_{Lam} \cup \Xi_{PZT}\,,\quad \Xi_{Lam} = \Omega_{Lam}(x, y) \times \left[-\frac{h}{2}, \frac{h}{2}\right] \tag{3}$$

$$\Xi_{PZT} = \Omega_{PZT}(x, y) \times \left[-\frac{h}{2} - \frac{t_{PZT}}{2}, -\frac{h}{2} \right] \cup \Omega_{PZT}(x, y) \times \left[\frac{h}{2}, \frac{h}{2} + \frac{t_{PZT}}{2} \right]$$

ΩLam denotes the surface area of the composite. ΩPZT is the surface area of the piezoelectric patch, h – the total thickness of the laminated, tPZT – the thickness of piezoelectric layers.

CONSTITUTIVE RELATIONS

Modeling of composite structures having smart piezoelectric actuators or sensors are very similar to that for conventional composite layered structures, however, there is one difference reflected in the constitutive laws in the form of the electromechanical coupling. It affects also the additional complexities in the FE formulation. The constitutive model for a 3-D lamina with embedded piezoelectric sensors/actuators (S/A(s)) is established in the local (fiber) rectangular coordinate system. In the matrix form it is given by:

$$[\hat{\sigma}] = \begin{bmatrix} [\sigma] \\ [D] \end{bmatrix} = [\hat{C}][\varepsilon] = \begin{bmatrix} [C] & [-e] \\ [e]^T & [\mu] \end{bmatrix} \begin{bmatrix} [\varepsilon] \\ [E] \end{bmatrix} \tag{4}$$

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \\ D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 & 0 & 0 & -e_{31} \\ C_{12} & C_{22} & C_{23} 0 & 0 & 0 & 0 & 0 & 0 & -e_{32} \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 & 0 & 0 & -e_{33} \\ 0 & 0 & 0 & C_{44} & 0 & 0 & 0 & -e_{24} & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 & -e_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e_{15} & 0 & \mu_{11} & 0 & 0 \\ 0 & 0 & 0 & e_{24} & 0 & 0 & 0 & \mu_{22} & 0 \\ e_{31} & e_{32} & e_{33} & 0 & 0 & 0 & 0 & 0 & \mu_{33} \end{bmatrix} \times \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{31} \\ \varepsilon_{12} \\ E_1 \\ E_2 \\ E_3 \end{Bmatrix}$$

The symbols having the bar over them have the standard mechanical interpretation, i.e. stresses [σ], a stiffness matrix [C] and strains [ε]. [D] is the vector of electric displacements (three components), [e] is the matrix of piezoelectric coefficients of size 6x3, [μ] is the permittivity matrix of size 3x3 and [E] is the applied electric field in three coordinate directions. The electric field is defined as the gradient of the electric potential Φel, i.e.:

$$[E] = -\text{grad } \Phi_{el}, \quad \{E_x, E_y, E_z\}^T = \left\{ -\frac{\partial \Phi_{el}}{\partial x}, -\frac{\partial \Phi_{el}}{\partial y}, -\frac{\partial \Phi_{el}}{\partial z} \right\}^T \tag{5}$$

There are several different models representing the input electric potential for such a piezoelectric layer. However, the assumed 2D simplification results in the further requirements with respect to the electric potential distribution along the z coordinate. Usually, it is assumed that that function can be written in the following form (Wang, 2001):

$$\Phi_{el} = \begin{cases} \left[1 - \left(\dfrac{2z-h}{2t_{PZT}}\right)^2\right]\varphi(x,y,t) + \dfrac{2z-h}{2t_{PZT}} V(x,y,t) & \dfrac{h}{2} \le z < \dfrac{h}{2} + t_{PZT} \\ \left[1 - \left(\dfrac{2z+h}{2t_{PZT}}\right)^2\right]\varphi(x,y,t) - \dfrac{2z+h}{2t_{PZT}} V(x,y,t) & -\dfrac{h}{2} - t_{PZT} \le z < -\dfrac{h}{2} \end{cases} \tag{6}$$

where the symbol $\varphi(x,y,t)$ denotes in-plane electric field induced by the deformation of the actuator layers on the mid-surface of piezoelectric layer – Fig. 1. They are unknown variables and are derived from the fundamental system of differential equations.

The above constitutive model is then transformed to the global coordinate system using the transformation matrix, which is given by:

$$[T] = \begin{bmatrix} [T_{11}] & [0] \\ [0] & [T_{22}] \end{bmatrix}, \quad [T_{11}] = \begin{bmatrix} c^2 & s^2 & 0 & 0 & 0 & -2sc \\ s^2 & c^2 & 0 & 0 & 0 & 2sc \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c & s & 0 \\ 0 & 0 & 0 & s & c & 0 \\ sc & -sc & 0 & 0 & 0 & c^2 - s^2 \end{bmatrix} \tag{7}$$

$$[T_{22}] = \begin{bmatrix} c^2 & s^2 & 0 \\ s^2 & c^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, s = \sin(\alpha), c = \cos(\alpha)$$

Here α is the fiber orientation of the individual lamina.

For most practical problems piezoelectric materials are located on plated or shell laminated multilayered structures, so that the analysis is reduced to 2-D one. For 2-D analysis, we normally employ a kinematical hypothesis to model plate/shell deformations. Assuming the validity the first order transverse shear hypotheses the 3-D components of displacements may be expressed in the following way:

$$U_1(x,y,z,t) = u(x,y,t) + z\psi_1(x,y,t), \quad U_2(x,y,z,t) = v(x,y,t) + z\psi_2(x,y,t) \tag{8}$$

$$U_3(x,y,z,t) = w(x,y,t)$$

where the comma after the symbol means the differentiation with respect to the variable, and the subscripts 1,2 denote x and y, respectively, whereas 3 correspond to z.

Considering only the geometrically linear theory of shell the relationship between deformation and strains can be represented in the following form:

$$\varepsilon_{11} = \frac{1}{H_1}\left(\frac{\partial U_1}{\partial x} + \frac{1}{H_2}\frac{\partial H_1}{\partial y}U_2 + \frac{1}{H_3}\frac{\partial H_1}{\partial z}U_3\right), \varepsilon_{12} = \frac{H_2}{H_1}\frac{\partial}{\partial x}\left(\frac{U_2}{H_2}\right) + \frac{H_1}{H_2}\frac{\partial}{\partial y}\left(\frac{U_1}{H_1}\right), \tag{9}$$

$$H_\gamma = A_\gamma(1+\frac{z}{R_\gamma}), \qquad H_3 = 1, \qquad \gamma = 1,2.$$

where $R\gamma$ means radius of curvature and $A\gamma$ denotes the Lame parameter. The subscripts 1, 2, 3 are subjected to so-called cyclic symmetry. For cylindrical shells R1 approaches infinity and the second radius is the constant radius (denoted by the symbol R) of cylindrical shell having a circular cross-section, A1=1 and A2=R.

Let us assume that:

$$\varepsilon_{ij}(x,y,z,t) = \ni_{ij}(x,y) + z\kappa_{ij}(x,y), \; \varepsilon_{i3}(x,y,z) = \ni_{i3}(x,y), \quad i,j = 1,2 \tag{10}$$

were components $\ni$ are called membrane strains and κ represent the parameters of the change of curvature.

The electric variables must also satisfy the Maxwell's equations which require that the divergence of the electric flux density vanishes at any point within the piezoelectric layers: $\mathrm{div}(D) = 0$.

OPTIMIZATION OF COMPOSITE STRUCTURES WITH PIEZOELECTRIC TRANSDUCERS

Optimal design of structures having smart sensors and actuators are very similar to conventional structures wherein numerical techniques (understood in the form of optimization algorithms) can be used. However, the optimization has to take care of additional complexities arising due to the material properties of smart materials. These are reflected in the constitutive laws in the form of the electromechanical coupling. From the optimization point of view, these complexities would lead to additional problems, similarly as for multilayered composite structures.

In the sense of so-called "no free lunch theorem" a particular optimization problem may be solved by an appropriate and consistent choice of three independent components, i.e.: 1) a set of design variables, 2) a definition of an objective function and 3) a selection of a proper optimization algorithm.

In general structures with actuators can be optimized using three computationally different strategies. First of all the volume or weight minimization of a structure has been considered here, in which:

$$\underset{\mathbf{s}}{\mathrm{Min}}\ \Xi(\mathbf{s}) \tag{11}$$

The next group of optimization problems is connected with controlling the energy consumed or dissipated by the system. A general form of the energy consumed by the system of piezoelectric actuators can be expressed as:

$$\mathrm{Energy}(\mathbf{s}) = [V]^T [Z][V] \tag{12}$$

where [V] denotes the vector of voltages applied to each individual actuators and [Z] is a positive defined symmetric weighting matrix. Thus the optimal control voltage distribution problem can be written in the following form:

$$\underset{\mathbf{s}}{\text{Min}}\ \text{Energy}(\mathbf{s}) \tag{13}$$

In addition, the most common optimization problem the objective function R to be maximized (minimized) is the response of the system, i.e.:

$$\underset{\mathbf{s}}{\text{Max}}\ R(\mathbf{s}) \quad \text{or} \quad \underset{\mathbf{s}}{\text{Min}}\ R(\mathbf{s}) \tag{14}$$

$$\text{subject to: } s_i^l \le s_i \le s_i^u, \quad i=1,...,ns \quad \text{and} \quad f_j(\mathbf{s}) \le 0, \quad j=1,...,nf \tag{15}$$

where **s** is the vector of design variables, s_i^l and s_i^u are the lower and upper limits of the design variables, respectively, $f_j(\mathbf{s})$ are the nf inequality constraint equations and ns is the total number of design variables. This group of optimization problems can deal with the analysis of structures with some response (denoted by R) requirements.

The response tuning is the next problem. It can be treated as the alternative or sometimes equivalent optimization problem to the above:

$$\underset{\mathbf{s}}{\text{Min}}\left[R_l(\mathbf{s}) - R_{l-1}(\mathbf{s})\right], l = 2,3,... \quad \text{or} \quad \underset{\mathbf{s}}{\text{Max}}\left[R_l(\mathbf{s}) - R_{l-1}(\mathbf{s})\right], l = 2,3,... \tag{16}$$

RESPONSE OF THE SYSTEM - MAXIMIZING DEFLECTION

Piezoelectric materials are characterized as being able to produce a mechanical strain when an electrical field is applied to the piezoelectric actuator. Piezoelectric actuators are desired to strain the host structure in a direction opposite to the strains developing in the host structure. So, deflection of the host structure can be used as the criterion for optimal placement of actuators.

Yang and Zhang (Yang, 2006) considered a simply supported rectangular plate subject to in-plane forces, resting on an elastic foundation and excited by a PZT actuator. The optimal placement of the piezoelectric actuator in terms of maximizing the plate deflection is calculated. It is observed that the optimal locations of the piezoelectric actuator can be represented by the combined position mode function, which is summation of position mode functions of participating modes (Yang, 2006).

Maximum plate deflection at a particular mode (j, k) of deformations can be expressed as a function of the product of two sine functions of actuator position coordinates (Yang, 2006):

$$\left|w_{max}\right| = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\left|A_{jk}\right| \sin^2(j\pi\alpha_0)\sin^2(k\pi\beta_0) \tag{17}$$

where $\alpha_0 = \dfrac{\alpha_1 + \alpha_2}{2}$, $\beta_0 = \dfrac{\beta_1 + \beta_2}{2}$.

Function A_{jk} depends upon plate and piezoelectric actuator material properties. α_1 and α_2 are normalized length coordinates of the sensor and the actuator. β_1 and β_2 denote normalized breadth coordinates of the sensor and the actuator. The product of two sine function in Eq. 14 is named as “position mode function” and can be partially differentiated to find optimal actuator location.

For the shape control of vibration of the plate structure Mota Soares et al. (Mota, 2006) assumed that the objective function can be written as follows

$$R(\mathbf{s}) = \sum_{i}^{np} [\gamma_i - w_i]^2 \tag{18}$$

where np represents the number of discrete nodal points used to control the shape of the plate, γ_i and w_i represent the desired transverse displacement and the actual transverse displacement corresponding to node i, respectively. In this case the design variables in the optimization problem are the electrical voltages that should be applied to the piezoelectric actuators. Now (in Eq. 15¹) the lower s_i^l and upper limits s_i^u of the design variables represent the upper and lower saturation voltages of the actuator, respectively. Next constrains (Eq. 15²) is written as

$$f_j(\mathbf{q},\mathbf{s}) \le 0, \quad j = 1,\ldots,nf \tag{19}$$

where **q** is the displacement vector.

Mota Soares et al. (Mota, 2006) considered the optimization problem of minimizing of the overall mass of the plate where the design variables are the thicknesses of both substrate and piezoceramic layers. A constraint is imposed on the transversal displacement, with a maximum allowable displacement of w_{max} = 1.5 mm and the lower and the upper limits are equal to 0.2 and 1.0 mm, respectively, are introduced to limit the possible variations of the layer thicknesses.

Kang and Tong (Kang, 2008) analyzed the shape control problem, in which the difference between the actual transverse displacements of given nodal points and the desired deformation pattern is to be minimized. A function measuring the shape error is defined as:

$$R = \sum_{i=1}^{nd} r_i^2 \ , \quad r_i = \mathbf{b}_i^T\mathbf{u} - q_i \ , \quad \mathbf{s} = (s_1, s_2, \ldots, s_n)^T \tag{20}$$

$$\mathbf{Ku} = \mathbf{F}^{ext} + \mathbf{F}^{pzt}(\mathbf{s}), \ \ \mathbf{s}^T\mathbf{Q}\mathbf{s} - E_{max} \le 0\ , \ \ s_j \in \{-V, 0, V\}, \quad j = 1,\ldots,n \tag{21}$$

$\mathbf{b}_i$ is a unit vector with the entry corresponding to the i-th concerned displacement equal to one and other entries are equal to zero. q_i denotes the desired nodal displacement value. nd is the number of concerned displacements. n is the number of elements. **K** is the global stiffness matrix. **u** is the nodal displacement vector $\mathbf{F}^{ext}$ and $\mathbf{F}^{pzt}$ are the equivalent force vector contributed by the external loads and the piezoelectric actuation, respectively. **Q** is a positive definite weighting matrix. Eq. 21² represents the constraint on the energy required by the piezoelectric actuation with E_{max} being the prescribed upper limit of the control energy. The applied actuation voltage for each individual element is a tri-level variable and its value can only be chosen from three discrete values –V, 0 and V.

Kang and Tong (Kang, 2008) confined problem as to search an element-wise distribution of actuation voltage that minimized the shape error of the controlled structure under a constraint on the consumed energy. They assumed that the applied voltage was attained by a single-channel voltage source.

RESPONSE OF THE SYSTEM - MAXIMIZING (MINIMIZING) FREE VIBRATIONS

In active vibration control, external source of the energy is utilized to cause deflection of the structure. The equations of motion of smart structure damping with respect to the reduced modal space are written as (Wang, 2001)

$$\{\ddot{\eta}\} + [\Omega]\{\eta\} = [F] + [B][\Phi_a] \quad (22)$$

where $\{\eta\}$ is the modal coordinate vector; $[\Omega]$ is the matrix of eigenvalues; [F] denotes the reduced force vector; [B] is the actuator influence matrix; $[\Phi_a]$ is the actuator voltage vector.

An optimal placement strategy of piezoelectric sensor/actuator pairs for vibration control of laminated composite plate was presented (Quek, 2003), where the active damping effect under a classical control framework was maximized using the finite element approach. The classical direct pattern search method was employed to obtain the local optimum, where two optimization performance indices based on modal and system controllability were studied. The start point for the pattern search was selected based on the maxima of integrated normal strains consistent with the size of the collocated piezoelectric patches used.

The simplest possible optimal performance indicators is based on modal controllability (Ryall, 2001), where the piezoelectric S/A pairs are placed to control a particular mode most effectively. It is assumed that maximizing the active damping ratio of this dominant mode of the transient vibration is effective in maximizing the active damping effect of the global system. The performance index can be written as

$$J = -\eta_i \quad (23)$$

where the active damping ratio of the i-th mode η_i is used to measure the effectiveness in controlling this mode.

A second optimal performance indicator considered in this study is based on the influence matrix of actuators, from the viewpoint of controllability (Sun, 1998). This is in view of the fact that regardless of the control algorithms adopted, a necessary condition for successful vibration control is that the control forces exerted by the piezoelectric actuators should affect all the vibration modes of interest (Ip, 2001). To allow for the maximum controllability of only a few modes up to the total number of the actuators, the indicator can be written as

$$J_2 = -W\left(\left|B_p\right|\right)^2 + \sum_{i=p+1}^{r} W_i \sum_{j=1}^{p} b_{ij}^2 \quad (24)$$

where B_p is the matrix comprising the entries in the first p lines of the influence matrix [B], corresponding to the first p modes; W the weight for the controllability of the first p modes; W_i the weight for the controllability of each of the other retained modes; and b_{ij} the corresponding element of the retained modes in the influence matrix [B], r the number of retained modes.

In practice, the voltages of the S/A pairs cannot exceed their breakdown voltages. This constraint on the input actuator voltages can be combined with Eq. 24. The optimization problem can be expressed in simple terms (Quek, 2003)

$$\min \quad R(\mathbf{z}), \quad R(\mathbf{z}) = R(z_1, z_2, \ldots, z_p) = J_2 \quad (25)$$

$$\text{subject to: } g_i(\mathbf{z}) < 0 \quad i = 1,2,\ldots,p, \qquad g_i(\mathbf{z}) = V_i(\mathbf{z}) - \varphi_i \quad (26)$$

z_j refers to the position of the j-th actuator defined by the x and y coordinates. $V_j(\mathbf{z})$ is the input voltage of the ith actuator and $\boldsymbol{\varphi}_i$ the allowable voltage for this actuator.

Piezoelectric circular unimorph and bimorph composite plates subject to electrical and differential pressure loads were studied by Papila et al. (Papila, 2008). The specific goal of the work (Papila, 2008) is to find the optimal circular plate configuration and to design for maximized volume displacement and bandwidth. The objective of the optimization study is to maximize is the volume displacement ΔVol. The mathematical representation of the optimization problem can be stated as

$$\max_{s=R_1,R_2,t_s,t_p} \Delta\mathrm{Vol}(s)\ ,\quad \Delta\mathrm{Vol} = 2\pi\int w(r)\big|_{p=0} r\mathrm{d}r \tag{27}$$

The design variables specify the composite plate geometry defined by the three radii, as well as the thickness of the shim t_s and piezoceramic patches t_p. In addition, among the three radii, the outer actuator radius R_3 was considered to be fixed at several specified values since applications often place overall size constraints on the actuator. The bounds for the design variables were determined by considering dimensions of commercially available products of piezoceramic composite plate/actuator providers, $s_{LB} \le R_1$, $t_p \le s_{UB}$ and allowable stresses.

RESPONSE OF THE SYSTEM - MAXIMIZING MODAL FORCES

To choose the correct position of the actuator depends on many factors, such as the method of control, or due to vibrational behavior.

Actuators can be so placed such that modal force available to modes of interest is maximized. When a plate is controlled using independent modal space control, modal force applied by an actuator to excite j-th mode depends upon the location of the actuator and is given by (Bin, 2000):

$$\mathrm{Max}\ Q_j(t) = -a_p b_p \frac{h + t_{PZT}}{2} L[\psi_j] V_j(t)\ ,\quad L = e_{31}\left\{\left(\partial^2/\partial x^2\right) + \left(\partial^2/\partial y^2\right)\right\} \tag{28}$$

where a_p – length of piezoelectric patch, b_p – breadth of piezoelectric patch, ψ_j –normalized modal function, $V_j(t)$ – voltage applied on j-th piezoelectric actuator

The operator L is defined for isotropic piezoelectric material. The vector, which gets multiplied by actuator control voltages, that is $L[\psi_j]$, is maximized to achieve maximum modal force objective. Optimal location thus obtained is where the sum of modal strains in x- and y-direction is maximum.

In general, the design variables can be divided into two groups: a) physical (material) variables produced by electro-mechanical properties characterized by a set of material constants in Eq. 4 – they have a great influence on values of terms in the stiffness matrix and b) topological (geometrical) variables describing distributions and geometrical dimensions of sensors/actuators located on host structures, i.e. understood in the sense of design variables associated with a structural geometry of constructions.

The physical response to external forcing performed by piezoelectric transducers depends on the laminate configuration. In this sense, there are 2N optimization variables, which are the orientations in each layer and the thickness of the individual layers of the laminate. The total number of design variables can be reduced to 12 independent parameters by introducing the so-called laminate parameters (Miki, 1986, Fukunaga, 1999). It is possible to further reduce design variables, depending on the present configuration of the laminate - for example, see Muc et al. (Muc, 2010, Muc, 2011, Muc, 2012, Kędziora, 2012).

Through these variables is defined location and geometrical dimensions piezoelectric transducers attached (bonded) to the analyzed structure.

The piezoelectric sensors/actuators have to be of suitable size and placement to ensure maximum effectiveness and efficiency. The problem of finding the optimal size and location of S/As is very challenging. The issues of sensor/actuator location and geometry, and their optimal selections with respect to certain performance criteria, have drawn much attention due to their importance in structural sensing and control.

Since in the case of material design variables (the case a) an arbitrary optimization problem is reduced rather to a parametric study, the further analysis will be limited to the application of topological design variables (the case b). Let us assume that piezoelectric patches are located symmetrically with respect to the host structure in the global normal direction denoted by the symbol z. Let us note that such o definition of topological design variables is different than commonly used in the literature.

On the upper/lower surface of the host structure piezoelectric S/As form an area closed by a curve Γ. That curve approximates boundaries of individual rectangular S/As distributed over the area Ω_{PZT}. In this way it is also possible to estimate the total number of individual S/As covering the area Ω_{PZT} – it is denoted by the symbol NP. In our opinion it is much better (from the optimization point of view) to use the curve Γ instead of the total number of individual rectangles since it is much more convenient to evaluate numerically different forms of curves and then insert them into the optimization algorithm. The boundary curve Γ can be easily build with the use of the classical B-spline functions and a finite number of key-points. In this way the curve is represented by a sequence of the key-point positions $\{P_0, P_1, \ldots, P_I\}$. It is worth to mention that it is possible to construct the family of curves fulfilling various additional constraints, e.g. the curve convexity or concavity. Fig. 2a and 2b shows the example of the geometrical construction (2a) of the convex curve Γ (2b).

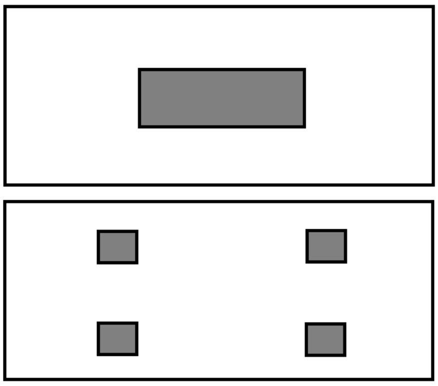

Fig.2a Location and geometrical dimensions of the piezoelectric actuators

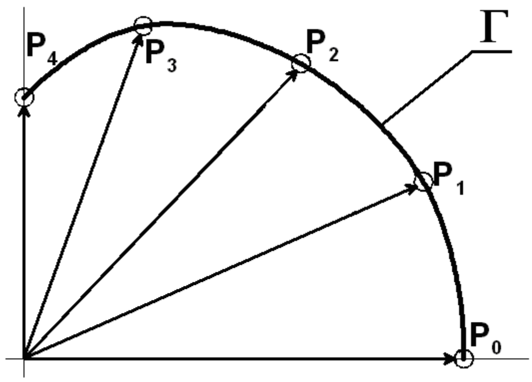

Fig.2b The geometrical construction of the convex curve Γ

For actuators the additional set of topological design variables may be introduced in the form of a voltage applied to each individual rectangular area in the thickness (normal z) direction. Finally, the total number of topological design variables is composed of I+1+NP real numbers where the first I are the numbers of key-points, and the rest NP are voltages applied to individual rectangular piezoelectric patches. If the applied voltages are equal then NP is assumed to be equal to zero.

Physical (material) variables are associated with electro-mechanical properties of the analyzed structure with piezoelectric transducers. Material properties depend inter alia on the structure, chemical composition, the process of manufacture.

Composite materials are formed from the combination of two or more components having different physical and chemical properties. There are two categories of components: matrix and reinforcement. The matrix material surrounds the reinforcement materials. The matrix polymer, ceramic or metal is used in the composite materials.

Piezoelectric materials are usually attached to the structure in the form of an additional layer or are embedded in the structure. In this case, the structure is composed of three components: the matrix, reinforcement and the piezoelectric material.

Often, in the analysis of such structures (e.g. Eq. 2) is used in the stiffness matrix [C]. The Young and Kirchoff modules and Poisson's ratios are included in elements of this matrix. In the case of isotropic materials it can be changed to other. Using composite materials it can also changed the component values of the stiffness matrix [C] by changing the direction of the fibers in the layer.

For the piezoelectric transducers generally, beside the stiffness matrix [C] it is used the matrix of piezoelectric coefficients [e], the permittivity matrix [μ] and the applied electric field [E] in three coordinate directions. The electric field is the gradient of the electric potential and will be expressed as the voltage divided by the layer thickness. It is assumed also that the electric field is applied through the thickness z direction only.

CONCLUSIONS

The optimization analysis of composite structures with piezoelectric layers has been presented. Optimal placements of piezoelectric actuators on structures depend upon the optimization criterion.

The following optimization problems can be taken into account:

- materials having different physical and chemical properties,
- change of the mechanical properties of the individual layers (by changing of the fiber direction),
- change of the volume fraction of the reinforcing material,
- selection of a unit cell used to model homogeneous mechanical properties - such as the shape of the reinforcement in the cell,
- change of the electric field [E].

Transverse deflection of the host structure is a function of actuator placement and can be used as criterion for optimal placement of actuators. Optimal position of actuator is where the system’s strain value is highest. Highest strain value corresponds to the position where structure’s curvature is highest.

Piezoelectric actuators are desired to strain the host structure in a direction opposite to the strains developing in the host structure. Piezoelectric actuators should be placed in the regions of high average strains and away from the areas of zero strain. Actuators can be so placed such that modal force available to modes of interest is maximized.

The shape of system of piezoelectric transducers, shape and number of PZT influence the quality and accuracy of detection of defects in the test of composite structures. For the actuator/actuator configuration, it was shown that the piezoelectric actuators induce piezoelectric forces produced by applying voltages. They significantly affect deformations and eigenfrequencies of the composite structure. When a voltage is applied, normal deflections decreases, however, it was demonstrated that the proper choice of the actuator area is more efficient in reducing deflections/eigenfrequencies. The effects of the fiber orientation, their material properties and their influence on the optimal design are also discussed.

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