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MATHEMATICAL MODELING OF A HIGH-SPEED DEFORMATION OF A VISCOPLASTIC PLATE

Volodymyr Hutsaylyuk[1(*)], Heorhiy Sulym[2], Iaroslav Pasternak[3], Igor Turchyn[4]
[1] Military University of Technology, Warsaw, Poland
[2] Bialystok University of Technology, Bialystok, Poland
[3] Lutsk National Technical University, Lutsk, Ukraine
[4] Ivan Franko National University of Lviv, Ukraine
[(*)] *Email:* vhutsaylyuk@wat.edu.pl

ABSTRACT

This study considers dynamic problem for deformation of a piecewise-homogeneous half-strip, which models experimental process of high-speed deformation of viscoplastic materials. Using the developed solution of corresponding elastic problem and the obtained experimental data, the input parameters of the problem are selected, which characterize loading rate and dissipation of the strain energy. Using these parameters the analysis is held for the influence of equipment inertia and external loading rate on the change in viscoplastic properties of the alluminium alloy 2024-T3 plate during the deformation process.

Keywords: elastic-plastic deformation, strain rate, viscoplastic material.

INTRODUCTION

Determination of the dynamic properties of materials, especially those related to their strength and ability to retain the shape, is a vital problem for modern engineering studies, especially those in the aircraft industry and airspace engineering (Meyers, 1994). Numerous experimental studies have revealed a number of peculiarities in the dynamic behavior of the materials at the yield point, which had not been observed under the quasi-static loading. One of the main peculiarities of the dynamic deformation is the influence of the strain rate on mechanical properties of the material. It is observed, that under the high-speed or impact loading the yield point of most of the structural materials is reached at essentially higher levels of the internal stress, comparing to slow loading.

Numerous studies have shown that the main reason of this behavior is caused by the sensitivity of the material to the strain rate. Several experimental studies allowed to plot and approximate the dependence of the yield stress on the strain rate for various materials. The majority of the materials used in modern structural elements possess mainly the same yield stress – strain rate dependence, which is plotted in Fig. 1.

Presented results feature the presence of three characteristic intervals of the strain rate: rapid growth of the yield stress (II) and its quasi-constant behavior (I, III). Moreover, the increase in the yield stress for the majority of structural materials is within 80-100%.

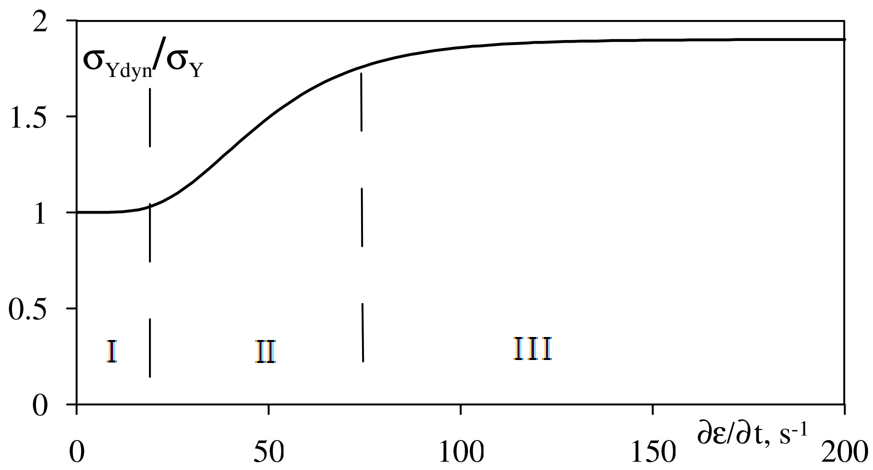

Fig.1. Dependence of the yield stress on the strain rate

Therefore, the processes, which feature intensive change in the strain rate due to the technological conditions or other external and internal factors, can be accompanied with the effects, which are not inherent to the classical understanding of material strength (Chausov and Pylypenko, 2005).

RESULTS

It is known (Achenbach, 1973) that in dynamic problems for finite solids the rate of external loading is crucial both for process identification (dynamic, quasi-static) and for determination of the intensity of transient stress-strain state and the strain rate. However, the studies of the authors (Hutsaylyuk et al., 2012) show that the value of strain rate is constant for the considered problem; therefore, the yield stress is constant too. This behavior is explained by the fact that this problem does not consider possible dissipation of strain energy, or its transformation. One of the ways to identify the dissipation of the elastic-plastic strain energy is to account for the inertia of measurement equipment, or in other words, to account for a perfect mechanical contact of a plate with a massive elastic medium during the deformation process.

Due to the above-mentioned, this study considers dynamic elastic-plastic problem for a rectangular plate, which is perfectly bonded with a half-strip made of other material (Fig. 2).

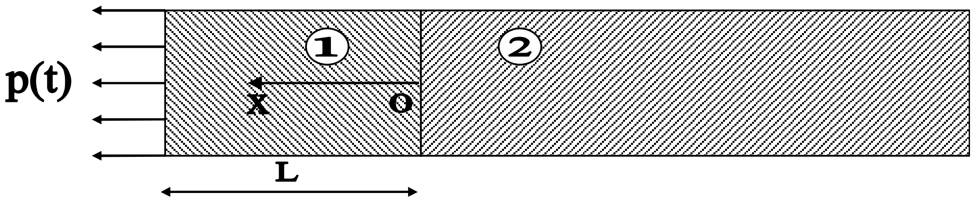

Fig. 2. The sketch of the problem

The source of transient processes in a plate is the loading of its edge with time-dependent traction $p(t)$.

The mathematical formulation of the problem lays in the determination of the solution of the initial-boundary value problem for the displacement function $u^{(i)}(x,\tau)$ in the plate $(i=1)$ and in the half-strip $(i=2)$, which consists of

- the equation of motion

$$\frac{\partial^2 u^{(1)}}{\partial x^2} = \frac{\partial^2 u^{(1)}}{\partial \tau^2},\ x \in [\,0;1\,],\quad \frac{\partial^2 u^{(2)}}{\partial x^2} = \tilde{c}^2 \frac{\partial^2 u^{(2)}}{\partial \tau^2},\ x \in (\,-\infty;0\,]\,; \tag{1}$$

- the boundary condition

$$x=1:\ \sigma_{xx}^{(1)} \equiv (\,\lambda^{(1)} + 2\mu^{(1)}\,)\frac{\partial u^{(1)}}{\partial x} = p(\,\tau\,)\,; \tag{2}$$

- coupling conditions

$$x=0:\ u^{(1)} = u^{(2)};\ (\,\lambda^{(1)} + 2\mu^{(1)}\,)\frac{\partial u^{(1)}}{\partial x} = (\,\lambda^{(2)} + 2\mu^{(2)}\,)\frac{\partial u^{(2)}}{\partial x}\,; \tag{3}$$

- Sommerfeld radiation condition

$$\lim_{x \to -\infty} u^{(2)} = 0 \tag{4}$$

- and initial conditions

$$u(\,x,0\,) = \dot{u}(\,x,0\,) = 0\,. \tag{5}$$

The following notation is used in Eqs. (1)-(5): $x = x_1 / L,\ \tau = c_1^{(1)} t / L^2$ are dimensionless coordinate and time; $c_1^{(i)} = \sqrt{\dfrac{\lambda^{(i)} + 2\mu^{(i)}}{\rho^{(i)}}}$, $i = 1,2$ are phase velocities of a plate and a half-strip, $\lambda^{(i)},\ \mu^{(i)}, \rho^{(i)}$ are their Lame constants and densities; $\tilde{c} = c_1^{(1)} / c_1^{(2)}$.

Using the Laplace integral transform (Sneddon, 1951) for a time variable, the following solution of the problem (1)-(5) is obtained in the transformed domain

$$\overline{u}^{(1)}(\,x,s\,) = \frac{\overline{p}(\,s\,)\left(\tilde{\mu}\,sinh(\,sx\,) + cosh(\,sx\,)\right)}{\left(\lambda^{(1)} + 2\mu^{(1)}\right)s\left(sinh(\,s\,) + \tilde{\mu}\,cosh(\,s\,)\right)},$$

$$\overline{u}^{(2)}(\,x,s\,) = \frac{\overline{p}(\,s\,)exp(\,s\tilde{c}x\,)}{\left(\lambda^{(1)} + 2\mu^{(1)}\right)s\left(sinh(\,s\,) + \tilde{\mu}\,cosh(\,s\,)\right)}, \tag{6}$$

where $\tilde{\mu} = \sqrt{\dfrac{\left(\lambda^{(2)} + 2\mu^{(2)}\right)\rho^{(2)}}{\left(\lambda^{(1)} + 2\mu^{(1)}\right)\rho^{(1)}}}$ and s is the Laplace transform parameter.

Inverse Laplace transform of Eq. (6) is obtained using the partial fraction expansions theorem (Sneddon, 1951). In particular, for the component $\varepsilon_{xx}^{(1)}(\,x,\tau\,)$ of the strain tensor the following equation is obtained

$$\varepsilon_{xx}^{(1)}(\,x,\tau\,) = \frac{1}{\lambda^{(1)} + 2\mu^{(1)}}\int_0^{\tau} p(\,\tau - t\,)g(\,x,t\,)dt\,, \tag{7}$$

$$g(\,x,t\,) = \frac{2\,exp(\,\alpha t\,)}{\sqrt{\tilde{\mu}^2 - 1}}\Big\{\left[\tilde{\mu}\,cosh(\,\alpha x\,) + sinh(\,\alpha x\,)\right]\sum_{k=0}^{\infty}(\,-1\,)^k\,cos(\,\beta_k x\,)\,sin(\,\beta_k t\,) +$$

$$+\left[\tilde{\mu}\,sinh(\,\alpha x\,) + cosh(\,\alpha x\,)\right]\sum_{k=0}^{\infty}(\,-1\,)^k\,sin(\,\beta_k x\,)cos(\,\beta_k t\,)\Big\},\ \alpha = arctanh(\,-\tilde{\mu}^{-1}\,)\,. \tag{8}$$

In instance, for the step-like loading $p(\tau)=p^{*}S_{+}(\tau)$ one can obtain

$$\varepsilon_{xx}^{(1)}(x,t)=\frac{2p^{*}}{\left(\lambda^{(1)}+2\mu^{(1)}\right)\sqrt{\tilde{\mu}^{2}-1}}\Big\{\left[\tilde{\mu}\cosh(\alpha x)+\sinh(\alpha x)\right]\sum_{k=0}^{\infty}\frac{(-1)^{k}}{\alpha^{2}+\beta_{k}^{2}}\cos(\beta_{k}x)\times$$
$$\times\left\langle\beta_{k}+\exp(\alpha\tau)\left[\alpha\sin(\beta_{k}\tau)-\beta_{k}\cos(\beta_{k}\tau)\right]\right\rangle+$$
$$+\left[\tilde{\mu}\sinh(\alpha x)+\cosh(\alpha x)\right]\sum_{k=0}^{\infty}\frac{(-1)^{k}}{\alpha^{2}+\beta_{k}^{2}}\sin(\beta_{k}x)\left\langle\exp(\alpha\tau)\left[\alpha\cos(\beta_{k}\tau)-\beta_{k}\sin(\beta_{k}\tau)\right]-\alpha\right\rangle\Big\}$$

In practice, dynamic impact loading is always continuous in time, and increase faster or slower from zero value to the limiting one. Therefore, the high-speed increase in load is approximated with the relation $p(t)=p^{*}(1-\exp(-at))^{2}$, which can be expressed in terms of the dimensionless time τ as follows

$$p(\tau)=p^{*}(1-\exp(-\tau_{0}\tau))^{2} \tag{9}$$

where $\tau_{0}=\left(L\cdot a\right)/c_{1}^{(1)}$. This relation allows agreeing initial and boundary conditions, and also it allows accurate approximation of the real dependence of dynamic load on time in many practically important cases.

For the selected time-dependence of loading the deformations inside the plate are defined by the following relations

$$\varepsilon_{xx}^{(1)}(x,t)=\frac{2p^{*}}{\left(\lambda^{(1)}+2\mu^{(1)}\right)\sqrt{\tilde{\mu}^{2}-1}}\Bigg\{\left[\tilde{\mu}\cosh(\alpha x)+\sinh(\alpha x)\right]\sum_{k=0}^{\infty}(-1)^{k}\cos(\beta_{k}x)\times$$
$$\times\Bigg(\frac{\beta_{k}+\exp(\alpha\tau)\left[\alpha\sin(\beta_{k}\tau)-\beta_{k}\cos(\beta_{k}\tau)\right]}{\alpha^{2}+\beta_{k}^{2}}-$$
$$-2\frac{\beta_{k}\exp(-\tau_{0}\tau)+\exp(\alpha\tau)\left[\left(\alpha+\tau_{0}\right)\sin(\beta_{k}\tau)-\beta_{k}\cos(\beta_{k}\tau)\right]}{\left(\alpha+\tau_{0}\right)^{2}+\beta_{k}^{2}}+$$
$$+\frac{\beta_{k}\exp(-2\tau_{0}\tau)+\exp(\alpha\tau)\left[\left(\alpha+2\tau_{0}\right)\sin(\beta_{k}\tau)-\beta_{k}\cos(\beta_{k}\tau)\right]}{\left(\alpha+2\tau_{0}\right)^{2}+\beta_{k}^{2}}\Bigg)+$$
$$+\left[\tilde{\mu}\sinh(\alpha x)+\cosh(\alpha x)\right]\sum_{k=0}^{\infty}(-1)^{k}\sin(\beta_{k}x)\Bigg(\frac{\exp(\alpha\tau)\left[\alpha\cos(\beta_{k}\tau)+\beta_{k}\sin(\beta_{k}\tau)\right]-\alpha}{\alpha^{2}+\beta_{k}^{2}}-$$
$$-2\frac{\exp(\alpha\tau)\left[\left(\alpha+\tau_{0}\right)\cos(\beta_{k}\tau)+\beta_{k}\sin(\beta_{k}\tau)\right]-\left(\alpha+\tau_{0}\right)\exp(-\tau_{0}\tau)}{\left(\alpha+\tau_{0}\right)^{2}+\beta_{k}^{2}}+$$
$$+\frac{\exp(\alpha\tau)\left[\left(\alpha+2\tau_{0}\right)\cos(\beta_{k}\tau)+\beta_{k}\sin(\beta_{k}\tau)\right]-\left(\alpha+2\tau_{0}\right)\exp(-\tau_{0}\tau)}{\left(\alpha+2\tau_{0}\right)^{2}+\beta_{k}^{2}}\Bigg)\Bigg\}. \tag{10}$$

Based on Eq. (10) the component $\dot{\varepsilon}_{xx}^{(1)}(x,\tau)$ of the strain rate tensor is determined, and afterwards, the time-dependence of the dynamic viscoplastic yield stress is determined for different values of the intensity of strain energy dissipation (relations between the physical and mechanical properties of the plate and the half-strip) and the rate of the external loading applied.

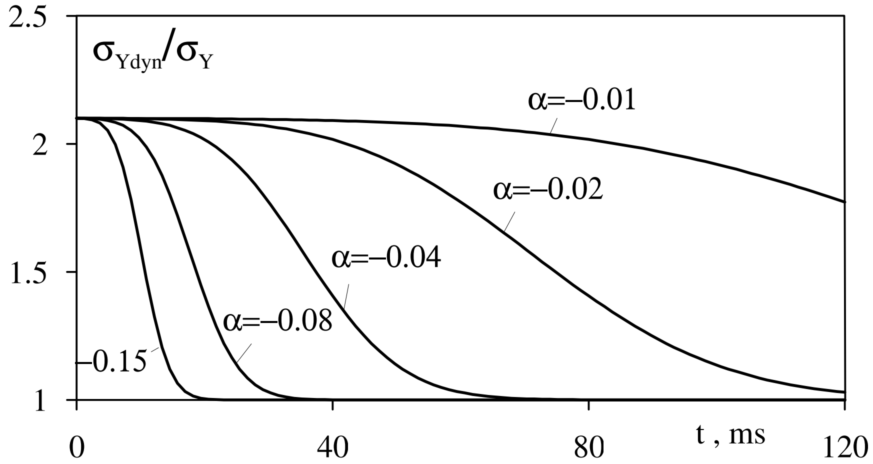

Fig. 3. Time dependence of the yield stress of the plate's material for various intensities of strain energy dissipation

One can see in Fig. 3 that the account of the permanent energy dissipation due to the equipment inertia causes the time-dependent behavior of the yield stress of the plate's material, and therefore, can cause its specific behavior under the limit (close to the fracture limit) loading applied. For values $\alpha \to 0$, which correspond to the case of rigidly fixed edge of the plate (elastic waves reflect from the edge of the plate without loss of their strain energy) the yield stress does not change during deformation process, and therefore, the effect of nonclassical behavior of the material close to the yield strength can be not observed. The increase in the parameter, which characterize strain energy dissipation, causes the decrease in the time interval of the yield strength change, which in instance for $\alpha = -0.15$ is only 10 % of the period of yield stress change for $\alpha = -0.02$. Furthermore, even for small values of the parameter α (intensive strain energy dissipation) there exists a time interval, where the yield strength changes slightly, which is in good agreement with the experimental data (Chausov and Pylypenko, 2005).

The same issues arise during the study of the influence of the load rate on the change in the yield stress. For example Fig. 4 depicts numerical results obtained for $\alpha = -0.02$. One should note that for a plate made of the alluminium alloy 2024-T3 ($E = 6.9$ GPa, $\nu = 0.3$) the value of the dimensionless parameter $\tau_0 = 1$ for the length of the plate $L = 0.15\,\mathrm{m}$ corresponds to the value $a = 50\ \mathrm{ms}^{-1}$, or in other words, for $\tau_0 = 1$ the external load reaches its stationary value in 0.1 ms. Furthermore, the numerical calculations held show that the results obtained for $\tau_0 = 10$ practically coincide with those obtained for step-like loading. With the decrease in the loading rate one can observe the decrease both in time needed for the change in the yield stress, and in the value at which it changes. For $\tau_0 = 0.2$ the material of the plate practically does not show its viscoplastic properties.

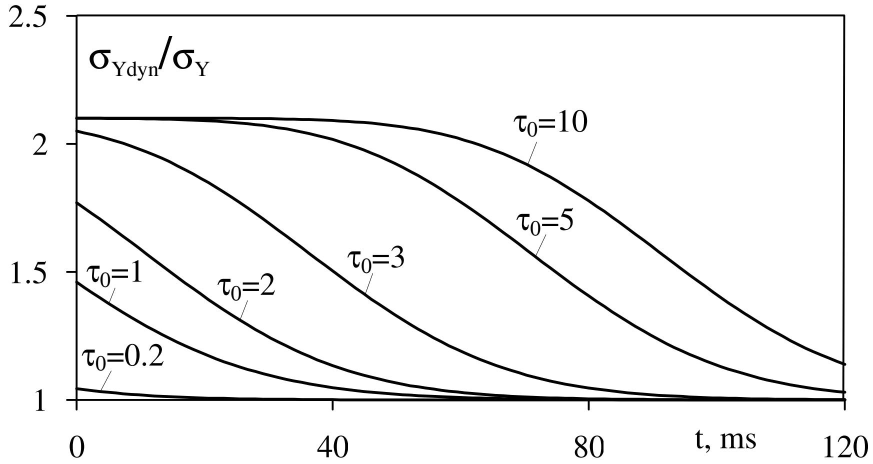

Fig. 4. Time dependence of the yield stress of the plate's material for various values of the external loading rate

CONCLUSIONS

Based on the mathematical model of the viscoplastic phenomenon and the elastic solution of the dynamic problem for a piecewise homogeneous half-strip the influence of the strain energy dissipation and the loading rate on the changes in yield stress is studied.

It is shown that the account of the equipment inertia (the phenomena of strain energy dissipation) causes the change in the yield stress during the deformation process and can be the basis for scientific explanation of the experimentally observed phenomenon of nonclassical behavior of materials under the limit high-speed loads.

It is also shown that the change in the yield stress during the process of dynamic deformation is essentially influenced by the speed of external loading (loading rate).

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