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# DEVELOPMENT ON MEASUREMENT TECHNOLOGY OF AXIAL LOAD IN TRUSS STRUCTURE MEMBER BY IMPACT SOUND

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## ABSTRACT

Truss structure with six members was manufactured and a relation between an axial stress of the member measured by a strain gauge and its characteristic frequency measured by FFT analysis through an impact sound was investigated. Axial stress in a member of the truss structure was calculated analytically, and a frequency equation was introduced for a beam which had an intermediate end support between a simple end support and a fixed end support. In this research, the intermediate end support is defined as bending moment at an end is proportional to a deflection angle at the end. The theoretical result was compared with the experiment data.

***Keywords:*** Characteristic Frequency, Truss Structure, Axial Stress, Intermediate End Support, Impact Sound

## 1. INTRODUCTION

In recent structures, a bar or a rod tend to be employed as a principal member as shown in Fig.1. Current cable-stayed bridges use many rods to suspend main passage. The stresses of members in the structure are related with each other. It is hard to adjust stress in the members which are fixed up into a flexible structure to be designated level. In Japan, the time to be paid a lot of attention for an aged deterioration is drawing near for structures fabricated during an economic boom in the 1960s. Many non-destructive inspection methods have been proposed (Wong, 1988). The demand to measure working stress in a member with ease and reliable accuracy is growing to secure integrity of structures.

Fig. 1 Prime member in recent structure

We have measured an axial load in a beam by an impact sound employing a relation between a characteristic frequency and an applied load. We researched the relation of a round bar for fixed ends or simply-supported ends experimentally and theoretically (Yoshida, 2010). We extended the result to a simple square plate to measure in-plane load (Yoshida, 2012).

In this research, six-member truss structure was manufactured to investigate a capability whether the above single bar result could be applied to members of the truss structure. Theoretical analysis was introduced for a beam supported by an intermediate condition between a simply end support and a fixed end support to explain experimental data.

## 2. THEORY

2.1 Characteristic frequency of beam with intermediate end support

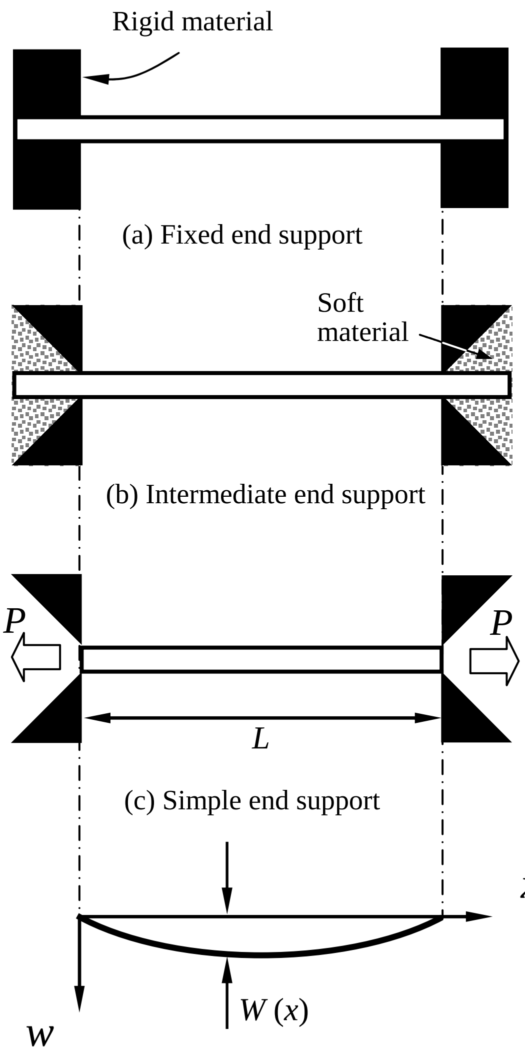
Most members in a practical structure are supported neither by a fixed end condition shown in Fig.1(a) nor a simple end condition shown in Fig.1(c), but by an intermediate condition between them.

One of such intermediate end support is shown in Fig1(b). The end of the beam is supported by the edge of rigid material and an end section of the beam extends into soft material from which the end section receives resistance. The circumstances are supposed to bring about the following end conditions.

(i) No deflection occurs at the end.

(ii) Reaction moment at the end is proportional to the deflection angle at the end.

In this paper, theoretical examination of a beam with the above end support condition was employed. The result was compared with experimental data. The formulation to analyze the characteristic frequency of the beam is explained as follows.




(d) Coordinate and deflection

Fig. 2 Beam with various end supports

Equation of motion of a beam with an axial force, $P$, as shown in Fig.1(c) is given by the next expression.

$$\frac{\partial^2 w}{\partial t^2} + \frac{EI}{\rho A}\frac{\partial^4 w}{\partial x^4} + \frac{P}{\rho A}\frac{\partial^2 w}{\partial x^2} = 0 \qquad \cdots(1)$$

where $w$ is a deflection of the beam, $x$ and $t$ are a coordinate and a time variable respectively. $A$ is a sectional area, $E$ is Young's modulus, $\rho$ is a density and $I$ is a moment of inertia of area .

A beam deflection is supposed to be represented by the next expression.

$$w(x) = W(x) \cdot e^{i\omega t} \qquad \cdots(2)$$

Here, $\omega$ is an angular frequency and $i$ is an imaginary unit.

Putting the expression (2) into (1) brings about the following expression.

$$\frac{d^4 W(x)}{d x^4} + \frac{P}{EI}\frac{d^2 W(x)}{d x^2} - \frac{\rho A \omega^2}{EI} \cdot W(x) = 0 \qquad \cdots(3)$$

The solution of the above ordinary differential equation is given by the next expression.

$$W(x) = C_1 \cdot \cos(\lambda_1 x) + C_2 \cdot \sin(\lambda_1 x) + C_3 \cdot \cosh(\lambda_2 x) + C_4 \cdot \sinh(\lambda_2 x) \qquad \cdots(4)$$

where the constants $C_1$ to $C_4$ are to be determined from the boundary conditions, and

$$\lambda_1 = \sqrt{\frac{\sqrt{\pi^4 \cdot \alpha^4 / L^4 + 4\omega^2 / v^2} + \pi^2 \cdot \alpha^2 / L^2}{2}}$$
$$\lambda_2 = \sqrt{\frac{\sqrt{\pi^4 \cdot \alpha^4 / L^4 + 4\omega^2 / v^2} - \pi^2 \cdot \alpha^2 / L^2}{2}} \quad \cdots(5)$$

$$v^2 = \frac{EI}{\rho A}, \quad \alpha^2 = \frac{PL^2}{EI\pi^2} \quad \cdots(6)$$

The boundary conditions for the intermediate end support are expressed as follows.
(i) At the location, $x = 0$

$$W(0) = 0 \quad \cdots(7a)$$

$$EI\left(\frac{d^2W}{dx^2}\right)_{x=0} = k\left(\frac{dW}{dx}\right)_{x=0} \quad \cdots(7b)$$

(ii) At the location, $x = L$

$$W(L) = 0 \quad \cdots(8a)$$

$$-EI\left(\frac{d^2W}{dx^2}\right)_{x=L} = k\left(\frac{dW}{dx}\right)_{x=L} \quad \cdots(8b)$$

The expressions (7b) and (8b) show that bending moment, $EI\,W''$ at the end is proportional to deflection angle, $\theta\ (\equiv W')$. Here, the constant of the reaction moment, $k$ is positive.

If $k$ were equal to zero, the bending moment at the end is zero. This corresponds to a simple end support shown in Fig.1(c). If $k$ were very large, $W'$ is nearly zero. This corresponds to a fixed end support as shown in Fig.1(a).

Substituting the expression (4) into the boundary conditions (7) and (8) brings about the next relations.

$$C_1 + C_3 = 0$$
$$-C_1\lambda_1^2 + \Lambda C_2\lambda_1 + C_3\lambda_2^2 + \Lambda C_4\lambda_2 = 0$$
$$C_1\cos(\chi_1) + C_2\sin(\chi_1) + C_3\cosh(\chi_2) + C_4\sinh(\chi_2) = 0 \quad \cdots(9)$$
$$\left(-\lambda_1^2\cos(\chi_1) + \Lambda\lambda_1\sin(\chi_1)\right)\cdot C_1 + \left(-\lambda_1^2\sin(\chi_1) - \Lambda\lambda_1\cos(\chi_1)\right)\cdot C_2$$
$$+\left(\lambda_2^2\cosh(\chi_2) - \Lambda\lambda_2\sinh(\chi_2)\right)\cdot C_3 + \left(\lambda_2^2\sinh(\chi_2) - \Lambda\lambda_2\cdot\cosh(\chi_2)\right)\cdot C_4 = 0$$

where

$$\Lambda = \frac{k}{EI}、\quad \chi_1 = \lambda_1 \cdot L、\quad \chi_2 = \lambda_2 \cdot L \quad \cdots(10)$$

The simultaneous equation (9) can also be written in a matrix form as follows.

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ -\lambda_1^2 & \Lambda\lambda_1 & \lambda_2^2 & \Lambda\lambda_2 \\ \cos(\chi_1) & \sin(\chi_1) & \cosh(\chi_2) & \sinh(\chi_2) \\ -\lambda_1^2\cos(\chi_1) & -\lambda_1^2\sin(\chi_1) & \lambda_2^2\cosh(\chi_2) & \lambda_2^2\sinh(\chi_2) \\ +\Lambda\lambda_1\sin(\chi_1) & -\Lambda\lambda_1\cos(\chi_1) & -\Lambda\lambda_2\sinh(\chi_2) & -\Lambda\lambda_2\cosh(\chi_2) \end{pmatrix} \cdot \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} = 0 \quad \cdots(11)$$

For a nontrivial solution of $C_1$ to $C_4$, the determinant of their coefficients must be zero. This brings about the following frequency equation.

$$\begin{vmatrix} 1 & 0 & 1 & 0 \\ -\lambda_1^2 & \Lambda\lambda_1 & \lambda_2^2 & \Lambda\lambda_2 \\ \cos(\chi_1) & \sin(\chi_1) & \cosh(\chi_2) & \sinh(\chi_2) \\ -\lambda_1^2\cos(\chi_1) & -\lambda_1^2\sin(\chi_1) & \lambda_2^2\cosh(\chi_2) & \lambda_2^2\sinh(\chi_2) \\ +\Lambda\lambda_1\sin(\chi_1) & -\Lambda\lambda_1\cos(\chi_1) & -\Lambda\lambda_2\sinh(\chi_2) & -\Lambda\lambda_2\cosh(\chi_2) \end{vmatrix} = 0 \quad \cdots(12)$$

By expanding the determinantal equation, we obtain the following expression (13).

$$\varphi\left(E, \rho, A, L, I, k, \omega, P\right) = 0 \quad \cdots(13)$$

We solved the equation (13) by numerical calculation for $P$ or $\omega$ under given material constants, $E$, $\rho$, given shapes and sizes of a beam, $A$, $L$, $I$, given reaction moment coefficient, $k$ employing measured characteristic angular frequency, $\omega$ or a designated axial load, $P$ respectively.

2.2 Reaction moment coefficient

In order to examine a relation between a reaction moment coefficient, $k$ and characteristic frequency, a round bar with 200 mm length, ϕ 8 diameter, made out of steel, under no axial load was analyzed.

Figure 3 shows the relation. Horizontal axis is a reaction moment coefficient, $k$ and a vertical axis is characteristic frequency. The relation represented by semi-log graph has a sigmoid shape between a simple end support and a fixed end support.

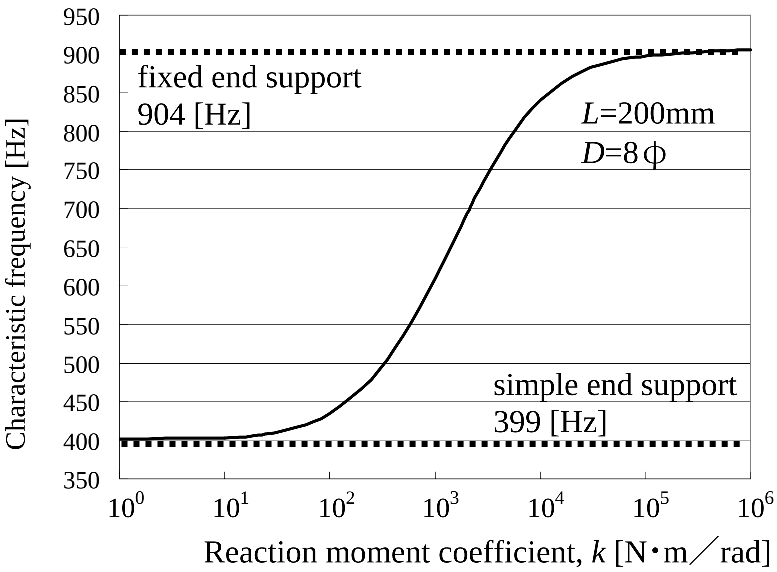


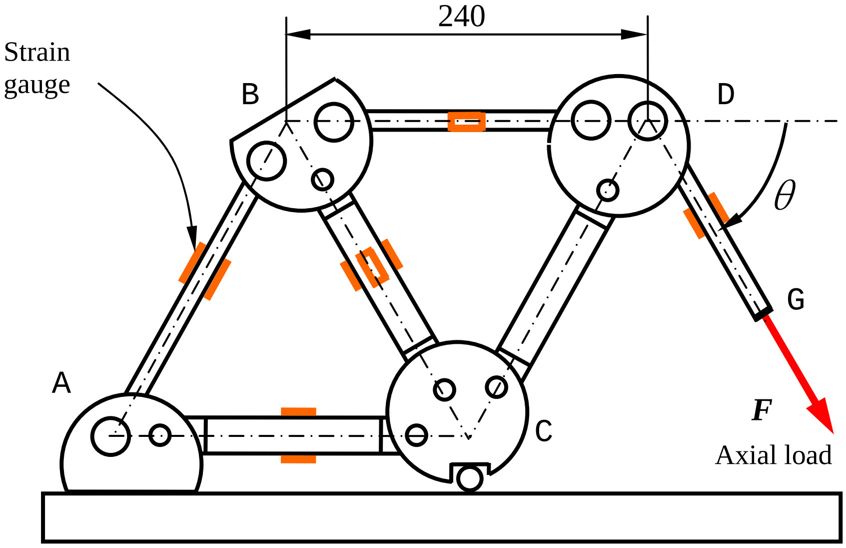

Fig. 3 Relation between reaction moment coefficient and characteristic frequency

## 3. EXPERIMENT

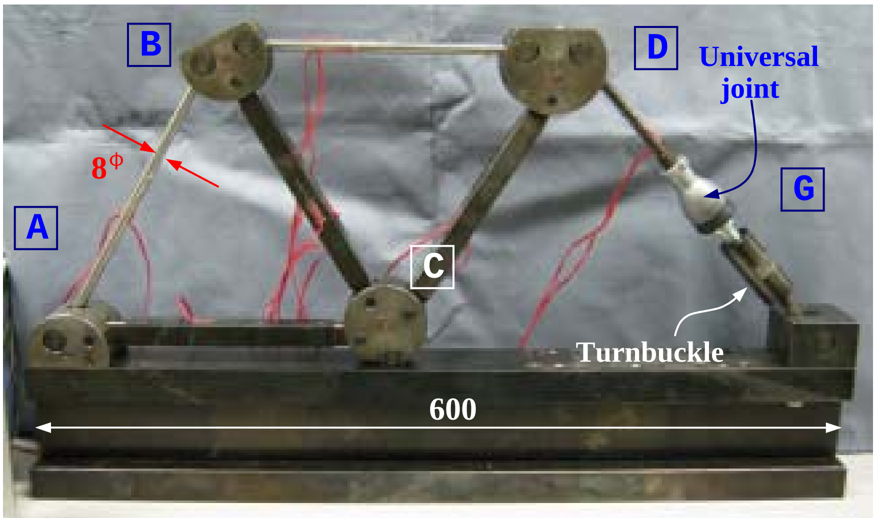
### 3.1 Experimental equipment set up

Six-member truss equipment was manufactured in order to examine whether the method to evaluate an axial load out of a characteristic frequency by an impact sound was applicable to a member of a structure.

The schematic drawing of the equipment is shown in Fig. 4(a). The truss configuration is consisted of two equilateral triangles, ABC and BCD. The length of the sides is 240 mm. Connections between members are designed to be pin-jointed. The structure has a hinged support at A and a movable support at C. Load was applied through member DG attached with turnbuckle. Stress in members varies with loading angle, $\theta$ at 48, 60 and 75 degree. A relation between an axial stress and a characteristic frequency of a member, AB and BD, was investigated. The members are made out of steel and have 8 mm diameter with around 216 mm length. Small compression stress is supposed to occur in other members which are made out of steel and have a square section. Strain gauges were attached to the members to measure




(a) Schematic drawing




(b) Photo of truss equipment

Fig. 4 Six-member truss equipment

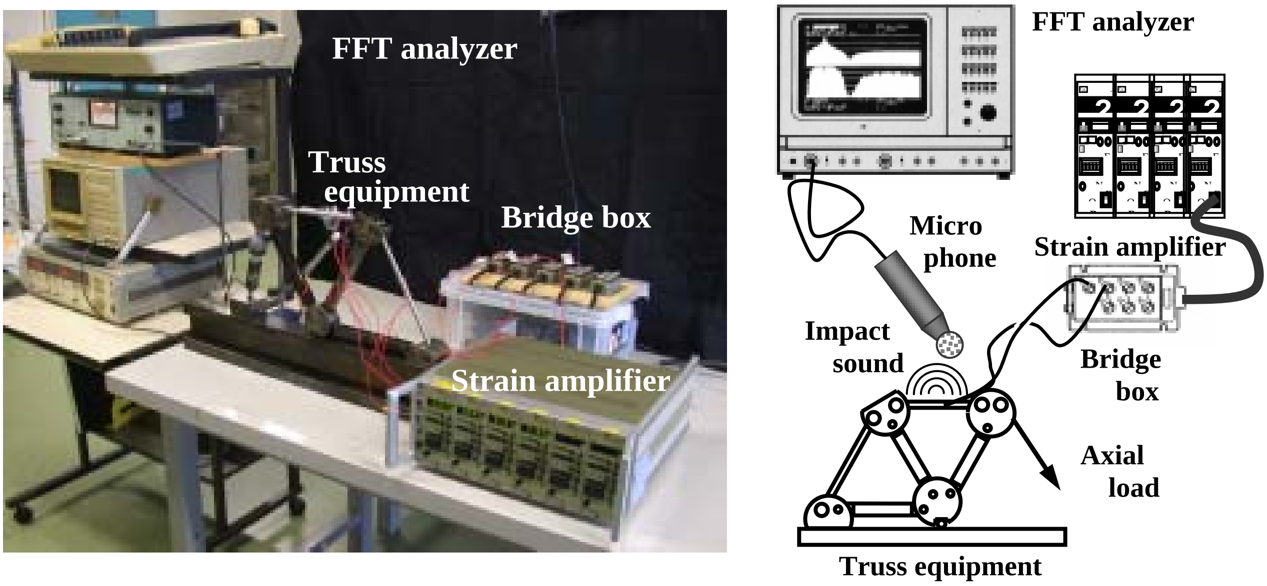



Fig. 5 Measurement system

stress. Characteristic frequencies of the members were measured using FFT analyzer for an impact sound as shown in Fig.5.

3.2 Axial working stress in member

Uniaxial loads in truss members shown in Fig.6 are calculated analytically. The axial load or stress obtained by the calculation are tabulated as *m* or *n* values in Table 1. When load or stress in member DG are *F* or $\sigma$, load or stress in other members are given as $m \times F$ or $n \times \sigma$.

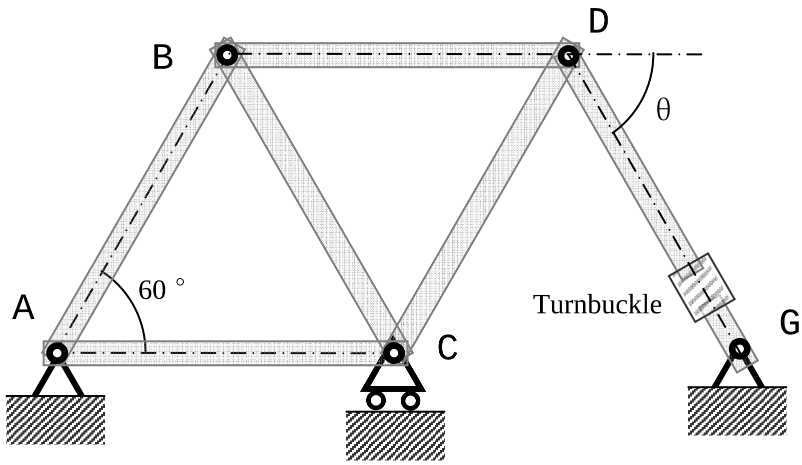



Fig. 6 Truss configuration

Table 1 Multiplier parameters, *m* and *n*

| | member AB,BD | | member AC | | member BC | | member CD | |
|---|---|---|---|---|---|---|---|---|
| θ [ ° ] | *m* | *n* | *m* | *n* | *m* | *n* | *m* | *n* |
| 48 | 1.13 | | -0.18 | -0.028 | -1.13 | -0.178 | -0.77 | -0.121 |
| 60 | 1.00 | | 0.00 | 0.00 | -1.00 | -0.157 | -1.00 | -0.157 |
| 75 | 0.82 | | 0.15 | 0.024 | -0.82 | -0.129 | -1.12 | -0.176 |
| | [N] | [MPa] | [N] | [MPa] | [N] | [MPa] | [N] | [MPa] |

## 4. RESULT

### 4.1 Stress in truss member

Figure 7 shows a relation between an axial stress in a loading member DG, plotted in abscissa, and those of other members in ordinate under a loading angle, θ = 48 degree. Various marks represent experiment data. Load in a member, AC being small, so is also the stress. Stress in other members increases linearly in accordance with stress in member DG. Solid line is a theoretical relation by an analytical truss calculation for members AB and BD given in Table 1. Theoretically, stresses in member AB and BD are same. In the experiment, stresses measured by strain gauges for the member, AB shown as □ and BD as ■ are nearly same in Fig.7. However the experiment data are half the theoretical value. Main reason of the difference seems to be caused by friction at a pin-joint connection. This result tells that stress in a truss member of practical structures may not be evaluated by a simple theoretical truss calculation.

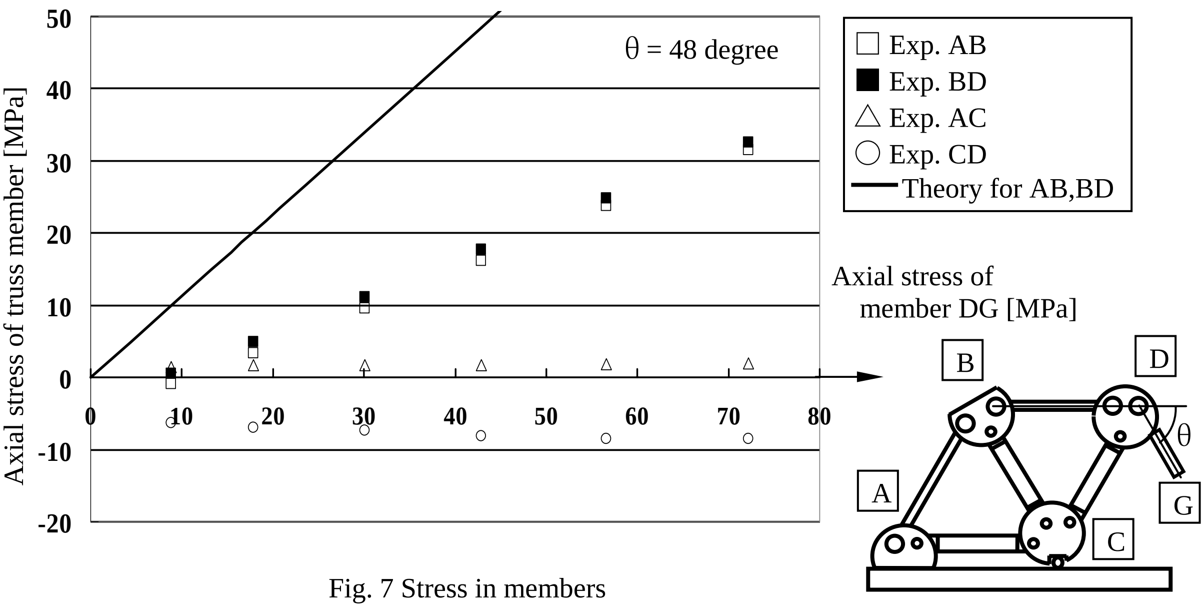
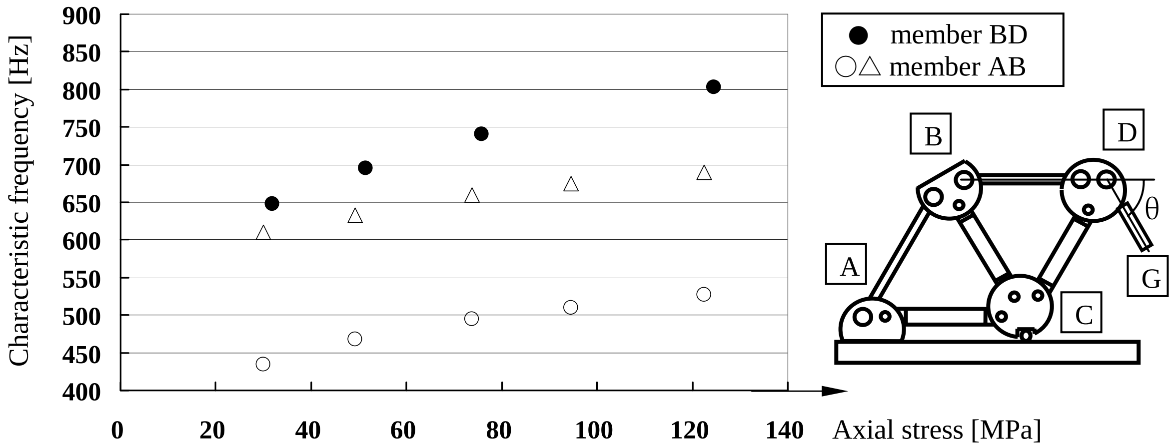



Fig. 7 Stress in members

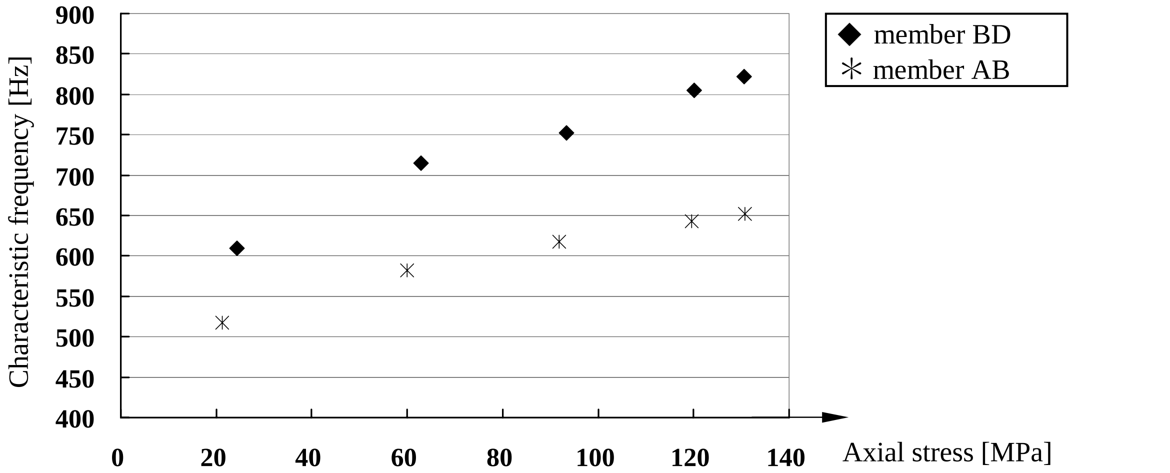
### 4.2 Relation between axial stress and characteristic frequency

Figure 8 shows a relation between an axial stress and the first mode characteristic frequency of members AB and BD under various loadings. There expected to occur two kinds of the vibration for the members with a round bar shape, one about a pin-joint axis with the frequency of simple end supports and another about an axis perpendicular to the pin-joint axis with the frequency of fixed end supports. Member AB had two characteristic frequencies, but varied with a loading angle. Member BD had only one relation for various loading angles, which seemed to be a vibration with the frequency near to fixed end support condition. Vibration with a characteristic frequency of a simple end support was not measured clearly. The relation between an axial stress and a characteristic frequency was much affected by circumstances how a member is supported at an end.

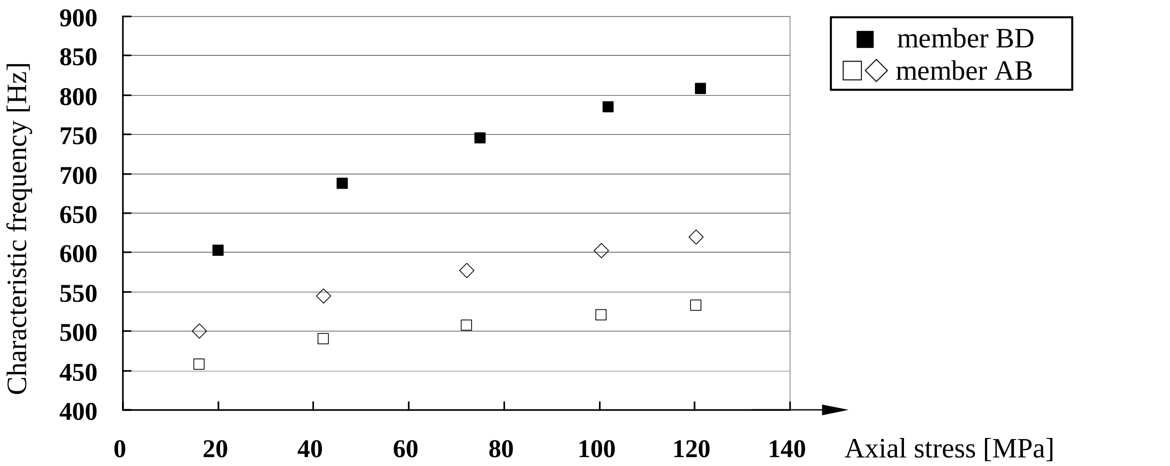
The experiment results shown in Fig.8 were collected all together into Fig.9. Theoretical relations for a fixed end and a simple end support are drawn by a broken or a solid line respectively. The experiment results lay between the above theoretical relations. Experiment data agreed with neither of them.




(a) θ = 75 degree




(b) θ = 60 degree




(c) θ = 48 degree

Fig. 8 Relation between axial stress and characteristic frequency

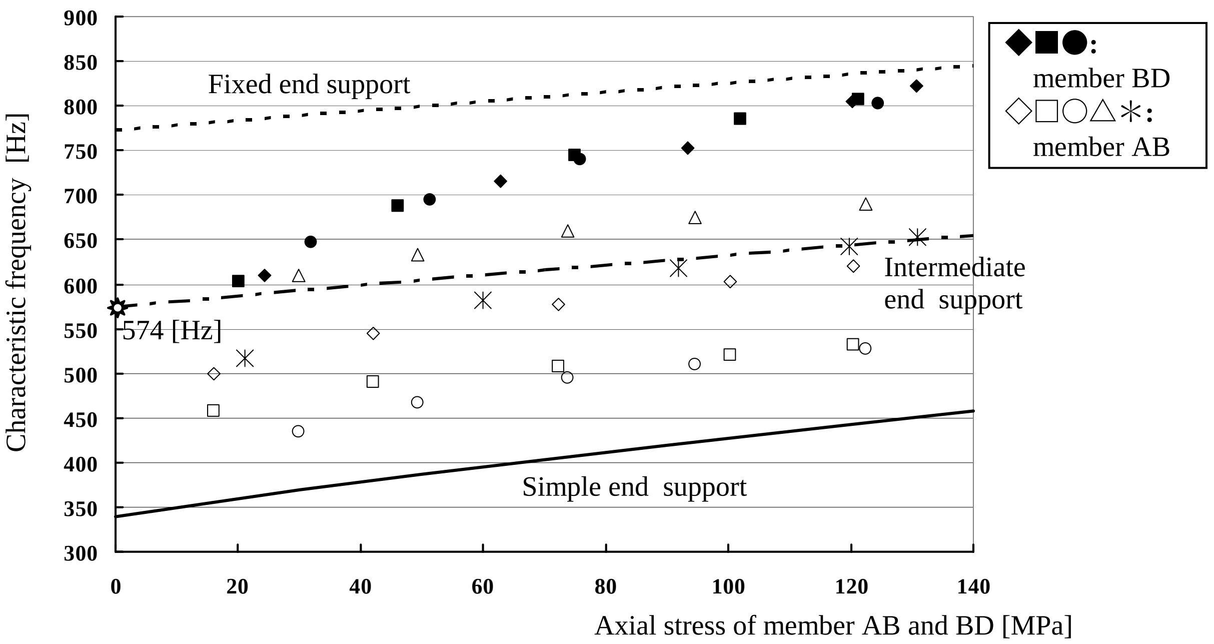



Fig. 9 Comparison between experiment data and theoretical result

In this research, a frequency equation for a beam with an intermediate support condition between a fixed end support and a simple end support explained in §2.1 was introduced. By the regression for the experimental data, the characteristic frequency with no axial stress for member BD was determined to be 574 [Hz]. Employing the frequency, reaction moment coefficient, $k$ was determined as $1.54\times10^3$ [N・m／rad] by the expression (13). Putting the $k$ into expression (13), the relation between an axial stress and a characteristic frequency was drawn as the chain line in Fig.9. A gradient of the relation between an axial stress and a characteristic frequency by the experiment and by the theory did not agree well. The gradient by the experiment was steeper than that by the theory and went up in proportion to an axial stress. Stresses occurred in the members AB and BD are nearly same as shown in Fig.7, but characteristic frequencies differed as shown in Fig. 9. Friction at pin-joints seemed to have given large effect not only on working stress in members but also on the relation between an axial stress and a characteristic frequency. Most joints in practical structures are supported neither by a simple end support nor a fixed end support. In order to evaluate stress in a member by its characteristic frequency, it is necessary to clarify support condition which largely affects the relation between an axial stress and a characteristic frequency.

## 5. CONCLUSION

A round bar member in a truss structure had two characteristic frequencies. One vibrates about a pin joint axis with a frequency near to a simple end support, and another about an axis perpendicular to the pin joint axis with a frequency near to a fixed end support. A relation by experiment results lay between a theoretical relation of a simple end support and a fixed end support. A theoretical relation for an intermediate end support was introduced, but the gradient of the relation by the experiment and by the theory differed. The relation between an axial stress and a characteristic frequency in the experiment was affected strongly by a friction load at an end depending upon how the member was supported. It is necessary to take into consideration the effect of a friction at the joint to develop the method to measure an axial stress in truss structure by an impact sound.

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