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SHAPE OPTIMIZATION OF A LIGHTWEIGHT TETRAPOD ELEMENT USING CAD/CAE TOOLS AND METAMODELS

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ABSTRACT

Nowadays lattice of tetrapod-shaped elements could be used for the synthesis of constructions with high stiffness properties using new types of modern lightweight materials. The possible application of the tetrapod lattice could be constructions with unconventional design and future space structures. Previously, time consuming topology optimization approach was employed for shape optimization of such elements.

It is known that metamodeling methods are efficient for shape optimization of CAD/CAE models. In this approach shape is produced by geometric boundaries defined as CAD based NURBS curves. Due to development of CAD/CAE software and advanced metamodeling techniques such approaches have become highly effective and popular in recent years. At the beginning developed technique of the shape optimization is verified on the test problem. Next in this work resource-saving technique is proposed for the shape optimization of the spatial tetrapod elements.

Keywords: shape optimization, design of experiment, metamodel, tetrapod.

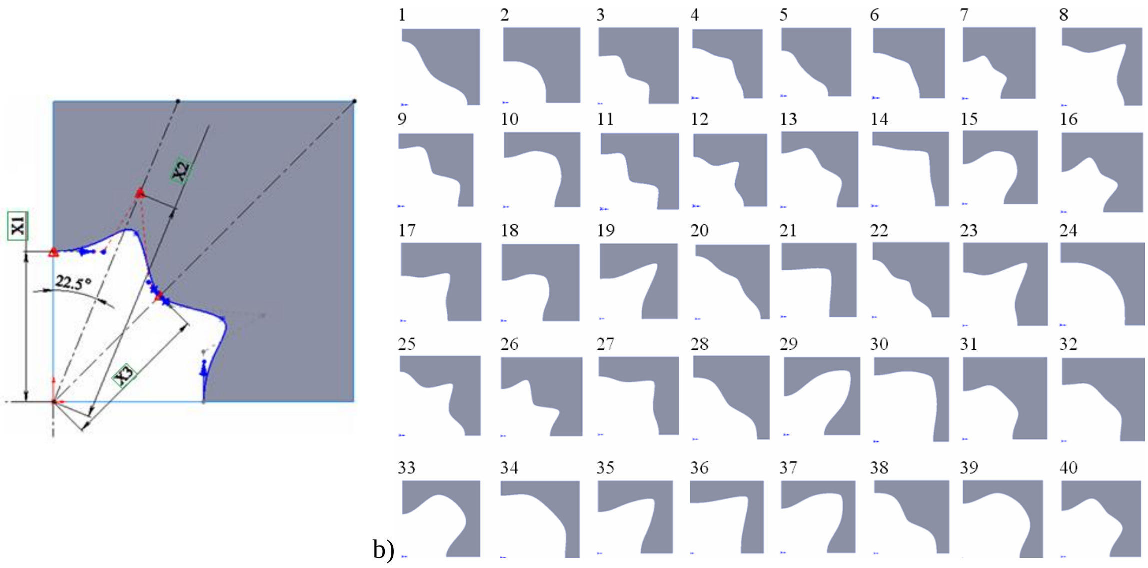
INTRODUCTION

For topology and shape optimization of structures the different realizations of homogenization method are vastly used [Arora, 2004; Bendsoe, 2003; Vanderplaats, 1999]. This method is highly effective for shell constructions. However it is very time consuming procedure because the number of design parameters can reach million and more. In case of solid bodies it frequently produces difficulty manufacturable shapes. As shown in work [Mullerschon, 2010] the Hybrid Cellular Automata method does not allow parallelization of computations and PBS queuing system has been used.

Quality of structures usually is estimated by calculating responses using FEA, which can be time consuming for complex mechanical objects. In such cases instead of full models the expedient metamodels are used. In nowadays, there are many examples of efficient use of metamodels for the shape optimizations of mechanical objects [Lee, 2007; Song, 2010]. As shown in [Janushevskis, 2010, 2012] the most efficient parameterization of geometric boundaries are obtained using control points of the NURBS polygons. The proposed approach can be used for shape optimization that includes following: 1) Planning positions of control points of NURBS polygons for obtaining smooth shape. 2) Creating geometrical models using CAD software in conformity with the design of experiment. 3) Calculation of responses for complete FEM model by CAE software. 4) Creating metamodels for responses obtained in pervious step. 5) Using metamodels for shape optimization. 6) Validation of optimal design by CAE software for complete FEM model.

TEST PROBLEM OF PLATE BENDING

At the beginning we demonstrate our approach on a simple test problem. A clamped square plate is considered under a concentrated load applied at center in a direction normal to its main surface. The isotropic material properties are: the Young's modulus E=1900 MPa, the Poisson's ratio μ=0.39 and dimensions are 200x200x4 mm. The shape optimization of the plate with constant thickness is carried out to minimize its volume in case of a single displacement constraint δ=0.5 mm. The cutout shape of the plate is defined with the control points of NURBS polygon shown in Fig. 1a: Due to symmetry only ⅛ of the plate is considered for cutout definition and ¼ of the plate for problem solution by FEM.



a) b)

Fig. 1 a) Technique for definition of cutouts b) Shapes of cutout in compliance with the design of experiment

Three parameters are stated to define location of the points. Parameters are varied in the following range: 50<X1<90; 50<X2<118; 50<X3<115 mm. At both end points two continuity vectors are defined additionally with direction normal to the side and to symmetry axis of the plate corresponding and with fixed length of 19 and 3 mm. The design of experiment for 3 factors and 40 trial points is calculated with mean-square error criterion (MSE) value 0.4262 by EDAOpt [Auzins, 2006, 2007] - software for design of experiments, approximation and optimization developed in Riga Technical University. This design of experiment also is available at http://www.mmd.rtu.lv. The geometrical models are developed using SolidWorks (SW) for all variants. The shapes are shown in Figure 1b. In the next step responses of these models are calculated using SW Simulation with 2 mm global size elements and total number of DoF ~100000. Then these responses are used for approximation by EDAOpt. For example, for approximation of response y by quadratic polynomial the following expression is used:

$$\hat{y} = \beta_0 + \sum_{i=1}^{d} \beta_i x_i + \sum_{i=1}^{d-1} \sum_{j=i+1}^{d} \beta_{ij} x_i x_j + \sum_{i=1}^{d} \beta_{ij} x_i^{2} + \varepsilon \qquad (1)$$

where there are d variables $x_1,\ldots, x_d$, $L=(d+1)(d+2)/2$ unknown coefficients β and the errors ε are assumed independent with zero mean and constant variance σ^2. In case of local approximation the coefficients $\beta=(\beta_1, \beta_2, \beta_3,..., \beta_L)$ depend on the point x_0 where prediction is calculated and are obtained by using of weighted least squares method:

$$\beta = \arg\min_{\beta} \sum_{j \in N_X} w(x_0 - x_j) \times (y_j - \hat{y}(x_j))^2 \tag{2}$$

The significance of neighboring points in the set N_x is taken into account by Gaussian kernel:

$$w(u) = \exp\left(-\alpha u^2\right) \tag{3}$$

where u is Euclidian distance from x_0 to current point and α is a coefficient that characterize significance.
Quality of the approximation is estimated by leave one out crossvalidation error:

$$\sigma_{err} = 100\% \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(\hat{y}_{-i}(x_i) - y_i)^2}}{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2}} \tag{4}$$

where root mean squared prediction error stands in numerator and mean square deviation of response from its average value stands in denominator, n is a number of confirmation points and $\sum_{i=1}^{n}\hat{y}_{-j}(x_i)$ denotes sum of responses calculated without taking into account j-th point. Using obtained locally weighted polynomial approximations by global search procedure [Janushevskis, 2004], implemented in EDAOpt, the optimal cutout shape are obtained (see Fig. 2). In the table 1 the results are summarized and compared with volume obtained in the work [Liang, 2001] by homogenization method. Variants correspond to shapes shown in Fig. 2. Value of Gaussian kernel parameter α of the local quadratic polynomial approximation is chosen to minimize relative leave one out crossvalidation error σ_{err} of approximations of appropriate responses, i.e. deflection δ and volume v of the plate. v_p is predicted volume calculated using approximations and v_a is the actual volume calculated using geometrical model. v_p and v_a in % show comparison of the appropriate volume respect to the volume obtained in [Liang, 2001]. Best results are achieved with variant “c”. This allows reducing the volume of the plate by 1.38 % in comparison with homogenization method.

To be fully confident that such approach will work for real objects, optimizations results of the plate bending problem were validated. Four samples for each different shape samples are manufactured. Plate (A) with the shape obtained by the topology optimization [Liang, 2001], plate (C) obtained by the current approach, the cross-shaped plate (D), and the simple square plate (E) (Fig. 5). Samples D and C have the same volume. The E was used to verify the quality of experiments.

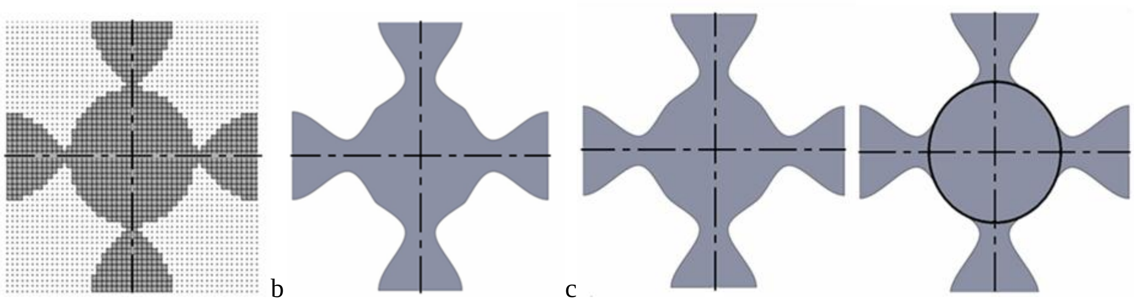


Fig. 2 Shape of plate obtained by a) homogenization method [Liang, 2001]; and by current approach, b) with the control points of NURBS polygon; c) same as "b" but with additionally optimized tangent weighting at the spline endpoints and same but with circle added

Table 1 Quantitative indices of the shape optimization of cutout for the plate bending problem

Variant	α	$\sigma_{err\ \delta}$ %	$\sigma_{err\ v}$ %	v_p mm^3	v_a mm^3	v_p %	v_a %
a	-	-	-	-	68750.00	-	-
b	15.6	9.81	0.03	68862.32	68721.98	0.16	-0.04
c	3.2	0.79	0.16	67797.524	67800.975	-1.385	-1.38

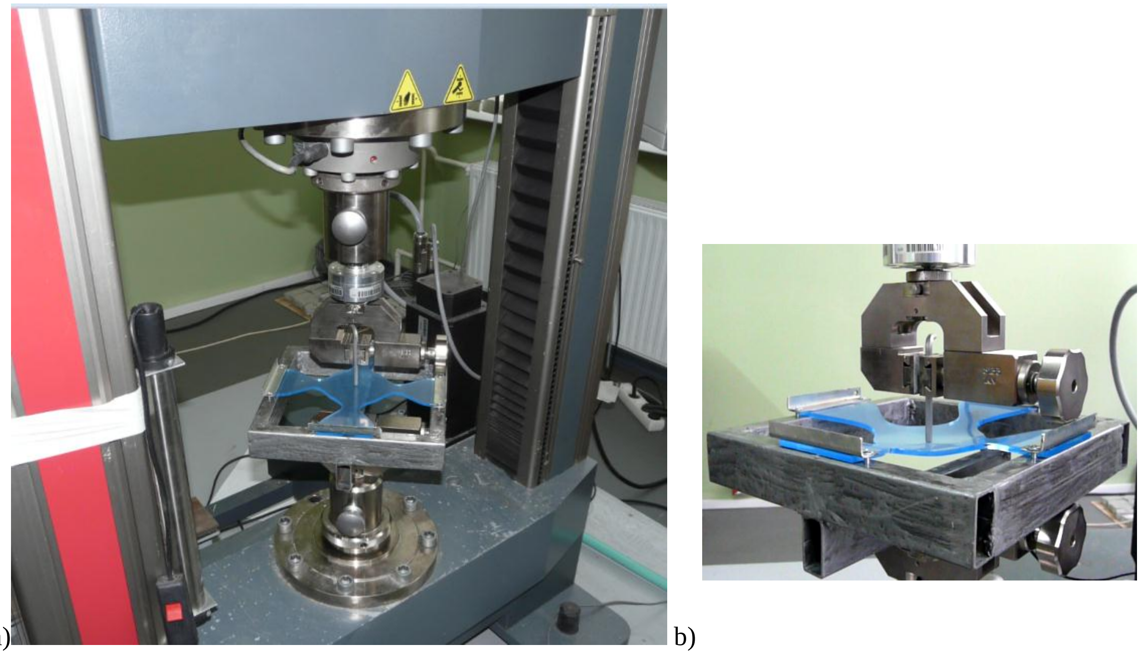


Fig. 3 Sample test: a) Zwick Roell Z 150 machine and b) Sample supporting construction

4 natural experiments are conducted for each shape of the sample on the Zwick Roell Z 150 testing machine (Fig. 3). Experiments are performed according to the loading and restraining model of FE analysis. Samples are slowly loaded up to a failure point. The result curves of experiments are shown in the Fig. 4 (left). The obtained data points of experiments are used to obtain averaged curves for each shape (Fig.4 right).

Table 2 Samples tests results

Sample variant	A	C	D	E
Volume, mm^3	68750	67800	67800	160000
Ultimate load (averaged), N	240.149	1433.46	235.39	1343.91
Relative Strength C, %	16.8	-	16.4	93.8
Max deflection δ (averaged), mm	5.02	27.12	5.81	11.35

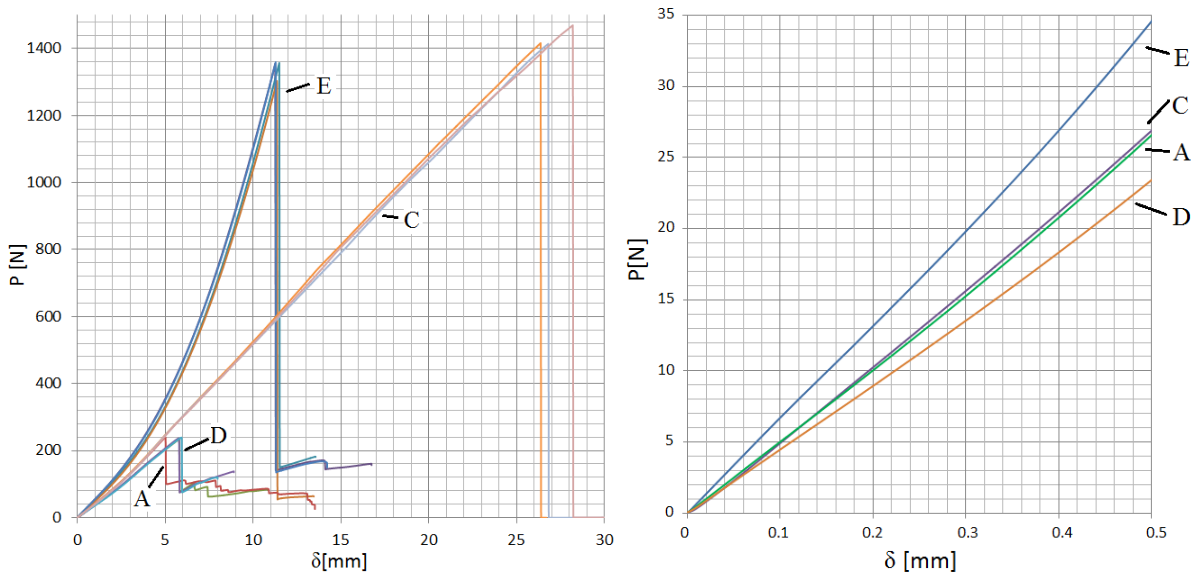


Fig. 4 Test of the samples (left) and the averaged curves of each type of samples (right)

The results (Table 2) show that samples with the shape C have higher averaged strength (for 6.2%) comparing with full plate E. On the other hand, E has for ~58% higher stiffness. C samples take loads more efficiently comparing with same volume D samples (Fig. 4 left). As a result, C shapes have ~ 11.3% lower averaged max deflection at the same load compared to D at the interval of the elastic deformation (δ<2.2 mm). The fracture behavior of the samples was consistent during repeated experiments (Fig. 5). Result plate of the topology optimization A has shown it optimal shape properties during elastic deformations and has almost the same curve as C (Fig. 4 right). However, as the plates A have many stress concentrators (Fig. 5A), it fractured at the beginning of deformations.

The classical analytical theory was used to calculate linear model for sample E and compared with experiments and FE model. Analytical and FE models show good agreement at the beginning interval (δ<2.2 mm) (Fig. 6II).

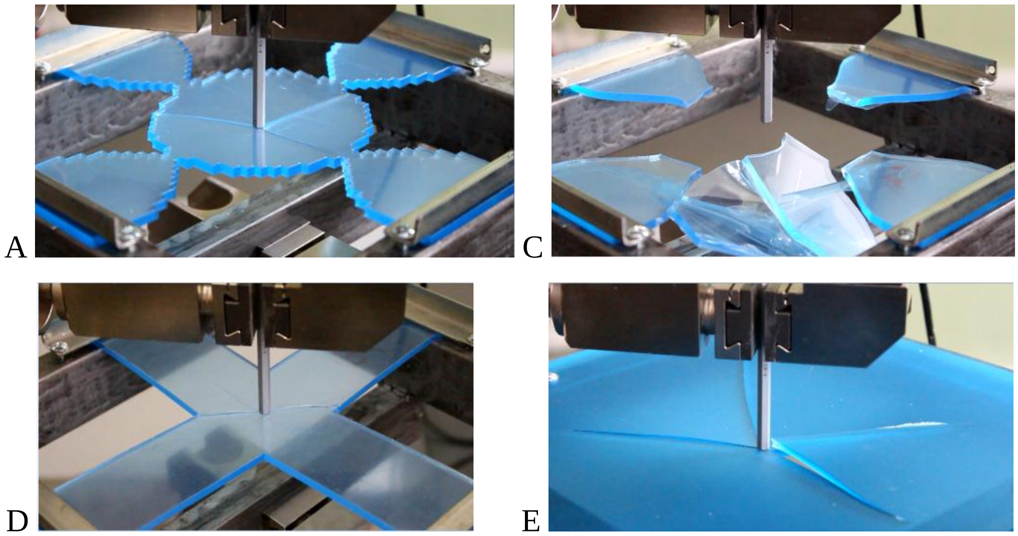


Fig. 5 Fracture behavior of the each sample: A triple crack from center; C center explosions type; D crack through center diagonal and E symmetric 4 cracks from center

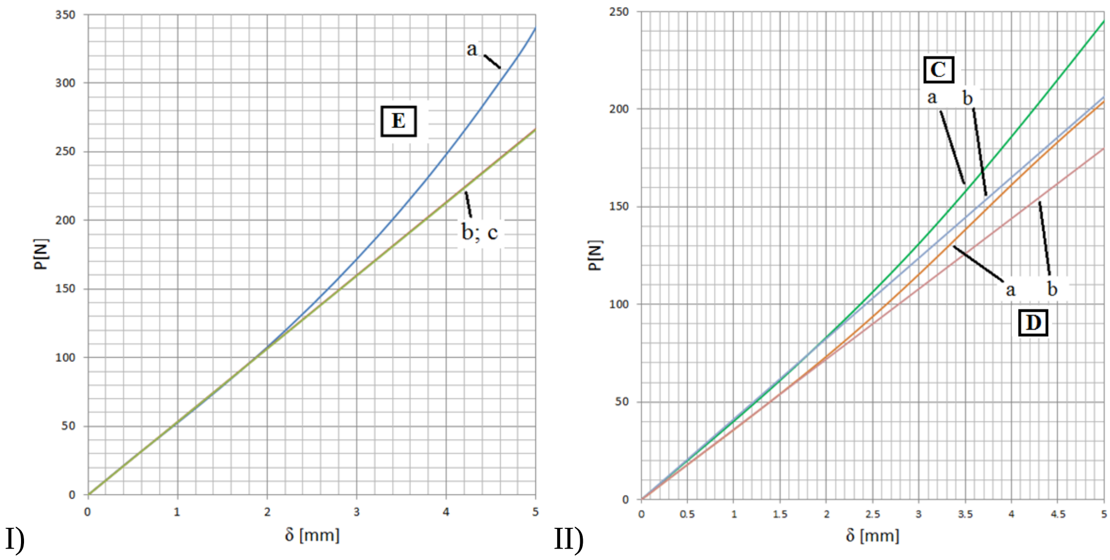


Fig. 6 Average curves of samples test and linear models: I) E: a) test (average), and b); c) – linear FE and analytical models; II) C and D: a) test (average), and b) linear FE model

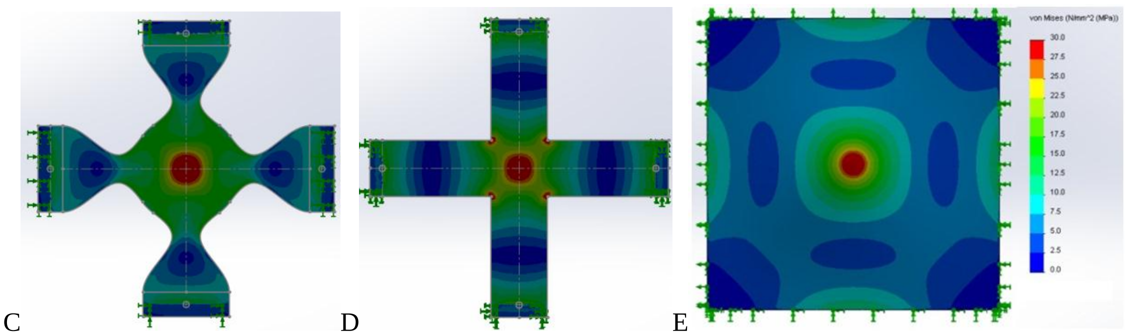


Fig. 7 Tetrahedral FE model. Equivalent stresses in the samples

The equivalent stresses of the samples as the FEM models are compared in Fig. 7. Samples D have visible stress concentrators between perpendicular edges (Fig. 7D). Stresses have smoother distributions in the C comparing with D and E. Square plate has highest stress level around the center. 2 types of FE were used for C and E strength calculations. The obtained FE results show that optimal shape C has the highest strength. Predicted and real strength is shown in the Table 3.

Table 3 C and E samples strength comparison

	FEM		Experiments
	Equivalent max stress [MPa]		
Samples	Tetrahedral element	Surface elements	
C	45	45	-
E	48.3	46.9	-
C higher strength that E, %	6.8	4.1	6.2

TETRAPOD SHAPE OPTIMIZATION

We use this shape optimizations technique for 3D constructions of joint element optimization. As was shown in the previous works [Bervalds, 2010; Yaghi, 2003], lattice of tetrapod-shaped elements could be used for the synthesis of constructions with high stiffness properties using new types of modern lightweight materials. One of the possible application of the tetrapod lattice could be constructions with unconventional design and future space structures. Previously, time consuming topology optimization approach was employed for shape optimization of such elements and optimal topology was found for cases of different criterions [Dobelis, 2010]. At this work only models of 1 element were considered and compressing loads were taken into account during topology optimization.

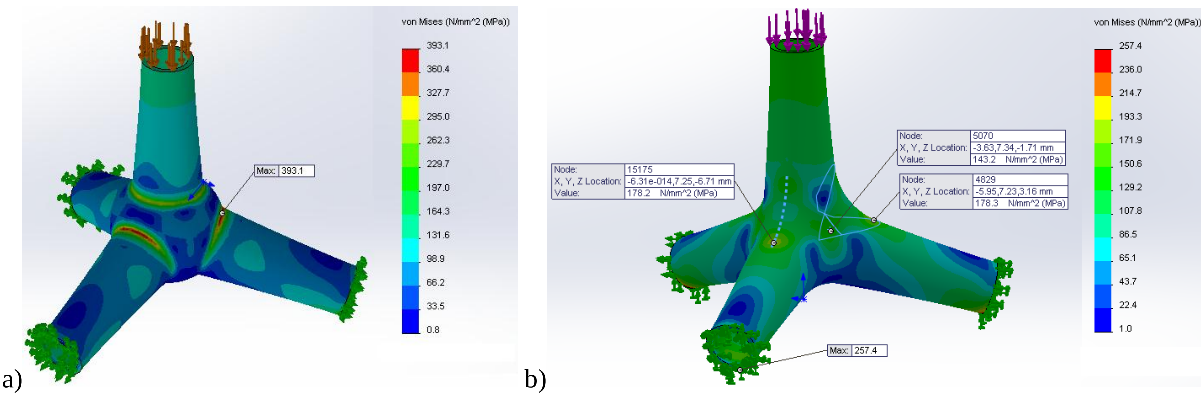


Fig. 8 Equivalent stresses concentrations at the hollow elements joints: a) Spherical and b) Continuous shape

There are many designs and techniques for connections pipe profiles in structural engineering. For example, Beijing National Aquatics Center [http://en.beijing2008.cn] unique walls and ceiling were built using special metal frames that are based on Weaire–Phelan structure [Weaire, 1994]. Such bubble shaped frame has many spherical 4 pipe joints (Fig. 8a). On the other hand, as we can see on the developed FE model (Fig. 8), continuous hollow shape could be more efficient for pipe element joints than spherical, because 4 pipe hollow spherical joint has a stresses concentration between pipe and spherical shape. As a result, such joint has 2.2 times higher equivalent stress level due to compressing loads, comparing with continuous (Fig. 8b). Consequently, it is proposed to use continuous shapes for pipe profiles joints - tetrapod-shaped elements (Fig. 9a). Such element's hexahedral lattice (Fig. 9b) could be used for basis of new constructions (Fig. 10), that take several compressive loads from under angle. Also, it is important to find optimal shape of the continuous connecting element, because it could significantly affect the entire construction stiffness and strength properties.

The tetrapod-shaped element could be manufactured from identical 3 parts (Fig. 11b-c). First, each part is molded and then is welded together. Element dimensions are chosen in compliance with pipe profile diameters. Pipes and tetrapods are connected with welding or thread if construction is needed to be dismountable. Connection type is not taken into account during optimization: the construction is assumed as 1 bonded part.

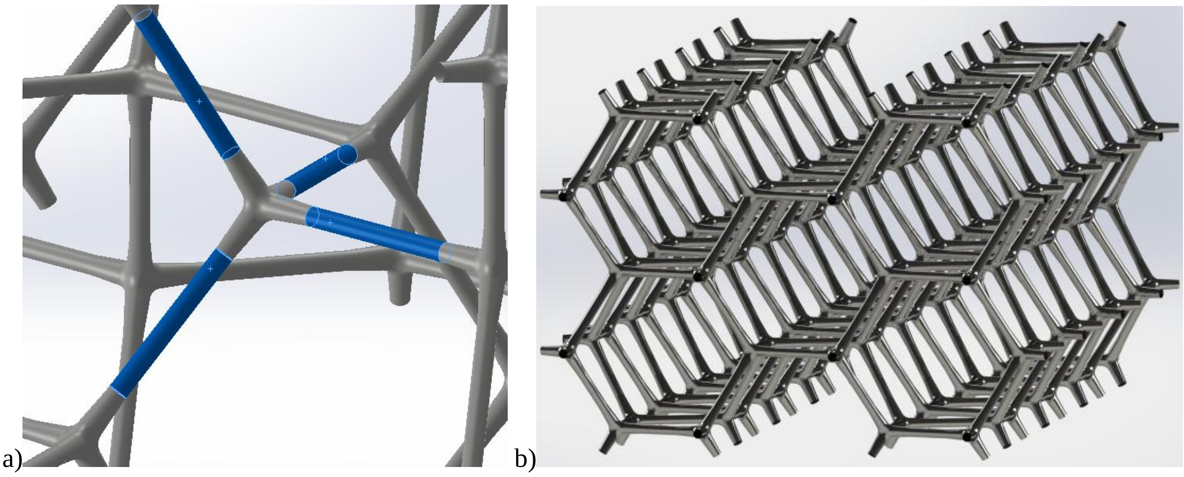


Fig. 9 Pipes profiles: a) Connection with tetrapod-shaped element and b) Hexahedral lattice

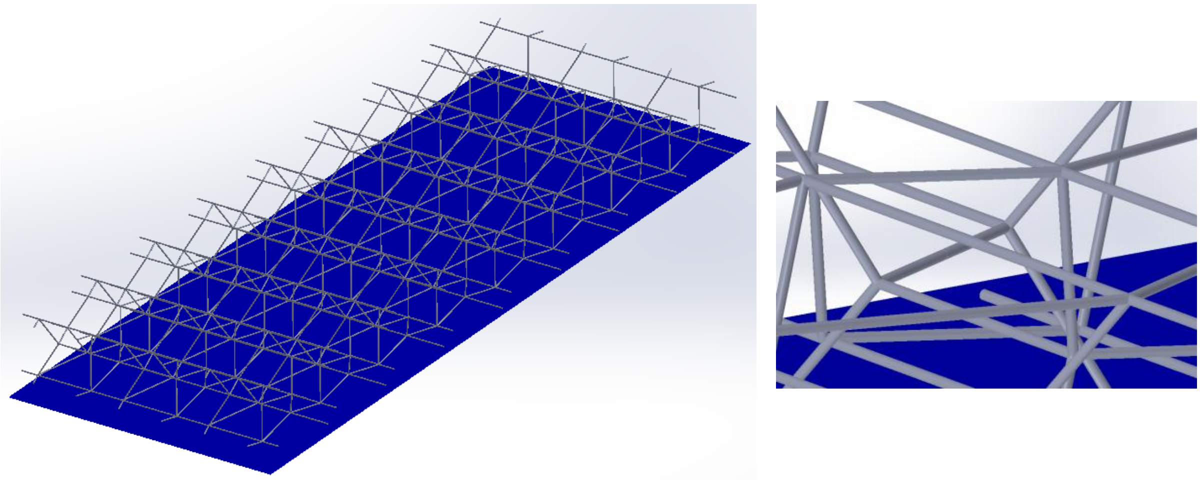


Fig. 10 Pipe profiles supporting construction with 4 pipe joints (bottom view)

At the beginning of optimization loop, we need to define minimal number of required parameters to specify accurately complex boundary shape of the element. Due to symmetry of the tetrapod the object boundary shape effective parameterization with 3 parameters is proposed as shown in Fig. 11a. The shape is controlled using small number of parameters that is important for successful optimization. The boundary shape is controlled by 2 control points of NURBS polygon and radius.

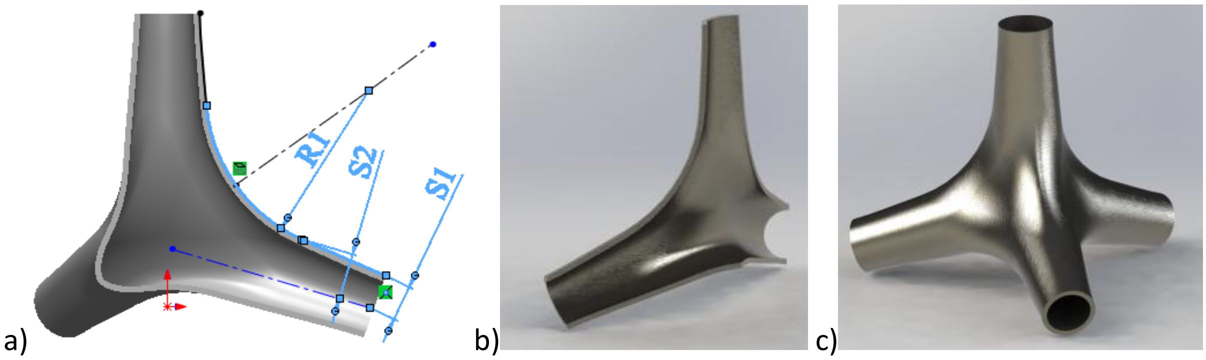


Fig. 11 Parameterization of the tetrapod-shaped element: a) Definition of shape with S1; S2 control points of NURBS polygon and radius R1, b) Smooth molded shape of the 1/3 tetrapod, and c) Welded initial model

Previously, it was found that tetrapod element has lower strength from compressive loads. According to this, compressive loading model is chosen for optimization. The considered FE model consists of 4 tetrapod-shaped elements (Fig. 12 left), that are cut out as shown in Fig. 10 construction. Only 1 element takes perpendicular compressive load other 3 element support the model. Curvature based surface FE mesh is used to describe accurately the complex NURBS shape of the element (Fig. 12 right).

The responses for the central element of the model are calculated by FEM. The parameters Fig. 11a are varied according to Latin Hypercube design of the experiments; the metamodel of the responses is constructed using local quadratic approximation with Gaussian kernel. The shape of the element is optimized using the metamodels. The objective minimization of maximal equivalent stresses with restrained volume and maximal displacement. The obtained shape of the element is shown on the Fig. 13b. The design is compared with initial variant Fig. 11c and Table 4. The maximal equivalent stress is reduced by 18.2 % comparing with initial design. The equivalent stresses distribution is shown on the Fig. 13a.

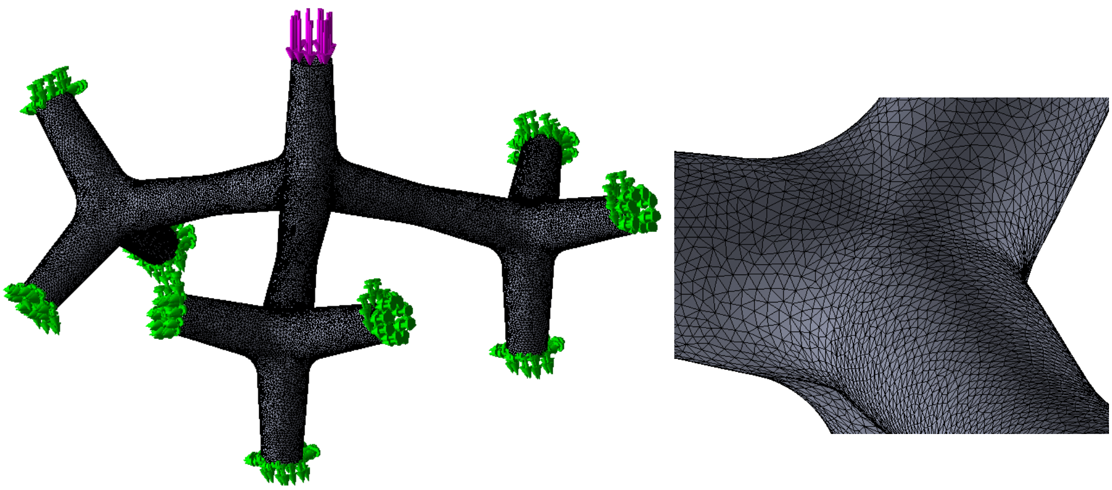
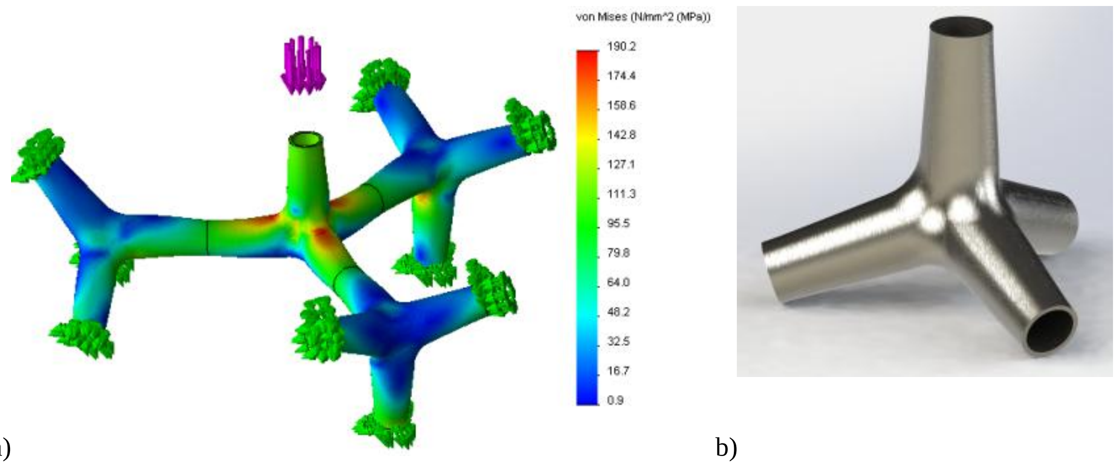


Fig. 12 Assumed simplified 4 tetrapod meshed model

Table 4 Results of the tetrapod-shaped element optimization (indices are fixed for central element of the design)

Indices	**Initial**	**Optimal**	
		Metamodel	**Actual Full Model**
Volume, mm^3	647.850	646.933	645.583
Max equivalent stress, MPa	232.503	170	190.2



a) b)

Fig. 13 Equivalent stresses from compressive load (a) and obtained result shape of the model (b)

CONCLUSIONS

The test problem validation has shown that developed shape optimization technique works effectively. The implemented shape optimization of the tetrapod element using NURBS polygon control point parameterization allows substantial diminishing of the number of design variables and obtaining smooth optimal shape. Metamodeling technique significantly reduces necessary optimization time in comparison with the homogenization method. The obtained shape of tetrapod element ensures ~18.2 % lower maximal equivalent stress level and 0.4% lower volume in comparison with the initial design.

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