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HOMOGENIZATION OF STRUCTURES WITH GENERALIZED PERIODICITY MADE OF ELASTOPLASTIC MATERIALS WITH HIGHLY CONTRASTED PROPERTIES

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ABSTRACT

This work presents the homogenization of structures with gerneralized periodicity made of elastoplastic materials. A general formulation of the heterogeneous problem in the framework of variational methods is developed and all micromechanics oriented requirements for the proposed microscopic fields are derived. The continuum problem is transformed to a discrete, constrained-optimization problem by applying an implicit backward Euler difference scheme. The homogenization scheme is demonstrated with a numerical example of a multilayer wavy stratified structure made of two bilinear elastoplastic materials.

Keywords: homogenization, J2 plasticity, wavy composites, computational scheme.

INTRODUCTION

Modeling the mechanical behavior of non-linear heterogeneous materials (functional graded materials or composites exhibiting highly contrasted properties) with complex geometry requires additional information from both mathematical and computational point of view. More specifically, if the properties and/or the geometry depend on a general periodicity function, for homogenization purposes a generalization of the two-scale convergence-based procedure (Allaire, 1992) is needed for passing to the limit in the weak form of the equations describing the problem. On the other hand, the incremental nature of the external agent and the iterative procedure charges seriously the computational procedure.

In this work we present the homogenization scheme of a wavy multilayer periodic structure made of elastoplastic materials. The homogenization of a similar structure made of elastic materials was treated in Tsalis et al (2012), where the analytical expressions of effective coefficients were presented. The non-linear character of the problem demands a general formulation of the heterogeneous problem in the framework of variational methods with an appropriate functional setting, and a discrete variational formulation in terms of two-scales.

We introduce the admissible deformation fields of materials with generalized periodicity and we present their properties. In the mathematical homogenization two scales are introduced. The first one is the macroscale denoted by $\mathbf{x}$ in Ω, where Ω is the volume occupied by the heterogeneous body, at which the heterogeneites are characterized by a very small ε. The second one is the microscale denoted by $\mathbf{y}'=\mathbf{x}/\varepsilon$, which is the scale for the heterogeneites. In the case of materials with generalized periodicity the choice of the representative volume element is made with respect to the generalized periodicity function $\boldsymbol{\rho}(\mathbf{x})$, and $Y=[0,y_1]x[0,y_2]x[0,y_3]$ is chosen to be the basic cell, where $\mathbf{y}=\boldsymbol{\rho}(\mathbf{x})/\varepsilon$. The dependence of functions on the micro-coordinate is performed in a non-periodic way via $y' = \mathbf{x}/\varepsilon$. We define the deformation fields (macro- and microvariables) in the framework of appropriate functional spaces. The new ingredient is that the actual displacement **w** within **Y** located at **x** has two properties: it is oscillating and it has a generalized periodicity, $w_i(\mathbf{x},\mathbf{y'},\mathbf{y})=E_{ij}y'_j+u^1_i(\mathbf{x},\mathbf{y})$, where $u^1_i(\mathbf{x},\mathbf{y})$ periodic with respect to $\boldsymbol{\rho}/\varepsilon$.

RESULTS AND CONCLUSIONS

We initially derive all micromechanics oriented requirements for the proposed fields, including the generalization of Hill's lemma and we show that the above fields are compatible with the heterogeneous problem. The gradient of the generalized periodicity function enters in both the macroscale and microscale problems, as well as in the formula of the effective tangent modulus. The microscopic stress and strain are obtained as the generalized two-scale limits of the corresponding heterogeneous quantities when the heterogeneity parameter ε tends to 0 in the equations describing the heterogeneous problem.

Next we transform the continuum problem, obtained as the two-scale limit of the above equations combined with the principle of maximum plastic dissipation in the framework of J_2-plasticity with linear isotropic hardening, to a discrete, constrained-optimization problem (Terrada and Kikuchi, 2001) by applying an implicit backward Euler difference scheme. The solution of the nonlinear macroequilibrium equation is obtained using the Newton-Raphson iterative method. The same method is used to solve the cell problem. The computational algorithm is composed from four subroutines, the macroscale, the microscale, the iterative plastic scheme and the effective tangent modulus problem.

Finally, we present a numerical example dealing with the cell problem of a multilayer wavy stratified structure made of two bilinear elastoplastic materials and we compare our results with the FVDAM method (Pindera et al, 2009).

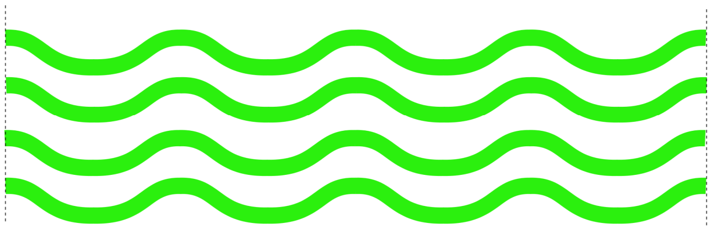

Fig. 1 - Periodic materials with wavy microstructure

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