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# HOMOGENIZATION OF MAGNETORHEOLOGICAL ELASTOMERS CONSIDERING GEOMETRICAL NONLINEARITIES

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## ABSTRACT

This work presents a homogenization framework for magnetorheological elastomers under large deformation processes. The macroscale and microscale magnetomechanical responses of the composite in material and spatial description are presented and the conditions for a well established homogenization problem in Lagrangian description are identified. It is shown that the use of kinematic and magnetic field potentials instead of kinetic field and magnetic induction potentials provides a more appropriate homogenization process. A computational homogenization scheme is presented and illustrated through appropriate numerical 2D and 3D examples.

*Keywords:* homogenization, magnetorheological elastomers, large deformations, finite element formulation.

## INTRODUCTION

Magnetorheological elastomers are magneto-sensitive composite materials whose mechanical behavior depends on the applied magnetic field. The increased research attention on elastomers filled with magnetic particles is due to the variety of applications that they can be incorporated, including adaptive engine mounts, vibration absorbers, suspensions and automotive bushing.

Magnetorheological elastomers can be considered as composites consisting of material components with general nonlinear magnetomechanical behavior. The two-scale homogenization process requires the description of macroscale and microscale responses in material and spatial description, as well as the connection between macroscopic and microscopic variables through appropriate volume averaging. In this work we introduce appropriate potentials in the microscale for the deformation gradient  $\mathbf{F}$ , the Piola stress  $\mathbf{P}$ , the magnetic field  $\mathbf{H}$  and the magnetic induction  $\mathbf{B}$  and we identify the conditions for a well established homogenization problem in Lagrangian description that leads to a large deformation process with meaningful space averages (Costanzo et al, 2005).

Using the Hill's lemma for large deformations (Hill, 1984) we obtain several magnetomechanical boundary conditions that satisfy the Hill-Mandel property. For these boundary conditions, the connection between the macroscopic magnetomechanical field variables obtained in Lagrangian description ( $\mathbf{P}$ , $\mathbf{H}$ , $\mathbf{B}$ ) and the volume averaging of the corresponding microscopic variables in the Eulerian description ( $\mathbf{p}$ , $\mathbf{h}$ , $\mathbf{b}$ ) is examined.

Using the deformation map **Y** and the scalar magnetic potential **Y**, which are connected with the deformation gradient and the Lagrangian magnetic field through the relations

 $\mathbf{F} = \operatorname{Grad} \mathbf{Y}$  and  $\mathbf{H} = \operatorname{Grad} \mathbf{Y}$ ,

respectively, we develop a numerical homogenization scheme. In this scheme the solution of the microscale problem provides information about the macroscopic stress, the macroscopic magnetic induction and the effective tangent magnetomechanical moduli. Special numerical examples in composites with 2D and 3D microstructure show the effect of the applied mechanical and magnetic loading on the macroscopic behavior of the magnetorheological elastomer.

## **RESULTS AND CONCLUSIONS**

This study shows that the use of kinematic and magnetic field potentials instead of kinetic field and magnetic induction potentials provides a more appropriate homogenization process (Chatzigeorgiou et al, 2013). A numerical homogenization scheme based on the former potentials can be easily implemented in a Finite Element code. The obtained numerical results indicate the influence of the effective stiffness on the applied magnetic field.



Fig. 1 - Macroscale and microscale of a magnetorheological elastomer in Lagrangian and Eulerian configuration.

## ACKNOWLEDGMENTS

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