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MEASUREMENT OF ENERGY LOSS COEFFICIENT OF ELECTROSTATICALLY ACTUATED MEMS RESONATORS

Marius Pustan[1(*)], Cristian Dudescu[2], Corina Birleanu[1]
[1]Department of Mechanical Systems Engineering, Technical University of Cluj-Napoca, Cluj-Napoca, Romania
[2]Department of Mechanical Engineering, Technical University of Cluj-Napoca, Cluj-Napoca, Romania
[(*)]*Email:* marius.pustan@omt.utcluj.ro

ABSTRACT

In this work the frequency response of MEMS resonators under electrostatic actuation is measured with the scope of energy loss coefficient estimation. The loss of energy in a vibrating structure is experimentally determined by measured the quality factor. The quality factor is a resonator qualifier that gives information about the energy dissipated during one cycle of oscillation. The total energy dissipation depends on the air damping and on the structural thermoelastic effect. MEMS resonators under investigations are polysilicon microcantilevers having changes of the geometrical dimensions and configuration (with holes and without holes)

Keywords: MEMS resonators, frequency response, quality factor, loss of energy

INTRODUCTION

The microresonator represents currently one of the important research areas of Microelectromechanical Systems (MEMS). The usual applications of microresonators are: radio-frequency MEMS devices, MEMS gyroscopes, vibrating MEMS sensors used in seismic or mass detection applications (Lobontiu, 2007; Pustan, 2011; Soboyejo, 2003; Tadayon, 2006).

The microresonators can be dynamically characterized based on their resonant frequency response and quality factor (Pustan, 2011; Yi, 2008). Quality factor is an expression of the cyclic energy loss in an oscillating system. These mechanical resonators are essential components in communication circuits because they generally exhibit orders of magnitude higher quality factor than electrical components. Such devices can be designed to vibrate over a very wide frequency range (higher than 1GHz), making them ideal for ultra stable oscillator and low filter functions for a wide range of transceiver types. Energy losses in high-frequency resonators are critical to designers (Yi, 2008).

Modifications in the environmental conditions have influence on the dynamic response of resonators by modifying the structural damping and the squeeze film coefficient [Pustan, 2011, Tadayon, 2006]. Indeed for the same loading conditions, the amplitude and velocity of oscillations increase if the temperature decreases (Pustan, 2011). In the same way for high temperature operating conditions, the amplitude and velocity of oscillations decrease due to the heat energy dissipation called thermal damping, and the material heat softening called temperature relaxation (Lobontiu, 2007).

The geometrical configurations and dimensions have a big influence on the sensitivity in response of microresonators. Sensitivity depends on the resonator stiffness that is strongly influenced by the beam length. Resonant frequency of the resonators changes as a function of geometrical dimensions. The effect of geometrical dimensions on the dynamical response of electrostatically actuated MEMS cantilevers is analyzed and discussed in this paper. Electrostatic actuation is the most common type of electromechanical energy conversion scheme in MEMS for a wide range of applications (Lobontiu, 2007; Pustan, 2011).

THEORETICAL FORMULATION

Stiffness and resonant frequency

MEMS resonators considered here are polysilicon microcantilevers fabricated in different geometrical dimensions. The samples are loading by an electrostatic force set up when a voltage is applied between the vibrating cantilever and the lower electrode (Fig.1).

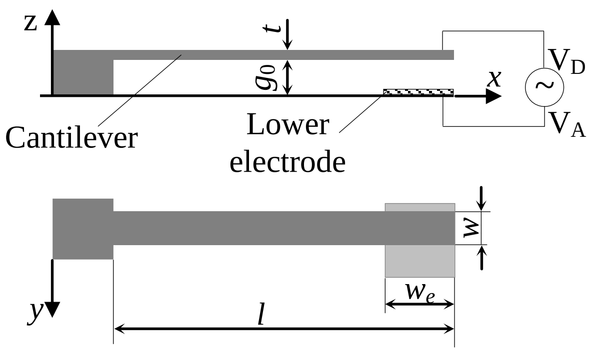


Fig. 1. Schematic representation of an electrostatically actuated MEMS cantilever.

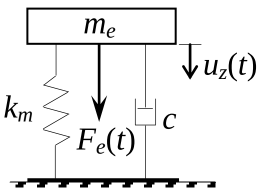


Fig. 2. A single degree of freedom model for electrostatically actuated MEMS cantilever.

When a DC voltage (V_{DC}) is applied between electrode and cantilever, the cantilever bends downwards and come to rest in a new position. To drive the resonator at resonance, an AC harmonic load of amplitude V_{AC} vibrates the cantilever at the new deflected position.

A single degree of freedom model is used to analyze the dynamic response of microresonator due to the V_{DC} and V_{AC} electric loadings as shown in Fig. 2.

In this model the proof mass of microcantilever is modeled as a lumped mass m_e, and its stiffness is considered as a spring constant k_m. This part forms one side of a variable capacitor - the movable part. The bottom electrode is fixed and considered as the second part of the sensor. If an voltage composed of DC and AC terms as

$$V(t) = V_{DC} + V_{AC}\cos(\omega t) \quad (1)$$

is applied between electrodes, the electrostatic force applied on the structure has a DC component as well as a harmonic component with the driving frequency ω such as:

$$F_e(t) = \frac{\varepsilon A V(t)^2}{2[g_0 - u_z(t)]^2} \quad (2)$$

where ε is the permittivity of the free space, $A=w_e \times w$ is the effective area of the capacitor, g_0 is the initial gap between flexible plate and substrate, and $u_z(t)$ is the displacement of the mobile plate.

When only a DC voltage is applied across the plates (V_{AC} = 0), the static force balance equation, including the electrostatic force and the spring force is:

$$k_m u_z = \frac{\varepsilon A V_{DC}^2}{2(g_0 - u_z)^2} \tag{3}$$

where u_z is the static displacement of the beam under a DC signal and k_m is the mechanical stiffness given by:

$$k_m = \frac{3EI_y}{l^3} \tag{4}$$

where E is the Young's modulus of cantilever material, I_y is the axial moment of inertia and l is the length of beam.

The equivalent stiffness k_e and resonant frequency ω_0 of microresonator under a DC actuation is obtained by linearizing the electric system around an equilibrium position $\tilde{u}_z$ as:

$$k_e = \frac{3EI_y}{l^3} - \frac{\varepsilon A V_{DC}^2}{(g - \tilde{u}_z)^3} \tag{5}$$

$$\omega_0 = \frac{1}{2\pi}\sqrt{\frac{k_m - \frac{\varepsilon A V_{DC}^2}{(g - \tilde{u}_z)^3}}{m_e}} \tag{6}$$

where m_e, the equivalent mass of system can be calculated using m_e=33m/140, with m - the effective mass of beam.

The free response of a mechanical resonator determines the resonant frequency in either the presence or the absence of damping. The forced response reveals the behavior of an undamped or damped mechanical system under the action of a harmonic excitation. In mechanical resonators, the phenomenon of resonance is important, and in such situation the excitation frequency matches the resonant frequency of the system.

Dynamical response and quality factor

The dynamic response of the microcantilever resonator shown in Fig. 1 subjected to a harmonic electrostatic force $F_e(t)$ with a driving frequency ω given by an AC voltage is governed by the equation of motion:

$$m \cdot \ddot{u}_z(t) + c \cdot \dot{u}_z(t) + k_m \cdot u_z(t) = F_e(t) \tag{7}$$

where $u_z(t)$ is the amplitude of beam oscillations and c is the damping factor.

The response of system under DC and AC voltages is given by equation (Lobontiu, 2007):

$$u_z(t) = \frac{u_z}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_0}\right)^2}} \tag{8}$$

where ξ is the damping ratio and ω_0 is the resonant frequency of microcantilever given by equation (6).

The damping ratio ξ is any positive real number. For value of the damping ratio $0 \le \xi < 1$, the system has an oscillatory response. The system damping controls the amplitude and velocity of the response when is excited at resonance.

Usually, the response is plotted as a normalized quantity $u_z(t)/u_z$. When the driving frequency equals the resonant frequency $\omega=\omega_0$, the amplitude ratio reaches a maximum value. At resonance, the amplitude ratio becomes:

$$\frac{u_z(t)}{u_z} = \frac{1}{2\xi} \tag{9}$$

An important qualifier of mechanical microresonators is the quality factor Q. At resonance, the quality factor is expressed as (Lobontiu, 2007):

$$Q_r = \frac{1}{2\xi} \tag{10}$$

and the normalized response given by equation (9) is exactly equal with Q_r.

The quality factor is also called sharpness at resonance, which is defined as the ratio

$$Q_r = \frac{\omega}{\Delta\omega} = \frac{\omega}{\omega_2 - \omega_1} \tag{11}$$

where $\Delta\omega = \omega_2 - \omega_1$ is the frequency bandwidth corresponding to $u_z(t)_{\max}/\sqrt{2}$ on the amplitude (velocity) versus frequency curves.

The loss coefficient of energy can be estimated using the quality factor. The total loss coefficient of a microresonator is influenced by two components as:

$$Q_{total}^{-1} = Q_e^{-1} + Q_i^{-1} \tag{12}$$

where the subscripts e denotes the extrinsic losses given by air damping and i is the intrinsic losses corresponding to thermoelastic damping.

The sample response in vacuum is used to determine the intrinsic losses. The total loss coefficient is experimentally determined when the sample is oscillated in ambient conditions.

EXPERIMENTAL INVESTIGATIONS

The scopes of experimental investigations are to measure the resonant frequency responses of investigated MEMS cantilevers, to estimate the quality factor and the energy dissipated during oscillations. The tests are performed under ambient conditions and in vacuum in order to estimate the damping effect on the velocity and amplitude of oscillations. The samples for experiments are polysilicon microcantilevers fabricated in different lengths (150μm, 175μm and 200μm), width of 30μm and 20μm, and a thickness of 1.9μm. The gap between flexible part and substrate is 2μm. Two different geometrical types of cantilevers, with holes and without holes, are chosen (Fig.3 and Fig.4). The holes effect on the dynamical response of microcantilevers tested in ambient conditions changes the amplitude and velocity of oscillations based on air damping decreases. The holes diameter is 3μm.

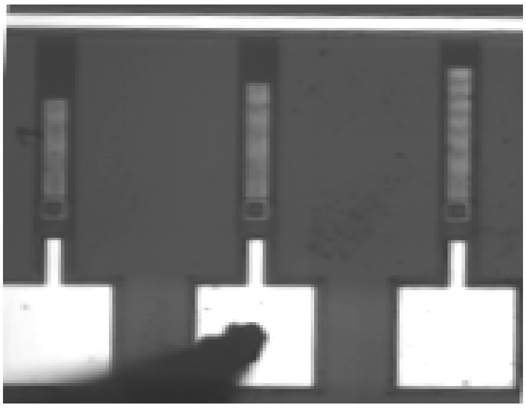


Fig. 3. MEMS cantilever with different lengths (without holes).

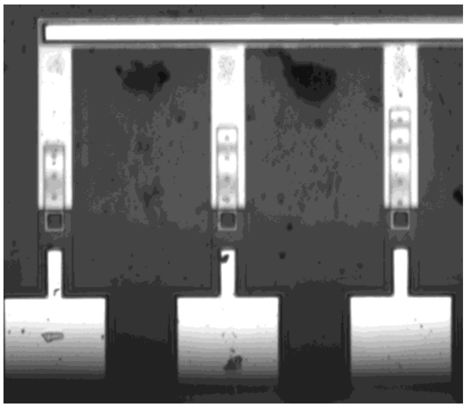
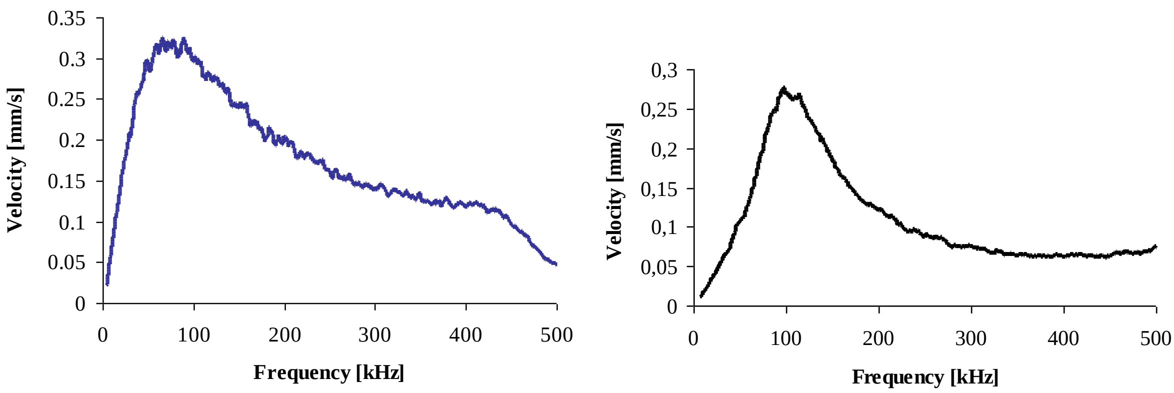


Fig. 4. MEMS cantilever with holes and different lengths.

The frequency response measurements were performed in the "Laboratoire de Techniques Aéronautiques et Spatiales (LTAS)" from University of Liege, Belgium, using a vibrometer analyzer and a white noise exciting signal of 5V offset current and 5V peak-to-peak amplitude of the driving current.

Firstly, the resonant frequency responses of investigated microcantilevers tested in air are measured. The resonant frequency decreases if the microcantilever length increases, respectively. If the experimental resonant frequency of the microcantilever with a length of 150μm is determined of 95 kHz (Fig.5), it decreases to 80kHz for the microcantilever with a length of 175μm (Fig.6) and to 60 kHz for the microcantilever with a length of 200μm (Fig.7). The dynamic response of the cantilever with a length of 150μm tested in vacuum ($5.2x10^{-4}$ mbar) is shown in Fig.8. Its resonant frequency is 98 kHz, a small shift comparative with ambient conditions tests, due to the reduced damping, could be noticed. The experimental results of resonant frequency are in good agreement with theoretical results (Fig.9) obtained using equation (6).



(a) (b)

Fig. 5. Frequency response of microcantilever with length of 150μm:
(a) width of 30μm; (b) width of 20μm.

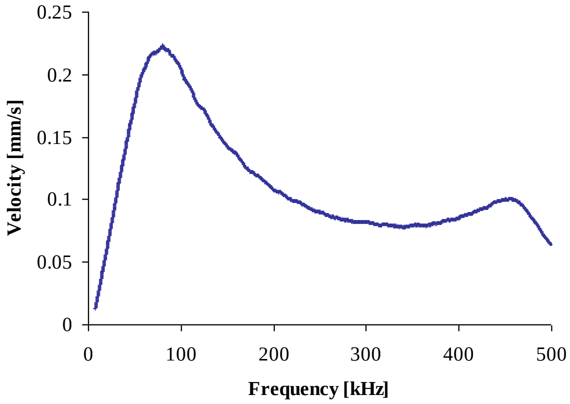


Fig. 6. Frequency response of microcantilever with length of 175μm and width of 30μm.

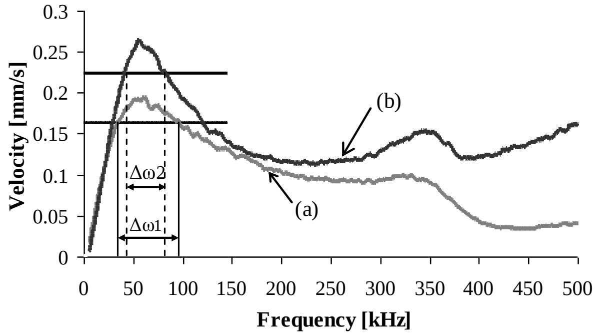


Fig. 7. Frequency responses of microcantilevers with length of 200μm and width of 30μm: (a) without holes; (b) with holes.

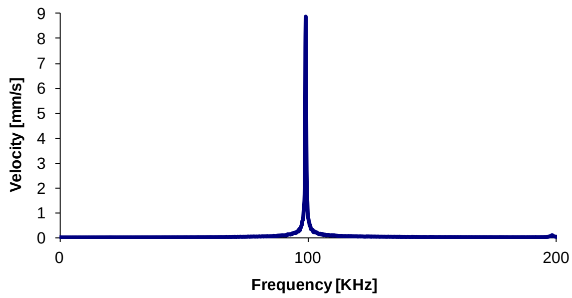


Fig. 8. Frequency response of microcantilever with length of 150μm, width of 30μm tested in vacuum

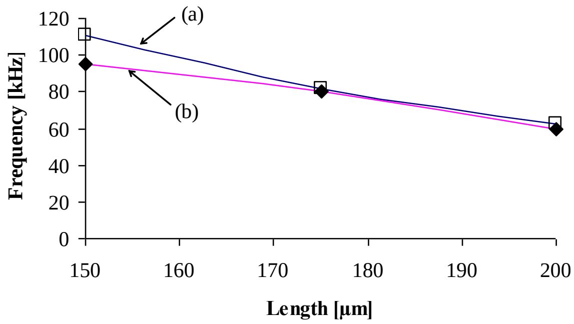


Fig. 9. Frequency response of microcantilevers (without holes as function of the lengths: (a) theoretical dependence; (b) experimental values.

Table 1. Velocity, quality factor, and loss coefficient of energy of investigated microcantilevers.

Length [μm]	**Velocity [mm/s]**	**Testing conditions**	**Quality factor Q**	**Total energy loss coefficient**
Microcantilever without holes (w=20μm, t=1.9μm)				
150	0.28	Air pressure	1.45	0.68
Microcantilevers without holes (w=30μm, t=1.9μm)				
150	0.33	Air pressure	0.91	1.09
150	9	5.2×10^{-4} mbar	283.85	0.0035
175	0.22	Air pressure	0.76	1.30
200	0.18	Air pressure	0.59	1.68
Microcantilever with holes (w=30μm, t=1.9μm)				
200	0.27	Air pressure	0.73	1.36

Secondly, using the band width $\Delta\omega$ given by the frequency response experimental curves, the quality factor of vibrating structures and the loss coefficient of energy is estimated based on equation (11) for microcantilevers with different lengths and widths. Table 1 shows the effect of the beam length and width on the dynamical response. If the length of microcantilevers increases, the quality factor decreases. On the other hand, if the width of microcantilevers decreases, the quality factor increases, respectively. The quality factor of microcantilever with a length of 150µm increases from 0.91 to 1.45 if the width of beam decreases from 30µm to 20µm.

Moreover, the effect of holes on the dynamical responses of beam is monitored. The holes increases the amplitude and velocity of oscillations because the damping given by air decreases. Fig. 7 shows the effect of holes on the dynamical response of the microcantilever with a length of 200µm and a width of 30µm. The band width of microcantilever without holes $\Delta\omega_1$ is different from the band width $\Delta\omega_2$ of microcantilever with holes (Fig.7), that increases the quality factor and decreases the loss energy coefficient, as it can be observed in the Table 1.

NUMERICAL SIMULATION

In order, to estimate the holes effect on the frequency response of investigated microcantilevers, modal analysis is performed.

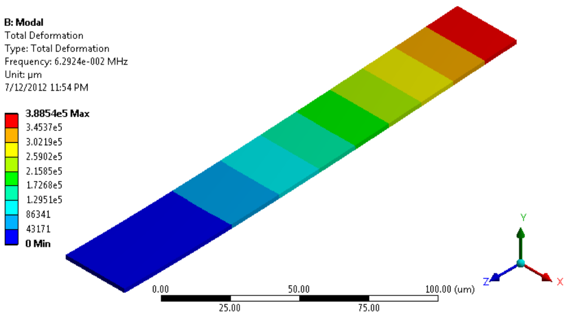


Fig. 10. Modal analysis of microcantilever with a length of 200µm (without holes)

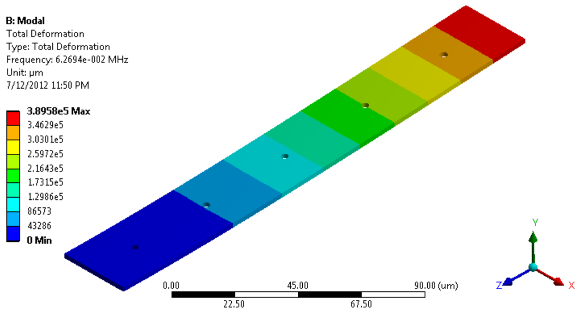


Fig. 11. Modal analysis of microcantilever with a length of 200µm (with holes)

Modal analysis of investigated microcantilever with a length of 200µm, width of 30µm and a thickness of 1,9µm without holes is shown in Fig. 10. The numerical value of resonant frequency is 62,92kHz close to the resonant frequency of the same microcantilever but with holes (Fig. 11). As a consequence, the effect of holes on the resonant frequency of beam is relatively small. A big influence of holes was experimentally observed on velocity and quality factor of oscillations.

CONCLUSIONS

The scale effect on the dynamical behavior of polysilicon microcantilevers is investigated and presented in this paper. Increasing the microcantilever length it can be noticed that the resonant frequency decreases and the loss of energy increases respectively. The loss of energy is estimated based on quality factor measured using the frequency response experimental curves. During experiments the total loss coefficient is monitored. Decreasing the beam width, the quality factor can be improved. In order to decrease the damping effect and to improve the dynamical response of beam operating in air, microresonators with small length and width are recommended to be used. The experimental results are in good agreement with the theoretical values. The difference between them is influenced by the accuracy of experimental tests.

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