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DYNAMIC CONTACT IN CONTINUA USING VARIATIONAL INEQUALITIES

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ABSTRACT

In this keynote lecture, attention is devoted to a class of problems where dynamic contact plays a major role in dictating the mechanical integrity of the component/system. Three aspects of the work are examined to overcome existing contact challenges: (i) the development of the appropriate dynamic variational inequalities expressions to accurately and consistently represent dynamic contact problems, (ii) the development of robust solution algorithms that guarantee the accurate imposition of the kinematic contact constraint and avoid interpenetration of the mating bodies, and (iii) evaluate the integrity of realistic design problems using the newly developed variational inequalities algorithms.

Keywords: dynamic, contact, nonlinear, variational inequalities.

INTRODUCTION

Dynamic contact plays an important role in dictating the integrity, performance and safety of many engineering systems/components. Despite their importance to the mechanical integrity of the systems examined, dynamic contact effects are frequently treated using oversimplifying assumptions, which neglect the main features of the problem. The reason is that modeling dynamic contact in solids poses mathematical and computational difficulties. With the application of loads to the bodies in contact, the actual surface on which these bodies meet, change with time, and the stresses at the surfaces *are* generally unknown and complex to determine. Analytical closed form solutions for contact problems were developed by Hertz in 1882. Hertz classical theory of contact was developed for *elastic quasi-static frictionless* bodies with the contact region being small compared with the dimensions of the contacting bodies. In spite of the fact that Hertz's theory of contact bas stood the test of time and has been a landmark in applied mechanics for many decades, it suffers from the above- mentioned severe restrictions.

RESULTS AND CONCLUSIONS

The finite element formulation of dynamic frictionless contact problems can be expressed as a constrained minimisation problem:

$$\boldsymbol{\pi} = \mathbf{\ddot{U}^T M U} + \tfrac{1}{2}\mathbf{U^T K U} - \mathbf{F^T U} \quad \textbf{subject to} \quad \mathbf{AU} \leq \mathbf{G} \tag{1}$$

where **A** represents the assembly of the kinematic contact conditions of the nodes on the candidate contact surface $\boldsymbol{\Gamma_c}$ *(see Fig. 1)* with the initial relative gap vector **G**. The matrices **K** and **M** are the assembled element stiffness and mass matrices, respectively. **U** and **Ü** represent the required displacement and acceleration vectors, respectively. The solution techniques adopted to solve these formulations were based upon the use of either the penalty function method or Lagrange multipliers in identifying the contact surface and imposing the contact constraints. In the Lagrange multipliers method, the kinematic contact constraints are imposed by introducing additional independent variables **Λ** representing the contact stresses into the following functional:

$$\boldsymbol{\pi}_L = \ddot{\mathbf{U}}^T\mathbf{MU} + \tfrac{1}{2}\,\mathbf{U}^T\mathbf{KU} - \mathbf{F}^T\mathbf{U} + \boldsymbol{\Lambda}^T\,(\mathbf{AU} - \mathbf{G}) \qquad (2)$$

The contact stiffness matrices and the corresponding contact forces are functions of the displacement vector **U** that must be solved iteratively. As a result, the contact conditions are implicitly imposed on the displacements of the nodes. The penalty method, on the other hand, enforces the contact compatibility conditions only approximately by means of a user-defined penalty parameter **α**. The resulting minimisation functional is given by:

$$\boldsymbol{\pi}_p = \ddot{\mathbf{U}}^T\mathbf{MU} + \tfrac{1}{2}\,\mathbf{U}^T\mathbf{KU} - \mathbf{F}^T\mathbf{U} + \tfrac{1}{2}\,(\mathbf{AU} - \mathbf{G})^T\,\boldsymbol{\alpha}\,(\mathbf{AU} - \mathbf{G}) \qquad (3)$$

In spite of its simplicity, the penalty method requires user-defined parameters for the contact stiffness and that contact stiffness should be large to avoid interpenetration. However, the use of excessively high values could lead to ill- conditioned global stiffness matrices. These high stiffness values also require the use of smaller time increments, which may introduce undesirable numerical oscillations. On the other hand, the use of smaller contact stiffness leads to interpenetration and incorrect estimates of the stick/slip regions. To overcome these challenges, several formulations combined Lagrange multipliers and penalty functions e.g., perturbed Lagrangian approach [2], and augmented Lagrangian approach [3]). Although these formulations are better than the above two approaches, they are still dependent upon user-defined parameters. In addition, mathematical inconsistencies associated with the use of Coulomb friction remain unresolved. In Figure 1, we provide a new expression for dynamic contact in terms of variational inequalities to be discussed in this keynote lecture.

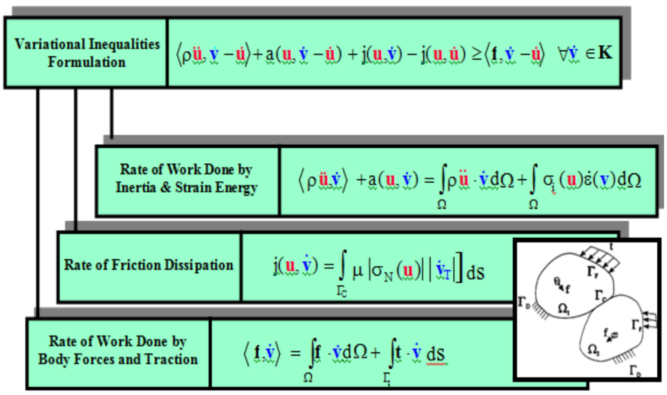

Fig. 1 - Variational inequalities formulation in dynamic contact

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