

Time-frequency analysis of railway bridges forced and free vibrations by wavelet transform

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Abstract. In the scope of the InBridge4EU project <https://inbridge4eu.eu/> a database has been developed containing information on more than 500 bridges, including the experimental response to approximately 2000 train passages. The database is accessible from Python and Matlab using API Token Authentication which allows the analysis of the structural behaviour of the bridges in a user-friendly way. In this work, the time-frequency analysis carried out on some of the bridges included in the database is presented. Bridges of different typologies have been analysed. The analysis has been carried out using the continuous wavelet transform with the mother wavelet Morse considering that this function presents a better correlation with the original signals. The results obtained have allowed to detect the response of the bridge in free vibration, after different train passages, and forced vibration. This aspect is relevant in the identification of the modal parameters of the structure, and in particular in that of structural damping, which is estimated from the free vibration data. Also, the analysis provides the predominant frequencies in the structural response before, during and after the passage of the train over the bridge. These frequencies are compared with the natural frequencies of the structure obtained from the response to ambient vibration and with the excitation frequencies depending on train characteristics and speed (bogie passing frequency, axle passing frequency and their harmonics). Conclusions about the influence of the passing vehicle on the bridges modal parameters are finally extracted.

Keywords: Wavelet · Modal parameters · Railway bridge.

1 Introduction

InBridge4EU project started on September 1st 2023. The project is funded by Europe's Rail Joint Undertaking under Horizon Europe research and innovation program. The overall objective of InBridge4EU is to develop a dynamic interface between railway bridges and rolling stock, proposing new methods compatible with existing regulations, namely INF TSI [1], LOC&PAS TSI [2], EN 15528 [3], EN 1990-Annex A2 [4] and EN 1991-2 [5], and approaching the analysis of existing infrastructures, which role is critical for the sustainability of the European rail system. The project is articulated in seven work packages including the following objective: identification of critical bridge parameters for the assessment of the economic impact of the new Dynamic Train Categories (DTCs).

During the first year of the project, an extensive and representative set of European railway bridges were selected, and relevant data was retrieved and stored in a database: <https://computeruse.us.es/>. Starting from the beginning of the second year, time-step calculation (TSC) transient dynamic analyses are being performed over the complete database under real train models/MU classes and under the new DTCs. From the analysis of the database, realistic worst-case combinations of critical parameters for use in parametric studies will be identified. From the detection and evaluation of the bridges that do not meet the economic technical acceptance criteria, an estimation of the resources required for implementing each DTC on a particular line will be presented. Relevant output will also be provided in relation to the appropriateness of different model updates and of the use of the classical beam line model for the analysis of certain bridge configurations.

So far, 10 railway lines from 5 EU countries have been selected covering different maximum design speeds. Relevant information from about 50 bridges per line has been retrieved with the intensive participation of the corresponding Infrastructure Managers, including technical drawings and experimental data. In parallel, a cloud database has been designed and made accessible to all the partners in the consortium. The database will accommodate the bridges data and the results of posterior analyses.

Here, the time-frequency analysis of some of the bridges included in the database is shown. The analysis has been carried out using the continuous wavelet transform. The analysis provides the predominant frequencies in the structural response before, during and after the passage of the train over the bridge. These frequencies are compared with the natural frequencies of the structure obtained from the response to ambient vibration and with the excitation frequencies depending on train characteristics and speed.

2 Continuous Wavelet transform

This section presents some basic ideas and definitions about wavelet transform [6]. A linear transformation of function $f(t)$ can be defined as [7]:

$$T = \int_{-\infty}^{\infty} \phi(t) \cdot f(t) \cdot dt \quad (1)$$

The resulting function T shows how similar is the original function to the integration function $\phi(t)$. When using the wavelet transform, function ϕ is a wavelet family defined through translation and dilatation of a function $\Psi(t)$:

$$\phi_{u,s}(t) = \frac{1}{\sqrt{s}} \Psi \left(\frac{t-u}{s} \right) \quad (2)$$

Parameter u is known as the translation parameter. Parameter s is defined as the scale of the wavelet transform. It defines the shrinking or stretching of the wavelet function. The resulting wavelet transformed function T will depend on scale s and translation parameter u .

The inverse of s can be interpreted as a pseudo-frequency, since it modulates the frequency content of the wavelet function. A value of the corresponding pseudo-frequency for each scale can be obtained from the following expression [8]:

$$F_a = \frac{F_c \cdot f_s}{a} \quad (3)$$

where F_a is the pseudo-frequency for scale a , f_s is the sampling frequency, and F_c is the center frequency of the wavelet. The center frequency is a convenient and simple characterization of the dominant frequency of the wavelet.

The wavelet transform gives time information of the frequency content of the signal. On the other hand, the wavelet analysis can provide higher time resolution for higher frequencies and lower time resolution for lower frequencies, since the convolution parameter (u/s) can be adapted according to the frequency.

The Continuous Wavelet Transform (CWT) of a function $f(t)$ can be defined as:

$$CWT_f(u, s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} f(t) \Psi^* \left(\frac{t-u}{s} \right) dt \quad (4)$$

where Ψ^* indicates the complex conjugate of the wavelet function. $CWT_f(u, s)$ indicates the content of the scaled wavelet shape in the original function $f(t)$ at a specific time. This can be understood in the sense of time evolution of frequency content.

In this paper, the analytic Morse wavelet [9] with the symmetry parameter $\gamma = 3$, and the time-bandwidth product equal to 120 has been used. This wavelet allows to analyse signals with wider variance in time. The wavelet coefficients are normalized to the maximum in each scale.

3 Results

Jabalón HSL Bridge (Figure 1), is a railway bridge composed by three identical S-S bays of 24.9 m equal spans. The structure crosses Jabalón River with a 134° skew angle. Each deck consists of a cast-in-situ concrete slab with dimensions $11.6 \text{ m} \times 0.3 \text{ m}$ (wide \times thickness). The slab rests over five prestressed concrete I girders with a height of 2.05 m separated 2.625 m. The girders rest on the supports through laminated rubber bearings. The slab carries two ballasted tracks with UIC gauge (1435 mm), UIC60 rails and mono-block concrete sleepers every 0.60 m. The deck has a linear mass of approximately 29000 kg/m. The substructure consists of two outer reinforced concrete abutments and two inner wall piers. Figure 2 shows the measurement points where accelerations were measured, all of them located at span 1.



Fig. 1. HSL bridge over Jabalón River ($38^\circ 53' 51.3'' \text{N}$ $3^\circ 57' 53.0'' \text{W}$).

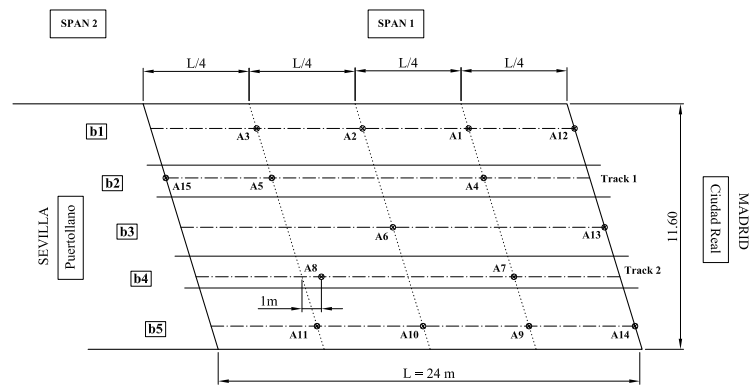
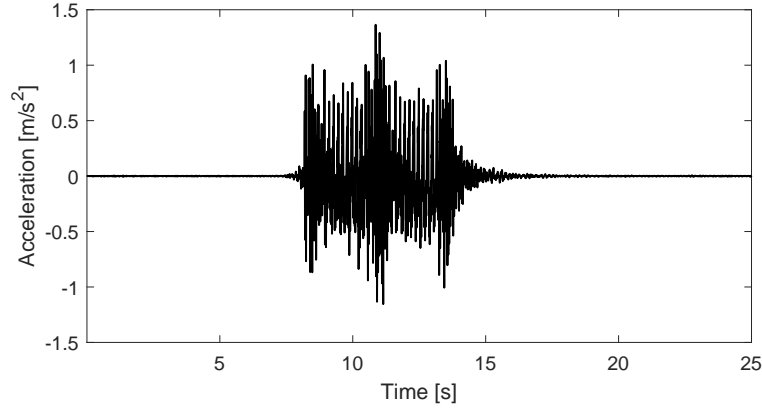


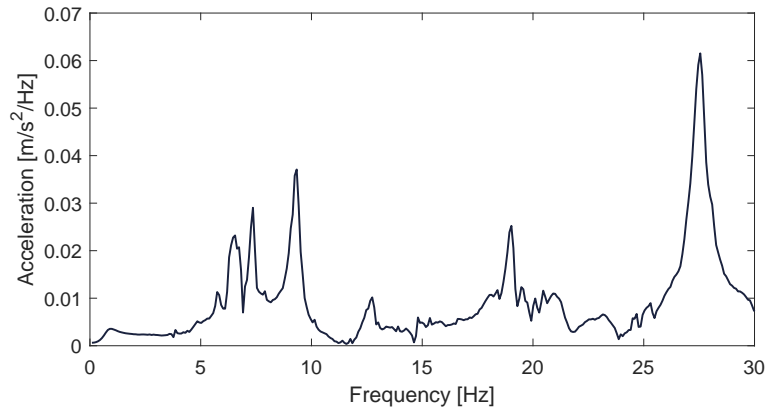
Fig. 2. Sensors layout at Jabalón HSL Bridge.

A comprehensive experimental study of this bridge was published in Reference [10] including the modal parameter identification of the structure.

Figure 3 shows the response of the bridge under the passage of S102 duplex train travelling along track 2 northbound for a non-resonant speed at a sensor located at a quarter of the span (accelerometer 9). The frequency content is computed from the free vibration response after the train passage. The bridge natural frequencies at 6.3, 7.2 and 9.3 Hz are visible in the response spectrum. Also it is lightly visible the excitation frequency associated to the axle distance (13.14 m) equal to 5.6 Hz.



(a)



(a)

Fig. 3. Jabalón HSL Bridge: (a) time history and (b) frequency content of the acceleration at point 9 induced by Renfe S102 duplex train circulating on track 2 at $v = 263$ km/h. The frequency content is computed from the free vibration response.

Figure 4 shows the wavelet transform of the bridge response. For comparison purposes, the frequency content of the forced and free vibration responses are shown. The forced vibration response is automatically detected by the maxima of the wavelet coefficients (dotted vertical lines). The behaviour is clearly different in both cases. The peak at 5.6 Hz strongly stands out in the forced vibration response. The peaks associated to the frequencies that contribute to the structural response move towards lower frequencies when the train is crossing the bridge is indicated by the wavelet time-frequency analysis. The change of the modal parameters can be due to the mass and damping of the vehicle when on the bridge.

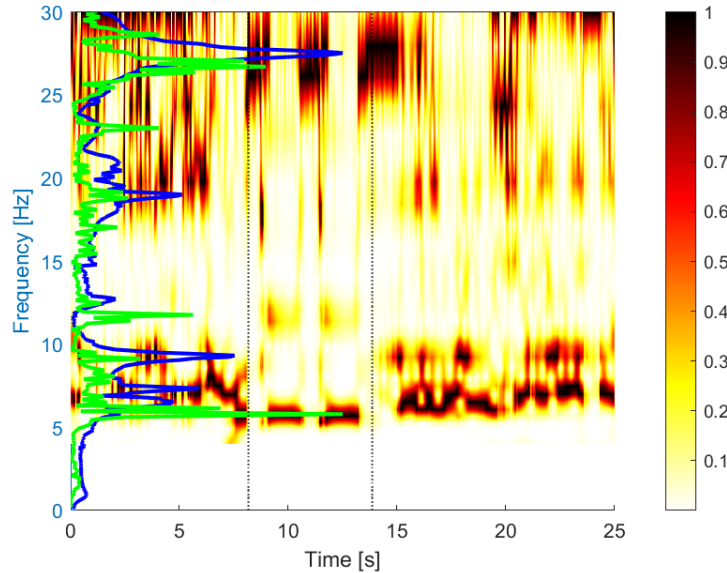


Fig. 4. Jabalón HSL Bridge: wavelet transform and frequency content of the acceleration at point 9 induced by Renfe S102 duplex train circulating on track 2 at $v = 263$ km/h. Frequency content of the (blue) free and (green) forced vibration responses are superimposed.

4 Conclusions

This preliminary work includes the time-frequency analysis carried out on one of the bridges included in the database of the InBridge4EU project. The analysis was performed using the continuous wavelet transform. The analysis allows to identify the response of the bridge in free and forced vibration. Moreover, the

methodology determines the predominant frequencies in the structural response during and after the passage of the train over the bridge concluding that these frequencies are modified by the effect of the vehicle. A more comprehensive study including other bridges, vehicles and speeds will be develop in future work.

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