A two-dimensional lattice with band gaps robust to mechanical variability

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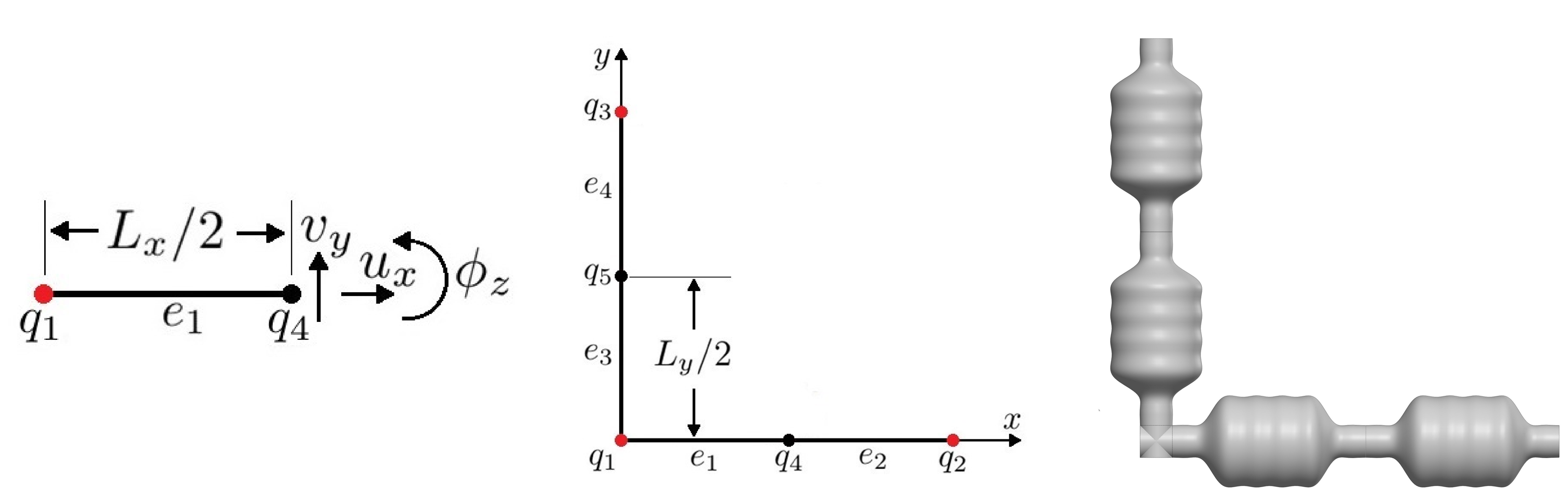
1. Introduction

Periodic structures can be applied as a solution to reduced undesired vibration levels that can cause structural damage and perceptual discomfort. Additive manufacturing has allowed the manufacture of more complex and lightweight structures, but they can present high variability, especially in the mechanical properties.

The dispersion diagram can be computed using a metastructure (supercell) or its representative unit cell, thus considerably reducing the necessary computational efforts. For two- and three-dimensional waveguides, the first Brillouin zone (FBZ) or the irreducible Brillouin zone (IBZ) contour can be considered.

2. Deterministic and the models

Using elementary rod and Timoshenko beam spectral elements, the frame element (Fig. **??**) can be assembled. The direct assembly can be used to obtain the global stiffness matrix ($\boldsymbol{D}_g(\omega)$. The dynamic condensation can be applied to eliminate some internal degrees-of-freedom (DOFs) and the condensed global stiffness matrix ($\boldsymbol{D}_c(\omega)$) can be obtained. For further details of the proposed unit cell, supercell, and metastructure see [**?**].



The Bloch-Floquet theorem is used to relate the DOFs of a unit cell or supercell of a periodic metastructure by considering their periodicity. The same theorem can be combined with the assumption of equilibrium of nodal forced in a node

$$\left\{\mathbf{q}_1\ \mathbf{q}_2\ \mathbf{q}_3\right\}^T = \left[\mathbf{I}_n\ \mathbf{I}_n\lambda_x\ \mathbf{I}_n\lambda_y\right]^T \mathbf{q}_1 \ \rightarrow\ \boldsymbol{q} = \boldsymbol{\Lambda}_R \boldsymbol{q}_1, \tag{1}$$

$$\boldsymbol{F}_1 + \boldsymbol{F}_2\lambda_x^{-1} + \boldsymbol{F}_3\lambda_y^{-1} = \mathbf{0} \ \rightarrow\ \left[\mathbf{I}_n\ \mathbf{I}_n\lambda_x^{-1}\ \mathbf{I}_n\lambda_y^{-1}\right]\left\{\boldsymbol{F}_1\ \boldsymbol{F}_2\ \boldsymbol{F}_3\right\}^T = \boldsymbol{\Lambda}_L\boldsymbol{F} = \mathbf{0}, \tag{2}$$

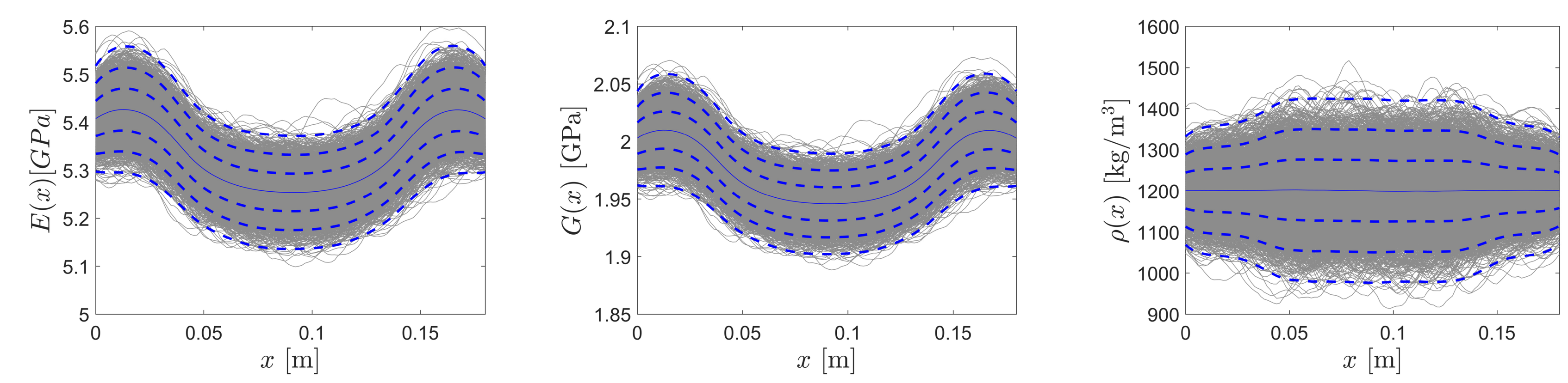
to obtain the polynomial equation with roots $\lambda_x = e^{-ik_xL_x}$ and $\lambda_y = e^{-ik_yL_y}$

$$\boldsymbol{\Lambda}_L\boldsymbol{D}_c(\omega)\boldsymbol{\Lambda}_R\mathbf{q}_1 = \mathbf{0}. \tag{3}$$

The stochastic fields of Young's modulus (E), shear modulus (G), and mass density (ρ) were defined as a function of the frame length.

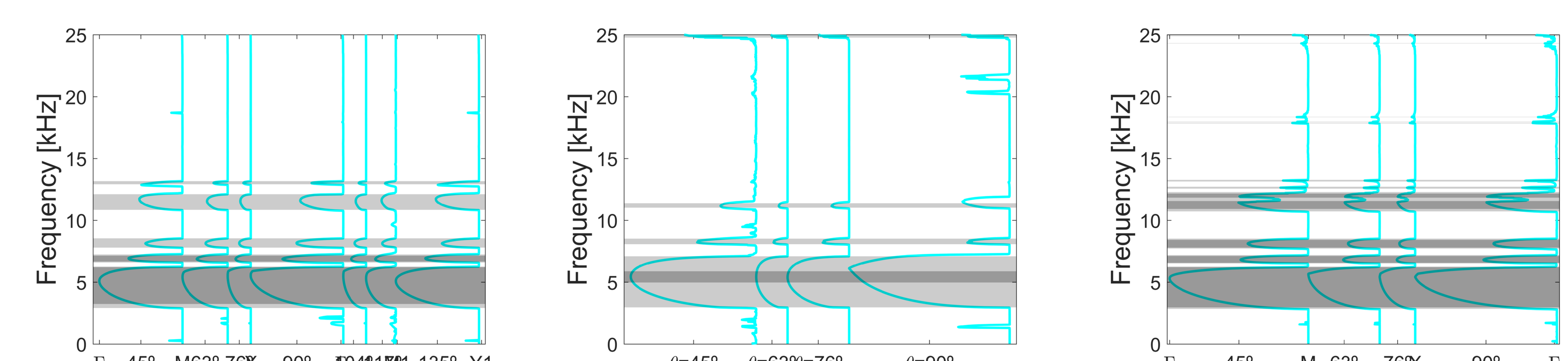
$$E(x) \sim N(\mu_E, \sigma_E), \qquad \rho(x) \sim N(\mu_\rho, \sigma_\rho), \qquad G(x) \sim \frac{E(x)}{(2(1+\nu))}, \tag{4}$$

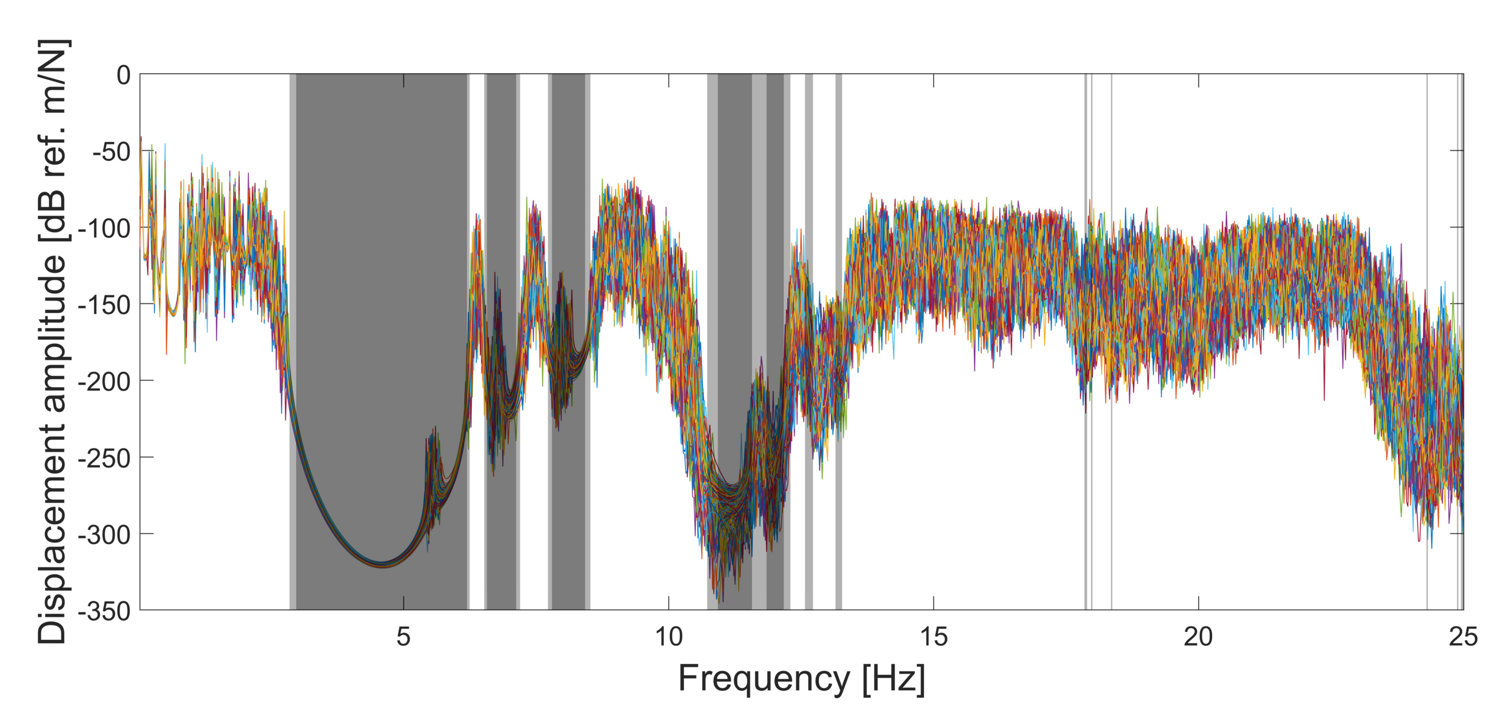
where $\mu_E = 5.5 + 0.5r_x^2 - 5r_x$, $\sigma_E = \mid (r_x - 0.05) + 0.1)/2 \mid$, $\mu_\rho = 1200 + 200r_x^2$, and $\sigma_\rho = 150 + 100(40r_x)$. The Monte Carlo method combined with the Kernel smoother was used to simulate the fields as shown in Fig. **??**, for more details, see [**?**].



3. Results

The robust attenuation bands are presented in Fig. **??** for the FBZ and IBZ contours of the unit cell and the IBZ contour of the supercell made of 9 unit cells. They mostly agree with the stochastic FRFs of a structure of 16 unit cells.





4. References

[1] Ribeiro, Luiz H. M. S., et al. "Investigating the stochastic dispersion of 2D engineered frame structures under symmetry of variability."Journal of Sound and Vibration 541 (2022): 117292.

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